CHAPTER 9

RAY OPTICS AND OPTICAL INSTRUMENTS

9.1 INTRODUCTION

1. *What is optics ? What are its two main branches ?*

Optics. *It is the branch of physics which deals with the study of nature, production and propagation of light.* The subject of optics can be divided into two main branches : *rays optics and wave optics.*

1. Ray or geometrical optics. It concerns itself with the particle nature of light and is based on

(*i*) the rectilinear propagation of light and

(*ii*) the laws of reflection and refraction of light.

It explains the formation of images in mirrors and lenses, the aberrations of optical images and the working and designing of optical instruments.

2. Wave or physical optics. It concerns itself with the wave nature of light and is based on the phenomena like

(*i*) interference

(*ii*) diffraction and

(*iii*) polarisation of light.

In the previous chapter, we have learnt that light is an electromagnetic wave in which the electric and magnetic fields vary harmonically in space and time. The visible light consists of waves with wavelengths ranging from 4000 Å to 7500 Å.

9.2 BEHAVIOUR OF LIGHT AT THE INTERFACE OF TWO MEDIA

2. *Name the different effects that may occur as light travels from one optical medium to another. State the laws of reflection of light.*

Behaviour of light at the interface of two media. When light travelling in one medium falls on the surface of a *second medium,* the following *three* effects may occur :

(*i*) A part of the incident light is turned back into the first medium. This is called ***reflection of light.***

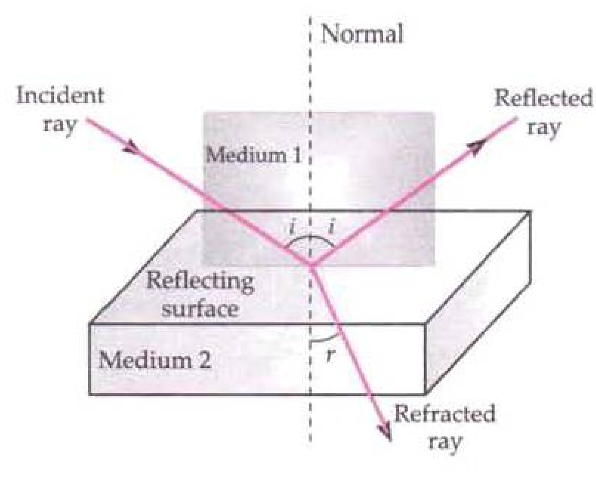


Fig. 9.1 Reflection and refraction of light.

(*ii*) A part of the incident light is transmitted into the second medium along a changed direction. This is called ***refraction of light***.

(*iii*) The remaining third part of light energy is absorbed by the second medium. This is called ***absorption of light***.

Laws of reflection of light. Reflection of light takes place according to the following *two* laws :

(*i*) The angle of incidence is equal to the angle of reflection, *i.e.*, $\angle i = \angle r$.

(*ii*) The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

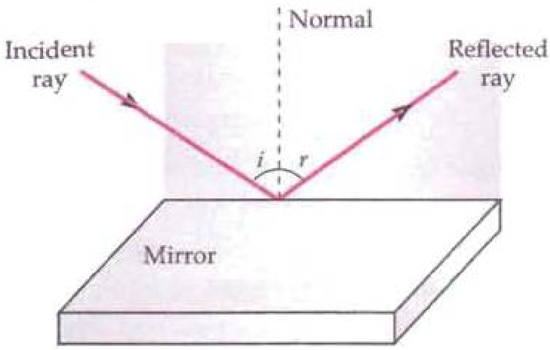


Fig. 9.2 The incident ray, reflected ray and the normal to the reflecting surface lie on the same plane.

The above laws of reflection are valid both in case of plane and curved reflecting surfaces.

9.3 SPHERICAL MIRRORS

3. *What are spherical mirrors ? What are their two types ?*

Spherical Mirrors. As shown in Fig. 9.3(*a*), consider a hollow glass sphere being cut by a plane. The section *APB*, cut by the plane, forms a part of a sphere and is known as a *spherical surface*. If either side of this spherical surface is silvered, we get a spherical mirror.

A **spherical mirror** is a *reflecting surface which forms part of a hollow sphere.*

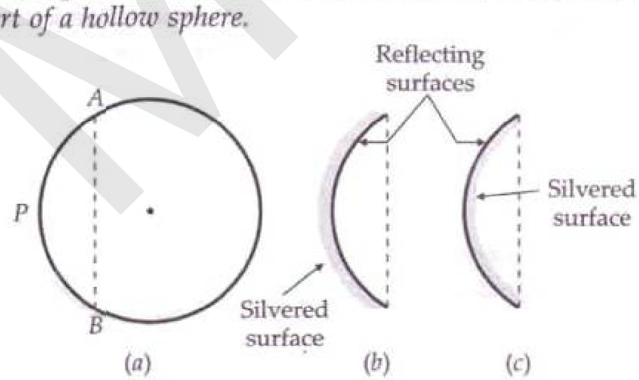


Fig. 9.3 (*a*) A hollow sphere cut by a plane (*b*) concave mirror (*c*) convex mirror.

Spherical mirrors are of *two* types :

(*i*) ***Concave mirror.*** *A spherical mirror in which the outer bulged surface is silvered polished and the reflection of light takes place from the inner hollow surface is called a concave mirror.*

(*ii*) ***Convex mirror.*** *A spherical mirror in which the inner hollow surface is silvered polished and the reflection of light takes place from the outer bulged surface is called a convex mirror.*

4. *Define pole, centre of curvature, radius of curvature, principal axis, linear aperture, angular aperture, principal focus, focal length, and focal plane of a spherical mirror.*

Definitions in Connection with Spherical Mirrors. In Fig 9.4, let *APB* be a *principal section* of a spherical

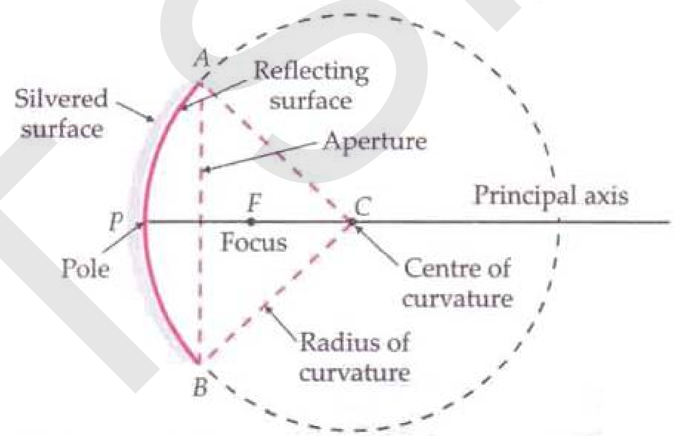


Fig. 9.4 Characteristics of a concave mirror.

mirror, *i.e.*, the section cut by a plane passing through pole and centre of curvature of the mirror.

1. **Pole.** *It is the middle point P of the spherical mirror.*
2. **Centre of curvature.** *It is the centre C of the sphere of which the mirror forms a part.*
3. **Radius of curvature.** *It is the radius* ($R = AC$ *or* BC) *of the sphere of which the mirror forms a part.*
4. **Principal axis.** *The line PC passing through the pole and the centre of curvature of the mirror is called its principal axis.*
5. **Linear aperture.** *It is the diameter AB of the circular boundary of the spherical mirror.*
6. **Angular aperture.** *It is the angle ACB subtended by the boundary of the spherical mirror at its centre of curvature C.*
7. **Principal focus.** *A narrow beam of light parallel to the principal axis either actually converges to or appears to diverge from a point F on the principal axis after reflection from the spherical mirror. This point is called the principal focus of the mirror.* A concave mirror has a real focus while a convex mirror has a virtual focus, as shown in Fig. 9.5.

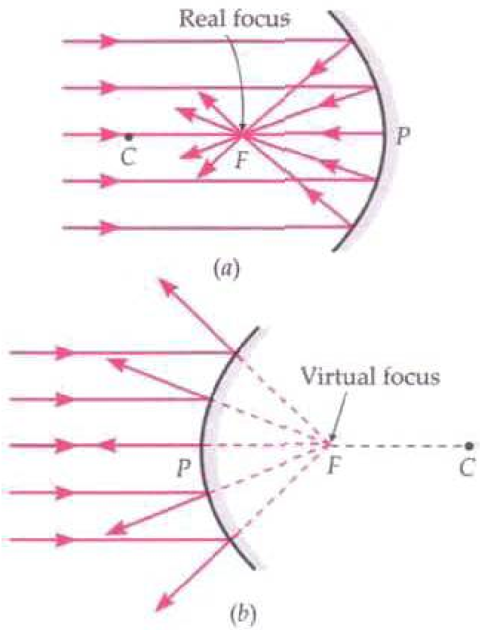


Fig. 9.5 Principal focus of (*a*) a concave mirror (*b*) a convex mirror.

8. **Focal length.** *It is the distance ($f = PF$) between the focus and the pole of the mirror.*
9. **Focal plane.** *The vertical plane passing through the principal focus and perpendicular to the principal axis is called focal plane.* When a parallel beam of light is incident on a concave mirror at a small angle to the principal axis, it is converged to a point in the focal plane of the mirror, as shown in Fig. 9.6.

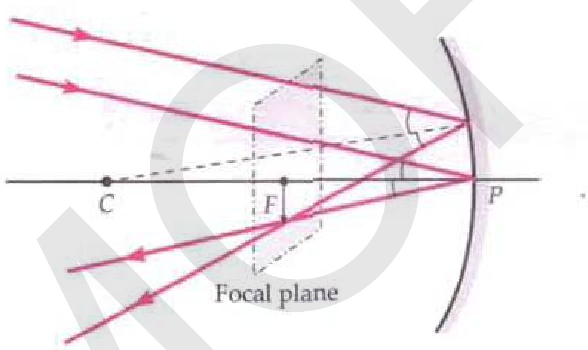


Fig. 9.6 Focal plane of concave mirror.

NOTE A line joining any point of the spherical mirror to its centre of curvature, will be *normal* to the mirror at that point.

5. *State the new cartesian sign convention used for spherical mirrors.*

New Cartesian Sign Convention for Spherical Mirrors. According to this sign convention :

1. All ray diagrams are drawn with the incident light travelling from *left* to *right*.
2. All distances are measured from the *pole* of the mirror.
3. All distances measured in the direction of incident light are taken to be *positive.*
4. All distances measured in the opposite direction of incident light are taken to be *negative.*
5. Heights measured *upwards* and perpendicular to the principal axis are taken *positive.*
6. Heights measured *downwards* and perpendicular to the principal axis are taken *negative.*

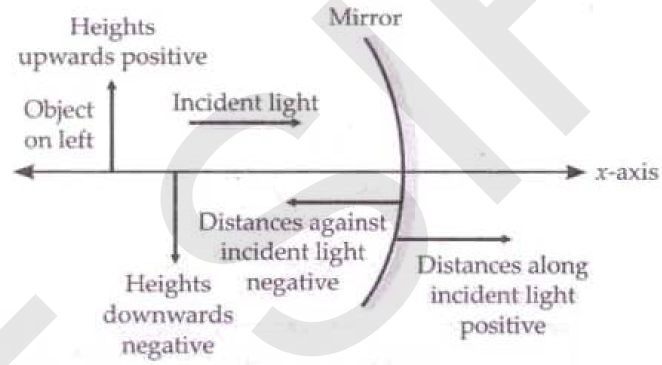


Fig. 9.6 New cartesian sign convention.

According to this sign convention, *the focal length and radius of curvature are negative for a concave mirror and positive for a convex mirror.*

6. *Derive a relationship between the focal length and radius of curvature of a spherical mirror.*

Relation between f and R. Consider a ray AB parallel to the principal axis, incident at point B of a spherical mirror (concave or convex) of small aperture. After reflection from the mirror, this ray converges to point F (in case of a concave mirror) or appears to

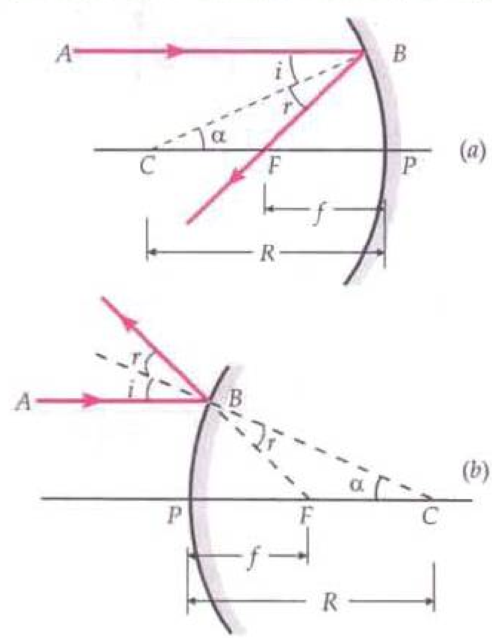


Fig. 9.8 Relation between f and R of (*a*) a concave mirror (*b*) a convex mirror.

diverge from point F (in case of a convex mirror), obeying the laws of reflection. Thus F is the focus of the mirror, C is the centre of curvature, CP = the radius of curvature and BC is a normal to mirror at point B.

According to the law of reflection,

$$\angle i = \angle r$$

As AB is parallel to PC,

$$\angle \alpha = \angle i$$

$\therefore$ In Δ BFC, $\quad \angle r = \angle \alpha$

Hence $\quad CF = FB$

For a mirror of small aperture,

$$FB \simeq FP \quad \therefore \quad CF \simeq FP$$

Hence $\quad CP = CF + FP = FP + FP = 2\,FP$

or $\quad R = 2f \quad$ or $\quad f = \dfrac{R}{2}$

or **Focal length = $\frac{1}{2}$ × Radius of curvature**

7. *State the rules used for drawing images formed by spherical mirrors. Draw ray diagrams showing the formation of images by concave and convex mirrors for different object positions on the principal axis.*

Rules for drawing images formed by spherical mirrors. The position of the image formed by spherical mirrors can be found by considering any two of the following rays of light coming from a point on the object.

(*i*) *A ray proceeding parallel to the principal axis will, after reflection, pass through the principal focus in the case of a concave mirror* [Fig. 9.9(*a*)], *and appear to come from focus in the case of a convex mirror* [Fig. 9.9(*b*)].

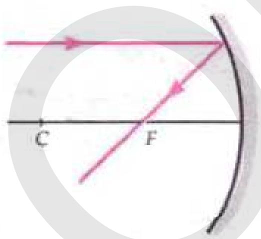


Fig. 9.9 (*a*) A ray parallel to the principal axis passes through F after reflection from a concave mirror.

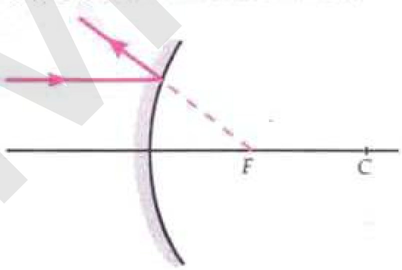


Fig. 9.9 (*b*) A ray parallel to the principal axis appears to come from F after reflection from a convex mirror.

(*ii*) *A ray passing through the principal focus in the case of a concave mirror* [Fig. 9.10(*a*)], *and directed towards the principal focus in the case of a convex mirror will* [Fig. 9.10(*b*)], *after reflection, become parallel to the principal axis.*

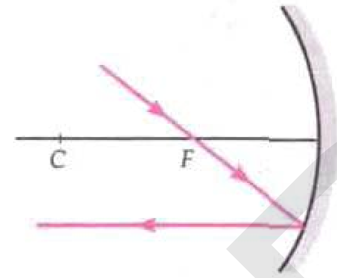


Fig. 9.10 (*a*) A ray through F becomes parallel to the principal axis after reflection from a concave mirror.

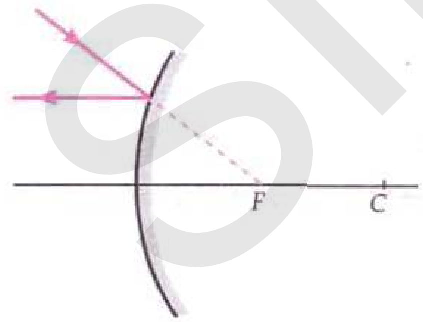


Fig. 9.10 (*b*) A ray directed through F becomes parallel to the principal axis after reflection from a convex mirror.

(*iii*) *A ray passing through the centre of curvature in the case of concave mirror* [Fig. 9.11(*a*)], *and directed towards the centre of curvature in the case of a convex mirror* [Fig. 9.11(*b*)] *falls normally* ($\angle i = \angle r = 0°$) *and is reflected back along the same path.*

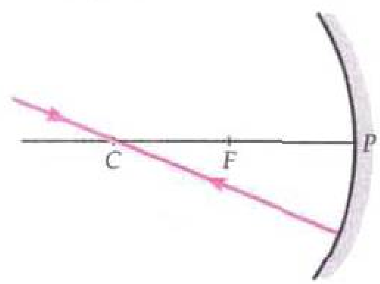


Fig. 9.11 (*a*) A ray passing through C is reflected back along of same path after reflection from

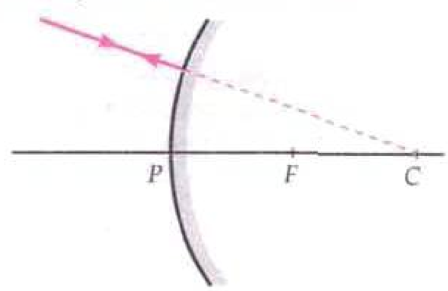


Fig. 9.11 (*b*) A ray directed towards C is reflected back along of same path after reflection from a convex mirror.

(*iv*) *For the ray incident at any angle at the pole, the reflected ray follows the laws of reflection.*

Formation of Images by Concave Mirrors :

(*a*) Object beyond *C*. The image is

1. Between *C* and *F*	2. Real
3. Inverted	4. Smaller than object.

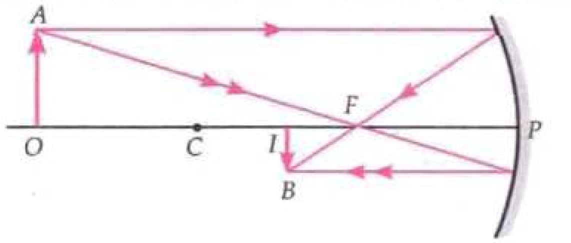


Fig. 9.12 (*a*)

(*b*) Object at *C*. The image is

1. At *C*	2. Real
3. Inverted	4. Same size as object.

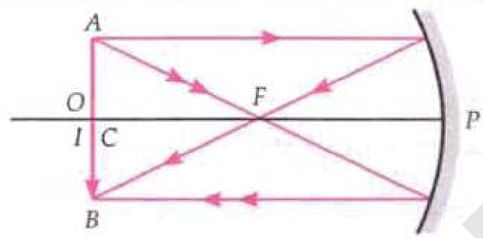


Fig. 9.12 (*b*)

(*c*) Object between *F* and *C*. The image is

1. Beyond *C*	2. Real
3. Inverted	4. Larger than object.

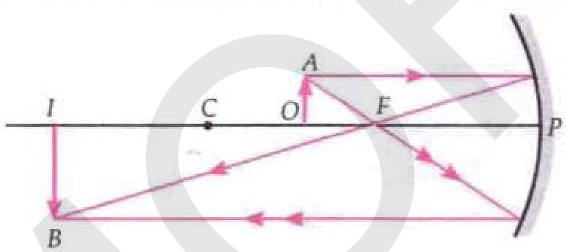


Fig. 9.12 (*c*)

(*d*) Object between *F* and *P*. The image is

1. Behind the mirror	2. Virtual
3. Erect	4. Larger than object.

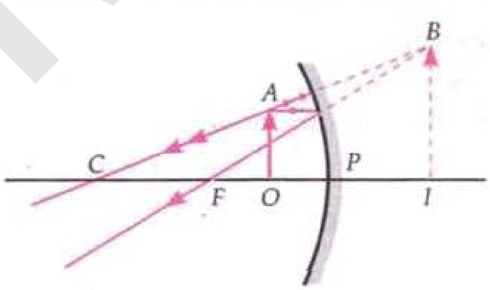


Fig. 9.12 (*d*)

Formation of image by Convex Mirror :

For any position of the object between ∞ and pole *P*, the image is

1. Behind the mirror	2. Virtual
3. Erect	4. Smaller than object.

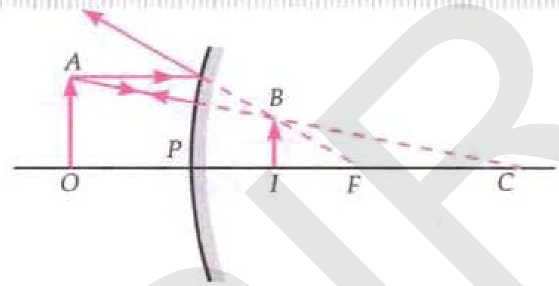


Fig. 9.12 (*e*)

9.4 THE MIRROR FORMULA

8. *State the mirror formula. Is the same formula applicable to both concave and convex mirrors ?*

Mirror formula. *The mirror formula is a mathematical relationship between object distance u, image distance v and the focal length f of a spherical mirror.* This relation is

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

In other words, we can say that

$$\frac{1}{\text{Object distance}}+\frac{1}{\text{Image distance}}=\frac{1}{\text{Focal length}}$$

This formula is applicable to all concave and convex mirrors, whether the image formed is real or virtual.

9(*a*) *By stating the sign convention and assumptions used, derive the relation between object distance u, image distance v and focal length f for a concave mirror, when it forms a real image of an object of finite size.*

Derivation of mirror formula for a concave mirror when it forms a real image. Consider an object *AB* placed on the principal axis beyond the centre of curvature *C* of a concave mirror of small aperture, as

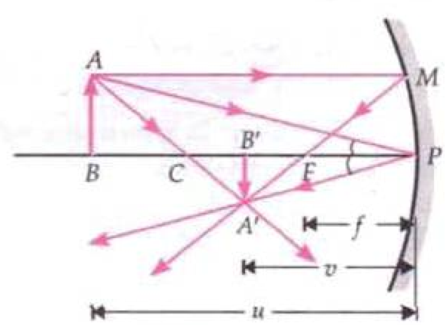


Fig. 9.13

shown in Fig. 9.13. A ray AM from the object travels parallel to the principal axis and after reflection from the mirror it passes through focus F. Another ray AP is incident on the pole P of the mirror and is reflected along PA' in accordance with the laws of reflection so that $\angle APB = \angle B'PA'$. The two reflected rays meet at point A'. Thus A' is the real image of A. The image of any point on AB will lie on a corresponding point of $A'B'$. Hence $A'B'$ is the real image of AB formed by reflection from the concave mirror.

Using cartesian sign convention, we find

Object distance,	$BP = -u$
Image distance,	$B'P = -v$
Focal length,	$FP = -f$
Radius of curvature,	$CP = -R = -2f$

Now $\Delta A'B'C \sim \Delta ABC$

$$\therefore \quad \frac{A'B'}{AB} = \frac{CB'}{BC} = \frac{CP - B'P}{BP - CP} = \frac{-R + v}{-u + R} \qquad ...(1)$$

As $\angle A'PB' = \angle APB$, therefore,

$\Delta A'B'P \sim \Delta ABP$.

Consequently,

$$\frac{A'B'}{AB} = \frac{B'P}{BP} = \frac{-v}{-u} = \frac{v}{u} \qquad ...(2)$$

From equations (1) and (2), we get

$$\frac{-R + v}{-u + R} = \frac{v}{u}$$

or $$-uR + uv = -uv + vR$$

or $$vR + uR = 2\,uv$$

Dividing both sides by uvR, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

But $$R = 2\,f$$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This proves the mirror formula for a concave mirror, when it forms a real image.

9(b) *Derive the relation between object distance u, image distance v and focal length f of a concave mirror when it forms a virtual image.*

Derivation of Mirror formula for a concave mirror when the image formed is virtual. Consider an object AB placed on the principal axis of a concave mirror (of small aperture) between its pole P and focus F. As shown in Fig. 9.14, a virtual and erect image $A'B'$ is formed behind the mirror, after reflection from the concave mirror.

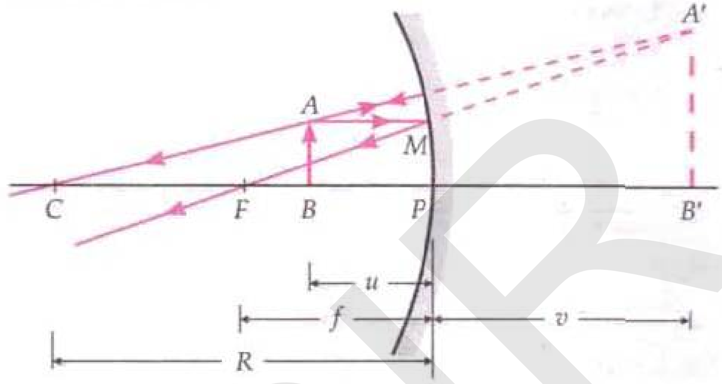


Fig. 9.14 Image formed by a concave mirror when the object lies between F and P.

Using the cartesian sign convention, we find that

Object distance,	$BP = -u$
Image distance,	$PB' = v$
Focal length,	$FP = -f$
Radius of curvature,	$CP = -R = -2f$

Now $\Delta ABC \sim \Delta A'B'C$, therefore

$$\frac{AB}{A'B} = \frac{CB}{CB'} = \frac{CP - BP}{CP + PB'} = \frac{-2f + u}{-2f + v} \qquad ...(1)$$

Also $\Delta MPF \sim \Delta A'B'F$, therefore,

$$\frac{MP}{A'B} = \frac{FP}{FB'} = \frac{FP}{FP + PB'}$$

or $$\frac{AB}{A'B} = \frac{-f}{-f + v} \qquad ...(2)$$

From equations (1) and (2), we get

$$\frac{-2f + u}{-2f + v} = \frac{-f}{-f + v}$$

or $$2f^2 - fu - 2fv + uv = 2f^2 - fv$$

or $$-fv - fu + uv = 0$$

or $$uv = fv + fu$$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This proves the mirror formula for a concave mirror when it forms a virtual image.

10. *Establish the relationship between object distance, image distance and radius of curvature for a convex mirror.*

Derivation of mirror formula for a convex mirror. Consider an object AB placed on the principal axis of a convex mirror of small aperture, as shown in Fig. 9.15. A ray AM from the object travels parallel to the principal axis and after reflection from the mirror, it appears to come from the focus F. Another ray AP is incident on the pole P of the mirror and is reflected along PQ in accordance with the laws of reflection, so that $\angle APB = \angle BPQ$. The two reflected rays appear to diverge from a common point A'. Thus A' is the virtual image of A. The image of any point on AB will lie on a corresponding point of $A'B'$. Hence $A'B'$ is the virtual image of AB formed by reflection from the convex mirror.

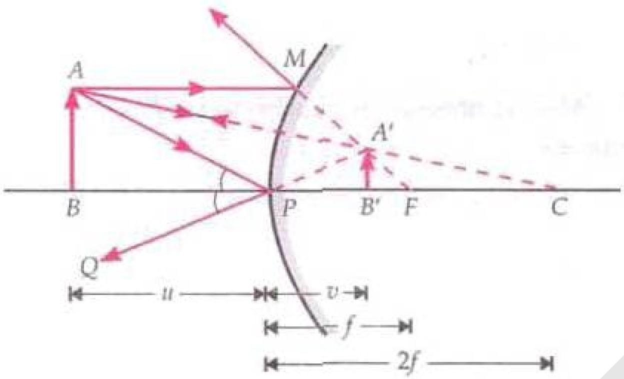


Fig. 9.15 To derive mirror formula for a convex mirror.

Using cartesian sign convention, we find

Object distance,	$BP = -u$
Image distance,	$PB' = +v$
Focal length,	$FP = +f$
Radius of curvature,	$PC = +R = +2f$

Now $\Delta A'B'C \sim \Delta ABC$

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'C}{BC} = \frac{PC - PB'}{BP + PC} = \frac{R - v}{-u + R} \qquad ...(1)$$

As $\angle A'PB' = \angle BPQ = \angle APB$,

Therefore, $\Delta A'B'P \sim \Delta ABP$.

Consequently,

$$\frac{A'B'}{AB} = \frac{PB'}{BP} = \frac{v}{-u} \qquad ...(2)$$

From equations (1) and (2), we get

$$\frac{R - v}{-u + R} = \frac{v}{-u}$$

or $\quad -uR + uv = -uv + vR$

or $\quad vR + uR = 2\,uv$

Dividing both sides by uvR, we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

But $\quad R = 2f$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This proves the mirror formula for a convex mirror.

11. *Define magnification. Write the expressions for magnification for (i) a concave mirror and (ii) a convex mirror. Express m in terms of u, v and f.*

Linear magnification. *The ratio of the height of the image to that of the object is called* ***linear or transverse magnification*** *or just* ***magnification*** and is denoted by m.

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{h_2}{h_1}$$

Concave mirror. Fig. 9.13 shows the ray diagram for the formation of image $A'B'$ of a finite object AB by a concave mirror.

Now, $\Delta APB \sim \Delta A'PB'$

$$\therefore \quad \frac{A'B}{AB} = \frac{B'P}{BP}$$

Applying the new cartesian sign convention, we get

$A'B' = -h_2$	*(Downward image height)*
$AB = +h_1$	*(Upward object height)*
$B'P = -v$	*(Image distance on left)*
$BP = -u$	*(Object distance on left)*

$$\therefore \quad \frac{-h_2}{h_1} = \frac{-v}{-u}$$

Magnification,

$$m = \frac{h_2}{h_1} = -\frac{v}{u}.$$

Convex mirror. Fig. 9.15 shows the formation of image $A'B'$ of a finite object AB by a convex mirror.

Now, $\quad \Delta A'B'P \sim \Delta ABP$

$$\therefore \quad \frac{A'B'}{AB} = \frac{PB'}{BP}$$

Applying the new cartesian sign convention, we get

$$A'B' = +h_2, \quad AB = +h_1$$

$$PB' = +v, \quad BP = -u$$

$$\therefore \quad \frac{h_2}{h_1} = \frac{v}{-u}$$

Magnification, $\quad m = \dfrac{h_2}{h_1} = -\dfrac{v}{u}$

Linear Magnification in terms of u and f. The mirror formula is

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Multiplying both sides by u, we get

$$1+\frac{u}{v}=\frac{u}{f}$$

or $$-\frac{u}{v}=1-\frac{u}{f}=\frac{f-u}{f}$$

∴ $$m=-\frac{v}{u}=\frac{f}{f-u}$$

Linear magnification in terms of v and f. As

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Multiplying both sides by v, we get

$$\frac{v}{u}+1=\frac{v}{f}$$

or $$-\frac{v}{u}=1-\frac{v}{f}=\frac{f-v}{f}$$

∴ $$m=-\frac{v}{u}=\frac{f-v}{f}$$

For Your Knowledge

- The same mirror formula is valid for both concave and convex mirrors whether the image formed is real or virtual.
- If $|m|>1$, the image is *magnified.*
- If $|m|<1$, the image is *diminished.*
- If $|m|=1$, the image is of the *same size* as the object.
- If m is *positive* (or v is positive), *the image is virtual* and *erect* .
- If m is *negative* (or v is negative), the image is *real* and *inverted* .

9.5 SPHERICAL ABERRATION

12. *What is spherical aberration in spherical mirrors ? How can it be reduced ?*

Spherical aberration. The *inability of a spherical mirror of large aperture to bring all the rays of wide beam of light falling on it to focus at a single point is called spherical aberration.* As shown in Fig. 9.16, only the paraxial rays are focussed at the principal focus F. The marginal rays meet the principal axis at a point closer to the pole than the principal focus. The different rays are reflected on to surface known as the *caustic curve.* This results in blurred image of the object.

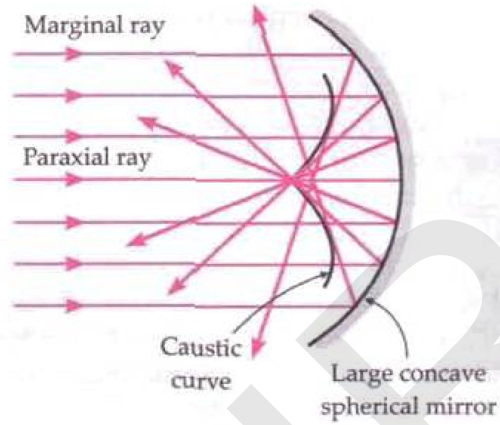


Fig. 9.16 Spherical aberration in a spherical mirror.

Spherical aberration can be reduced by following methods :

1. By using spherical mirrors of small apertures.
2. By using stoppers so as to cut off the marginal rays.
3. By using parabolic mirrors.

As shown in Fig. 9.17, a parabolic mirror focusses all the rays in a wide parallel beam to a single point on the principal axis and thus spherical aberration is reduced.

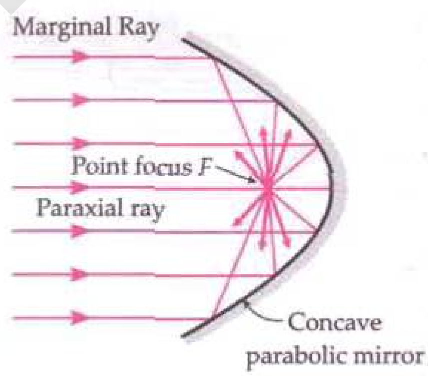


Fig. 9.17 No spherical aberration in a parabolic mirror.

9.6 USES OF CURVED MIRRORS

13. *Give some uses of spherical and parabolic mirrors.*

Uses of concave mirrors :

1. A concave mirror is used as shaving or make-up mirror because it forms a magnified and erect image of the face when it is held closer to the face.
2. Doctors use concave mirrors as *head mirror*. The mirror is strapped to the doctor's forehead and light from a lamp after reflection from the mirror is focussed into the throat or ear of the patient.

3. A small concave mirror with a small hole at its centre is used in the doctor's *ophthalmoscope*. The doctor looks through the hole from behind the mirror while a beam of light from a lamp reflected from it is directed into the pupil of patient's eye which makes the retina visible.
4. Concave mirrors are used *as reflectors* in head-lights of cars, railway engines, torch lights, etc. The source is placed at the focus of a concave mirror. The light rays after reflection travel over a large distance as a parallel intense beam.

Uses of convex mirrors :

A convex mirror is used as a *rear view mirror* in automobiles. The reason is that it always forms a small and erect image and it has a larger field of view than that of a plane mirror of the same size.

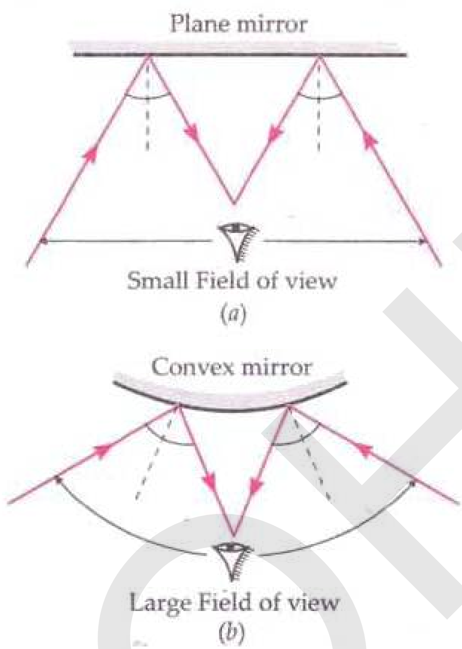


Fig. 9.18 Field of view of (*a*) a plane mirror (*b*) a convex mirror.

Uses of parabolic mirrors :

1. A concave parabolic mirror can focus a wide parallel beam to a single point. This property is used by dish antennas to collect and bring to focus microwave signals from satellites.
2. When a source of light is placed at the focus of a paraboloidal mirror, the reflected beam is accurately parallel and is thrown over a very large distance. Due to this property, paraboloidal mirrors are used as reflectors in search lights, car head lights, etc.
3. They are used in astronomical telescopes of large aperture for overcoming spherical aberration.

Examples based on Formation of Images by Spherical Mirrors

Formulae Used

1. For any spherical mirror, $f = R/2$
2. Mirror formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R}$
3. Magnification, $m = \frac{h_2}{h_1} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$
4. Magnification m is –ve for real images and +ve for virtual images.
5. f and R are –ve for a concave mirror and +ve for a convex mirror.
6. For a real object u is –ve, v is –ve for real image and +ve for virtual image.
7. Do not give any sign to unknown quantity. The sign will automatically appear in the final result.

Units Used

The quantities f, u, v, h_1 and h_2 are all in m or cm while magnification m has no units.

Example 1. *An object is placed (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature and magnification of the image in each case.* [NCERT]

Solution. As R is negative for a concave mirror, so

$$f = \frac{R}{2} = -\frac{15}{2} = -7.5 \text{ cm}$$

(*i*) Here object distance, $u = -10$ cm

By mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-0.75} - \frac{1}{-10}$$

$$= \frac{-2.5}{7.5 \times 10} = -\frac{1}{30}$$

or $\quad v = \mathbf{-30\ cm}$

As v is –ve, a real image is formed 30 cm from the mirror on the same side as the object.

Magnification, $m = -\frac{v}{u} = -\frac{-30}{-10} = \mathbf{-3}$

The image is magnified, real and inverted.

(*ii*) Here object distance, $u = -5$ cm

By mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-7.5} - \frac{1}{-5} = \frac{-5+7.5}{7.5 \times 5} = \frac{1}{15}$$

or $\quad v = \mathbf{+15\ cm}$

As v is +ve, a virtual image is formed 15 cm behind the mirror.

Magnification, $m = -\frac{v}{u} = -\frac{15}{-5} = 3$

The image is magnified, virtual and erect.

Example 2. *If you sit in a parked car, you glance in the rear view mirror $R = 2$ m and notice a jogger approaching. If the jogger is running at a speed of 5 ms^{-1}, how fast is the image of the jogger moving when the jogger is (a) 39 m (b) 29 m (c) 19 m (d) 9 m away ?* [NCERT]

Solution. As the rear view mirror is convex, so

$$R = +2 \text{ m}, \quad f = R/2 = +1 \text{ m}$$

From mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \quad \therefore \quad v = \frac{fu}{u - f}$$

When, $u = -39$ m,

$$v = \frac{1 \times (-39)}{-39 - 1} = \frac{39}{40} \text{ m}$$

As the jogger moves at a constant speed of 5 ms^{-1}, the position of the jogger after 1 s,

$$u = -39 + 5 = -34 \text{ m}$$

Position of the image after 1 s,

$$v' = \frac{1 \times (-34)}{-34 - 1} = \frac{34}{35} \text{ m}$$

Difference in the position of the image in 1 s is

$$v - v' = \frac{39}{40} - \frac{34}{35} = \frac{1365 - 1360}{1400}$$

$$= \frac{5}{1400} = \frac{1}{280} \text{ m}$$

$\therefore$ Average speed of the image $= \frac{1}{280}$ ms^{-1}.

For $u = -29$ m, -19 m and -9 m, the speeds of image will be

$$\frac{1}{150} \text{ ms}^{-1}, \ \frac{1}{60} \text{ ms}^{-1} \text{ and } \frac{1}{10} \text{ ms}^{-1} \text{ respectively.}$$

The speed becomes very high as the jogger approaches the car. The change in speed can be experienced by anybody while travelling in a bus or a car.

Example 3. *A 5 cm long needle is placed 10 cm from a convex mirror of focal length 40 cm. Find the position, nature and size of the image of the needle. What happens to the size of image when the needle is moved farther away from the mirror ?* [CBSE Sample Paper 11]

Solution. Here $h_1 = +5$ cm,

$$u = -10 \text{ cm},$$

$$f = +40 \text{ cm}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{40} + \frac{1}{10} = \frac{5}{40} = \frac{1}{8}$$

or $\quad v = +8$ **cm**

As v is +ve, the image is virtual and erect and is formed at 8 cm behind the mirror.

Magnification, $\quad m = \frac{h_2}{h_1} = -\frac{v}{u} = -\frac{+8}{-10} = +0.8$

Size of image, $\quad h_2 = 0.8 \times h_1 = 0.8 \times 5 = \mathbf{4\ cm}$

As the needle is moved farther away from the mirror, the image shifts towards the focus and its size goes on decreasing. When the needle is far off, it appears almost as a point image at the focus.

Example 4. *A square wire of side 3.0 cm is placed 25 cm away from a concave mirror of focal length 10 cm. What is the area enclosed by the image of the wire ? (The centre of the wire is on the axis of the mirror, with its two sides normal to the axis).* [NCERT]

Solution. Here, $u = -25$ cm, $\quad f = -10$ cm

As $\quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} + \frac{1}{25}$$

$$= \frac{-5 + 2}{50} = -\frac{3}{50}$$

or $\quad v = -\frac{50}{3}$ cm

Now $\quad m = -\frac{v}{u} = -\frac{50}{3 \times 25} = -\frac{2}{3}$

$$= \frac{\text{Side of image of wire } (h_2)}{\text{Side of square wire } (h_1)}$$

$\therefore$ Side of image of wire, h_2

$$= -\frac{2}{3} \times h_1 = -\frac{2}{3} \times 3 = -2 \text{ cm}$$

Area enclosed by the image of wire $= (2)^2 = \mathbf{4\ cm^2}$.

Example 5. *A concave mirror of focal length 10 cm is placed at a distance of 35 cm from a wall. How far from the wall should an object be placed to get its image on the wall ?*

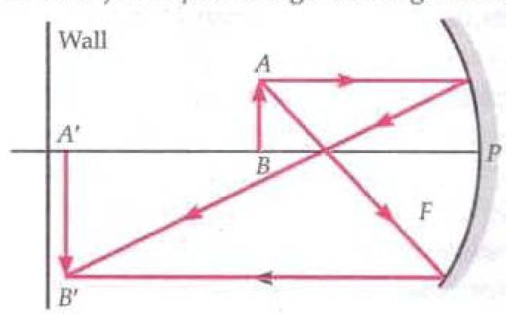


Fig. 9.19

Solution. Here, $f = -10$ cm, $v = -35$ cm

From mirror formula,

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v} = -\frac{1}{10} + \frac{1}{35} = -\frac{1}{14}$$

or $\quad u = -14$ cm

$\therefore$ Distance of the object from wall

$$= 35 - 14 = \mathbf{21\ cm.}$$

Example 6. *An object is placed at a distance of 40 cm on the principal axis of a concave mirror of radius of curvature 30 cm. By how much does the image move if the object is shifted towards the mirror through 15 cm?*

Solution. *In first case :*

$u = -40$ cm, $R = -30$ cm or $f = -15$ cm

From mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{15} + \frac{1}{40}$$

$$= -\frac{1}{24} \quad \text{or} \quad v = -24 \text{ cm}$$

In second case : The object is shifted towards the mirror by 15 cm, so

$$u' = -(40 - 15) = -25 \text{ cm}$$

From mirror formula,

$$\frac{1}{v'} = \frac{1}{f} - \frac{1}{u'} = -\frac{1}{15} + \frac{1}{25} = -\frac{2}{75}$$

or $\quad v' = -37.5$ cm

Distance through which the image shifts

$$= v' - v = -37.5 + 24 = \mathbf{-13.5\ cm}$$

i.e., the image shifts 13.5 cm farther from the mirror.

Example 7. *An object is placed exactly midway between a concave mirror of radius of curvature 40 cm and a convex mirror of radius of curvature 30 cm. The mirrors face each other and are 50 cm apart. Determine the nature and position of the image formed by successive reflections first at the concave mirror and then at the convex mirror.*

Solution. The image formation is shown in Fig. 9.20.

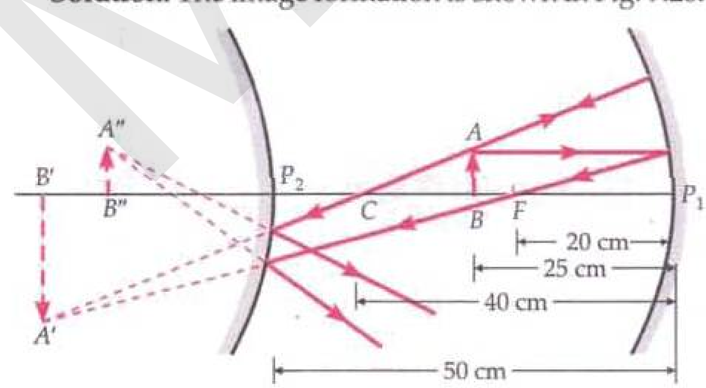


Fig. 9.20

(*i*) *For concave mirror.* $u_1 = -25$ cm, $f_1 = -20$ cm

From mirror formula,

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = -\frac{1}{20} + \frac{1}{25} = -\frac{1}{100}$$

$\therefore \quad v_1 = -100$ cm

As v_1 is negative, the image $A'B'$ is real and is formed in front of concave mirror such that $P_1B' = 100$ cm.

(*ii*) *For convex mirror.* The image $A'B'$ acts as virtual object

$\therefore \quad u_2 = +(100 - 50) = 50$ cm, $f_2 = +50$ cm

Hence $\quad \dfrac{1}{v_2} = \dfrac{1}{f_2} - \dfrac{1}{u_2} = \dfrac{1}{15} - \dfrac{1}{50} = \dfrac{7}{150}$

or $\quad v_2 = \mathbf{+21.43\ cm}$

As v_2 is positive, the final image $A''B''$ is virtual and is formed behind the convex mirror such that $P_2B'' = 21.43$ cm.

Example 8. *An object is placed at a distance of 36 cm from a convex mirror. A plane mirror is placed in between so that the two virtual images so formed coincide. If the plane mirror is at a distance of 24 cm from the object, find the radius of curvature of the convex mirror.*

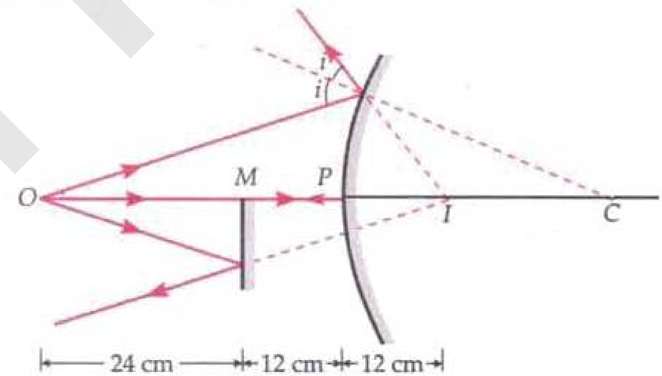


Fig. 9.21

Solution. The image I of the object O formed by plane mirror should be at 24 cm behind the mirror or 12 cm behind the convex mirror. For no parallax between the images formed by the two mirrors, the image formed by the convex mirror should also lie at I. Therefore, for convex mirror

$$u = OP = -36 \text{ cm}; \quad v = PI = +12 \text{ cm}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{36} + \frac{1}{12} = \frac{-1+3}{36} = \frac{1}{18}$$

or $\quad f = 18$ cm

Radius of curvature of convex mirror = **36 cm.**

Example 9. *An object is kept in front of a concave mirror of focal length 15 cm. The image formed is three times the size of the object. Calculate the two possible distances of the object from the mirror.* [CBSE D 98]

Solution. As the mirror is concave, so

$$f = -15 \text{ cm}$$

When the image formed is real

$$m = \frac{h_2}{h_1} = -\frac{v}{u} = -3 \quad \text{or} \quad v = +3u$$

As $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\therefore \quad \frac{1}{u} + \frac{1}{3u} = -\frac{1}{15}$

or $\quad \frac{4}{3u} = -\frac{1}{15}$

or $\quad u = -\frac{15 \times 4}{3} = $ **– 20 cm.**

When the image formed is virtual

$$m = \frac{h_2}{h_1} = -\frac{v}{u} = +3 \quad \text{or} \quad v = -3u$$

As $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\therefore \quad \frac{1}{u} - \frac{1}{3u} = -\frac{1}{15} \quad$ or $\quad \frac{2}{3u} = -\frac{1}{15}$

$\therefore \quad u = -\frac{15 \times 2}{3} = $ **– 10 cm.**

Example 10. *When the distance of an object from a concave mirror is decreased from 15 cm to 9 cm, the image gets magnified 3 times than that in first case. Calculate the focal length of the mirror.*

Solution. Magnification, $m = \frac{f}{f-u}$

In *first case*,

$u = -15$ cm $\quad \therefore \quad m = \frac{f}{f+15}$

In *second case*,

$u = -9$ cm $\quad \therefore \quad m' = \frac{f}{f+9}$

But $\quad m' = 3m$

or $\quad \frac{f}{f+9} = \frac{3 \times f}{f+15}$

or $\quad f + 15 = 3f + 27$

or $\quad f = $ **– 6 cm.**

Example 11. *Two objects A and B when placed one after another in front of a concave mirror of focal length 10 cm, form images of same size. Size of object A is 4 times that of B. If object A is placed at a distance of 50 cm from the mirror, what should be the distance of B from the mirror ?*

Solution. For object *A*,

$$m = \frac{h_2}{h_1} = \frac{f}{f-u_1}$$

For object *B*,

$$m' = \frac{h_2'}{h_1'} = \frac{f}{f-u_2}$$

$$\therefore \quad \frac{m}{m'} = \frac{h_2}{h_1} \times \frac{h_1'}{h_2'} = \frac{f-u_2}{f-u_1}$$

As $h_1 = 4h_1'$, $h_2 = h_2'$, $f = -10$ cm and $u_1 = -50$ cm, therefore,

$$\frac{1}{4} = \frac{-10-u_2}{-10+50} \quad \text{or} \quad u_2 = \mathbf{-20\ cm.}$$

Example 12. *A thin rod of length f/3 is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. What will be the magnification ?* [IIT 91]

Solution. The image of the rod placed along the optical axis will touch the rod only when one end of the rod *AC* is at the centre of curvature of the concave mirror ($PC = 2f$, $AC = f/3$). Then the image of the end *C* of the rod will be formed at the same point *C*.

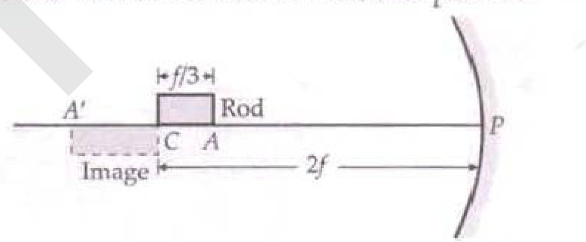


Fig. 9.22

For the end *A* of the rod, we have

$$u = PA = PC - AC = 2f - \frac{f}{3} = \frac{5f}{3}$$

From mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{f} - \frac{3}{5f} = \frac{2}{5f}$.

Thus, the image of *A* is formed at *A'* at a distance $5f/2$ from the pole P ($PA' = 5f/2$).

Length of the image

$$= A'C = PA' - PC = \frac{5f}{2} - 2f = \frac{f}{2}.$$

$\therefore$ Magnification $= \frac{CA'}{CA} = \frac{f/2}{f/3} = $ **1.5.**

Problems For Practice

1. An object is placed at a distance of 10 cm from a concave mirror of radius of curvature 40 cm. Find the nature, position and magnitude of the image. [Himachal 96]

(Ans. Virtual, erect image at 20 cm behind the mirror, $m = -2$)

2. An object is placed at a distance of 15 cm from a convex mirror and image is formed at a distance of 5 cm from the mirror. Calculate the radius of curvature of the mirror. (Ans. 15.0 cm)
3. A candle flame 3 cm high is placed at a distance of 3 m from a wall. How far from the wall must a concave mirror be placed so that it may form 9 cm high image of the flame on the same wall ? Also find the focal length of the mirror. (Ans. 4.5 m, – 1.125 m)
4. A dentist concave mirror has a radius of curvature of 30 cm. How far must it be placed from a small cavity in order to give a virtual image magnified five times ? (Ans. 12 cm)
5. Calculate the distance of an object of height h from a concave mirror of focal length 10 cm, so as to obtain a real image of magnification 2. **[CBSE D 08]** (Ans. $u = -15$ cm)
6. A concave mirror forms a real image four times as tall as the object placed 10 cm in front of mirror. Find the position of the image and the radius of curvature of the mirror. (Ans. 40 cm in front of the mirror, $R = -16$ cm)
7. When an object is placed at a distance of 60 cm from a convex spherical mirror, the magnification produced is 1/2. Where should the object be placed to get a magnification of 1/3 ? (Ans. – 120 cm)
8. An object of 1 cm^2 face area is placed at a distance of 1.5 m from a screen. How far from the object should a concave mirror be placed so that it forms 4 cm^2 image of object on the screen ? Also, calculate the focal length of the mirror. (Ans. – 1.5 m, – 1 m)

HINTS

3. Refer to Fig. 9.19. Let $BP = x$.

Then $A'P = 3 + x$ metre

So $u = -x$ m and $v = -(x+3)$ m

As $m = \frac{h_2}{h_1} = -\frac{v}{u}$ $\therefore$ $\frac{-9\text{ cm}}{3\text{ cm}} = -\frac{x+3}{x}$

or $x = 1.5$ m

$\therefore$ $u = -1.5$ m and $v = -$ **4.5 m**

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{1.5} - \frac{1}{4.5} = -\frac{8}{9}$$

or $f = -$ **1.125 m.**

5. Here $f = -10$ cm and $m = -2$ for real image

But $m = \frac{f}{f-u}$ $\therefore$ $-2 = \frac{-10}{-10-u}$

or $20 + 2u = -10$ or $u = -$ **15 cm.**

7. Here $u = -60$ cm.

In first case,

$$m = -\frac{v}{u}$$

$\therefore$ $\frac{1}{2} = -\frac{v}{-60}$ or $v = +30$ cm

Now $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{60} + \frac{1}{30} = \frac{1}{60}$

or $f = +60$ cm

In second case,

$$m = \frac{1}{3} = -\frac{v}{u} \quad \text{or} \quad v = -\frac{u}{3}$$

As $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ $\therefore$ $\frac{1}{u} - \frac{3}{u} = \frac{1}{60}$ or $u = -$**120 cm.**

8. Refer to Fig. 9.23.

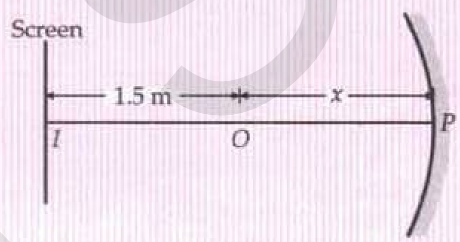


Fig. 9.23

Let $u = OP = -x$ m and $v = IP = -(x + 1.5)$ m

$$\text{Areal magnification} = \frac{4\text{ cm}^2}{1\text{ cm}^2} = 4$$

$\therefore$ Linear magnification $= -\sqrt{4} = -2$

(Negative sign for real image)

As $m = -\frac{v}{u}$

$\therefore$ $-2 = -\frac{-(x+1.5)}{-x}$ or $x = 1.5$ m

$\therefore$ $u = -1.5$ m, $v = -3$ m

Now $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{1.5} - \frac{1}{3} = -1$ $\therefore$ $f = -$**1 m.**

9.7 REFRACTION OF LIGHT

14. *What is meant by refraction of light ?*

Refraction of light. When light travels in the same homogeneous medium, it travels along a straight path. However, when it passes obliquely from one transparent medium to another, the direction of its path changes at the interface of the two media. This is called ***refraction of light***.

The phenomenon of the change in the path of light as it passes obliquely from one transparent medium to another is called refraction of light.

The path along which the light travels in the first medium is called *incident ray* and that in the second medium is called *refracted ray*. The angles which the incident ray and the refracted ray make with the normal at the surface of separation are called *angle of incidence* (i) and *angle of refraction* (r) respectively.

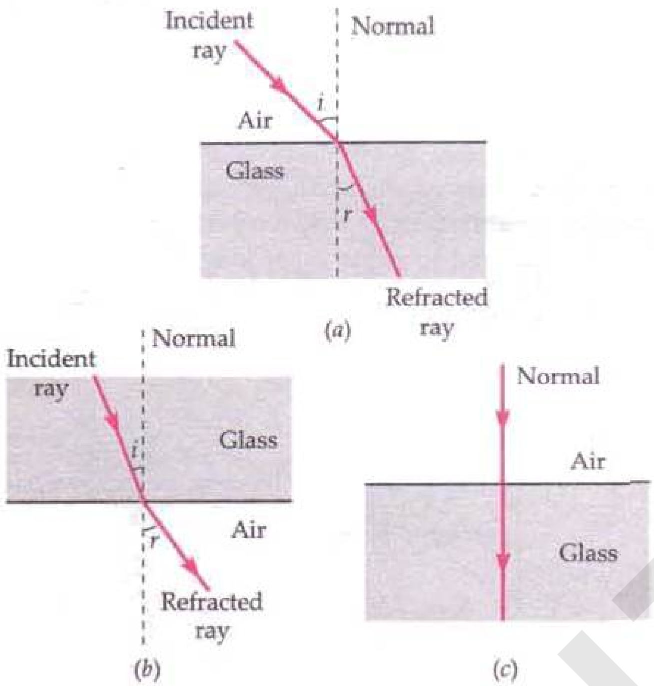


Fig. 9.24 Refraction of light (*a*) from rarer to denser medium (*b*) from denser to rarer medium (*c*) no refraction for normal incidence.

It is observed that

1. When a ray of light passes from an optically rarer medium to a denser medium, it bends towards the normal ($\angle r < \angle i$), as shown in Fig. 9.24(*a*).
2. When a ray of light passes from an optically denser to a rarer medium, it bends away from the normal ($\angle r > \angle i$), as shown in Fig. 9.24(*b*).
3. A ray of light travelling along the normal passes undeflected, as shown in Fig. 9.24(*c*). Here $\angle i = \angle r = 0°$.

9.8 LAWS OF REFRACTION OF LIGHT

15. *State the laws of refraction of light.*

Laws of refraction of light. The phenomenon of refraction of light obeys the following *two* laws :

First law. *The incident ray, the refracted ray and the normal to the interface at the point of incidence all lie in the same plane.*

Second law. *The ratio of the sine of the angle of incidence and the sine of the angle of refraction is constant for a given pair of media.*

Mathematically, $\frac{\sin i}{\sin r} = {}^1\mu_2$, a constant.

The ratio ${}^1\mu_2$ is called ***refractive index*** of second medium with respect to first medium. The second law was first deduced by a Dutch scientist *Willibord Snell* in 1621, so it is also known as ***Snell's law of refraction***.

9.9 REFRACTIVE INDEX

16. *Define refractive index of a medium in terms of (i) speed and (ii) wavelength, of light. What is relative refractive index ?*

Refractive index in terms of speed of light. The refractive index of a medium may be defined in terms of the speed of light as follows :

The refractive index of a medium for a light of given wavelength may be defined as the ratio of the speed of light in vacuum to its speed in that medium.

$$\text{Refractive index} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$

or $$\mu = \frac{c}{v}$$

Refractive index of a medium with respect to vacuum is also called ***absolute refractive index***.

Refractive index in terms of wavelength. Since the frequency (ν) remains unchanged when light passes from one medium to another, therefore,

$$\mu = \frac{c}{v} = \frac{\lambda_{vac} \times \nu}{\lambda_{med} \times \nu} = \frac{\lambda_{vac}}{\lambda_{med}}$$

The refractive index of a medium may be defined as the ratio of wavelength of light in vacuum to its wavelength in that medium.

Relative refractive index. The relative refractive index of medium 2 with respect to medium 1 is defined as the ratio of speed of light (v_1) in medium 1 to the speed of light (v_2) in medium 2 and is denoted by ${}^1\mu_2$.

Thus $${}^1\mu_2 = \frac{v_1}{v_2}$$

As refractive index is the ratio of two similar physical quantities, so it has no units and dimensions.

17. *State the factors on which the refractive index of a medium depends.*

Factors on which the refractive index of a medium depends. These are as follows :

1. Nature of the medium.
2. Wavelength of the light used.
3. Temperature.
4. Nature of the surrounding medium.

It may be noted that refractive index is a characteristic of the pair of the media and also depends on the wavelength of light, but is independent of the angle of incidence.

For Your Knowledge

➢ Optical density is a quantity quite different from mass density. Optical density is the ratio of the speed of light in two media while mass density is the mass per unit volume. Interestingly, an optically denser medium may have mass density less than an optically rarer medium. For example, the mass density of turpentine is less than that of water but turpentine is optically more denser than water.

9.10 CAUSE OF REFRACTION

18. *Describe the cause of refraction of light.*

Cause of refraction of light. Light travels with different speeds in different media. *The bending of light or refraction occurs due to the change in the speed of light as it passes from one medium to another.* Larger the change in the speed of light as it passes from one medium to another, the more is the bending due to refraction. The Snell's law of refraction may be written as

$$^1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

From the above equation, we can note the following results :

(*i*) If $v_1 > v_2$, then $^1\mu_2 > 1$ and $\sin i > \sin r$ or $i > r$ *i.e.*, the refracted ray bends towards the normal. The medium 2 is said to be ***optically denser*** than medium 1. Hence *a ray of light bends towards the normal as it refracts from a rarer medium into a denser medium.*

(*ii*) If $v_1 < v_2$, then $^1\mu_2 < 1$ and $\sin i < \sin r$ or $i < r$ *i.e.*, the refracted ray bends away from the normal. The medium 2 is said to *optically rarer* than medium 1. Hence *a ray of light bends away from the normal as it refracts from a denser medium into a rarer medium.*

19. *Give the physical significance of refractive index.*

Physical significance of refractive index. The refractive index of a medium gives the following *two* informations :

(*i*) The value of refractive index gives information about the direction of bending of refracted ray. It tells whether the ray will bend towards or away from the normal.

(*ii*) The refractive index of a medium is related to the speed of light. It is the ratio of the speed of light in vacuum to that in the given medium. For example, refractive index of glass is 3/2. This indicates that the ratio of the speed of light in glass to that in vacuum is 2 : 3 or the speed of light in glass is two-third of its speed in vacuum.

9.11 PRINCIPLE OF REVERSIBILITY OF LIGHT

20. *State the principle of reversibility of light. Use this principle to show that the refractive index of medium 2 with respect to medium 1 is reciprocal of the refractive index of medium 1 with respect to medium 2.*

Principle of reversibility of light. *This principle states that if the final path of a ray of light after it has suffered several reflections and refractions is reversed, it retraces its path exactly.*

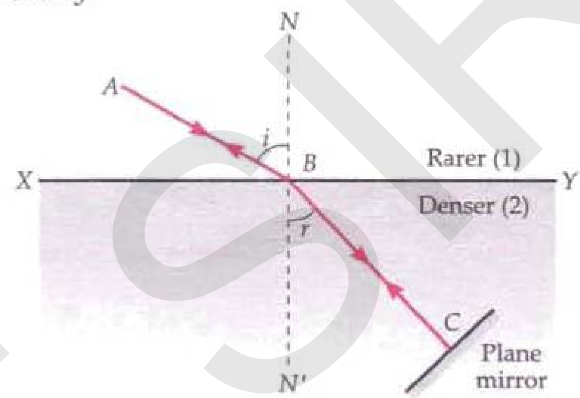


Fig. 9.25 Principle of reversibility of light.

As shown in Fig. 9.25, consider a ray of light *AB* incident on a plane surface *XY*, separating rarer medium 1 (air) from denser medium 2 (water). It is refracted along *BC*.

Let angle of incidence,

$$\angle ABN = i$$

and angle of refraction,

$$\angle CBN' = r$$

From Snell's law of refraction

$$\frac{\sin i}{\sin r} = {}^1\mu_2 \quad ...(1)$$

Suppose a plane mirror is placed perpendicular to the path of ray *BC*. This reverses the beam along its own path. Therefore, for the reversed ray, we have

Angle of incidence, $\angle CBN' = r$

Angle of refraction, $\angle ABN = i$

Again, from Snell's law

$$\frac{\sin r}{\sin i} = {}^2\mu_1 \quad ...(2)$$

Multiplying equations (1) and (2), we get

$$\frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = {}^1\mu_2 \times {}^2\mu_1$$

or

$$1 = {}^1\mu_2 \times {}^2\mu_1 \text{ or } {}^1\mu_2 = \frac{1}{^2\mu_1}$$

Thus the refractive index of medium 2 with respect to medium 1 is reciprocal of the refractive index of medium 1 with respect to medium 2.

9.12 REFRACTION THROUGH A RECTANGULAR GLASS SLAB AND LATERAL SHIFT

21. *Discuss the refraction through a glasss lab and show that emergent ray is parallel to the incident ray but laterally displaced.*

Refraction through a rectangular glass slab. Consider a rectangular glass slab *PQRS*, as shown in Fig. 9.26. A ray *AB* is incident on the face *PQ* at an angle of incidence i_1. On entering the glass slab, it bends towards normal and travels along *BC* at an angle of refraction r_1. The refracted ray *BC* is incident on face *SR* at an angle of incidence i_2. The emergent ray *CD* bends away from the normal at an angle of refraction r_2.

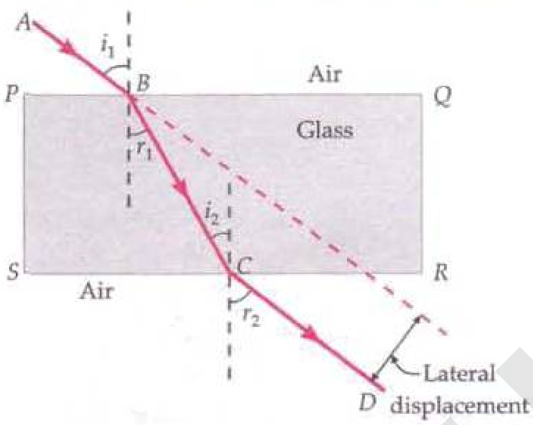


Fig. 9.26

Using Snell's law for refraction at face *PQ*,

$$\frac{\sin i_1}{\sin r_1} = {}^a\mu_g \quad ...(1)$$

For refraction at face *SR*,

$$\frac{\sin i_2}{\sin r_2} = {}^g\mu_a = \frac{1}{{}^a\mu_g} \quad ...(2)$$

Multiplying (1) and (2), we get

$$\frac{\sin i_1}{\sin r_1} \times \frac{\sin i_2}{\sin r_2} = 1$$

As *PQ* || *SR*, therefore, $i_2 = r_1$; hence

$$\frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} = 1$$

or $\sin i_1 = \sin r_2$ or $i_1 = r_2$

Thus the emergent ray *CD* is parallel to the incident ray *AB*, but it has been laterally displaced with respect to the incident ray. *This shift in the path of light on emerging from a refracting medium with parallel faces is called lateral displacement.*

Hence ***lateral shift*** *is the perpendicular distance between the incident and emergent rays, when light is incident obliquely on a refracting slab with parallel faces.*

22. *A ray of light is incident at angle i on a rectangular slab of thickness t and refractive index μ. Obtain an expression for the lateral displacement of the emergent ray. Can lateral displacement exceed t ?*

Expression for lateral displacement. Fig. 9.27 shows the path of the ray undergoing refraction through the slab *PQRS*. Let *t* be the thickness of the slab and *x*, the lateral displacement of the emergent ray. Then from right Δ *BEC*, we have

$$\frac{x}{BC} = \sin(i - r) \quad \text{or} \quad x = BC \sin(i - r)$$

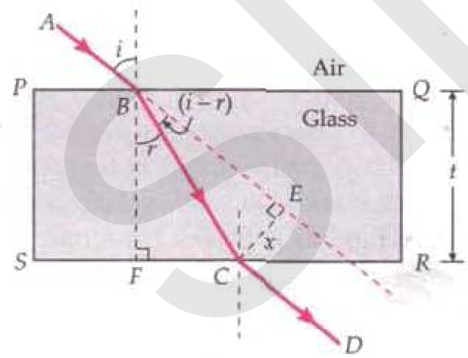


Fig. 9.27 Calculation of lateral displacement.

From right Δ *BFC*, we have

$$\frac{BF}{BC} = \cos r \quad \text{or} \quad BC = \frac{BF}{\cos r} = \frac{t}{\cos r}$$

$$\therefore \quad x = \frac{t}{\cos r}\sin(i - r) \quad ...(1)$$

$$= \frac{t}{\cos r}[\sin i \cos r - \cos i \sin r]$$

$$= t\left[\sin i - \frac{\cos i \sin r}{\cos r}\right]$$

From Snell's law,

$$\mu = \frac{\sin i}{\sin r} \quad \text{or} \quad \sin r = \frac{\sin i}{\mu}$$

and $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{\sin^2 i}{\mu^2}}$.

Hence $x = t\left[\sin i - \dfrac{\cos i . \sin i}{\mu\left(1 - \dfrac{\sin^2 i}{\mu^2}\right)^{1/2}}\right]$

or $$x = t \sin i\left[1 - \frac{\cos i}{(\mu^2 - \sin^2 i)^{1/2}}\right] \quad ...(2)$$

Clearly, *x* tends to a maximum value when $i \to 90°$, so that $\sin i \to 1$ and $\cos i \to 0$. Thus

$$x_{max} = t \sin 90° = t$$

i.e., the displacement of the emergent ray cannot exceed the thickness of the glass slab.

From equation (2), it may be noted that the lateral shift produced by a glass slab increases with

(*i*) the increase in the thickness of the glass slab,

(*ii*) the increase in the value of the angle of incidence, and

(*iii*) the increase in the value of the refractive index of the slab.

9.13 REFRACTION THROUGH A COMBINATION OF MEDIA

23. *For a ray of light undergoing refraction through a combination of three media, show that*

$$^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1.$$

Refraction through a combination of media. Fig. 9.28 shows the refraction of a ray of light from air (1) to water (2), glass (3) and finally to air. As all boundaries are parallel planes, emergent ray is parallel to the incident ray. Thus the angle of emergence is equal to the angle of incidence.

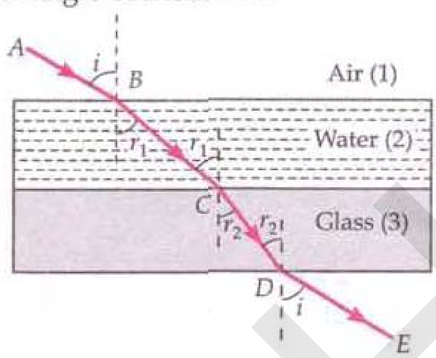


Fig. 9.28 Refraction through a combination of media.

For the ray going from medium 1 to medium 2,

$$^1\mu_2 = \frac{\sin i}{\sin r_1}$$

For the ray going from medium 2 to medium 3,

$$^2\mu_3 = \frac{\sin r_1}{\sin r_2}$$

For the ray going from medium 3 to medium 1,

$$^3\mu_1 = \frac{\sin r_2}{\sin i}$$

Multiplying the above three equations, we get

$$^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1$$

Moreover, $$^2\mu_3 = \frac{1}{^1\mu_2 \times {}^3\mu_1}$$

or $$^2\mu_3 = \frac{^1\mu_3}{^1\mu_2} \qquad \left[\because\ {}^3\mu_1 = \frac{1}{^1\mu_3}\right]$$

or $$^w\mu_g = \frac{^a\mu_g}{^a\mu_w}$$

Thus, by knowing the refractive indices of any two media like glass and water with respect to air, the refractive index of glass with respect to water or *vice versa* can be calculated.

9.14 PRACTICAL APPLICATIONS OF REFRACTION

24. *Why is the apparent depth of an object placed in a denser medium less than the real depth ? For viewing near the normal direction, show that the apparent depth is real depth divided by the refractive index of the medium. What is normal shift ? Write an expression for it.*

Real and apparent depths. It is on account of refraction of light that the apparent depth of an object placed in denser medium is less than the real depth.

Fig. 9.29 shows a point object *O* placed at the bottom of a beaker filled with water. The rays *OA* and *OB* starting from *O* are refracted along *AD* and *BC*, respectively. These rays appear to diverge from point *I*.

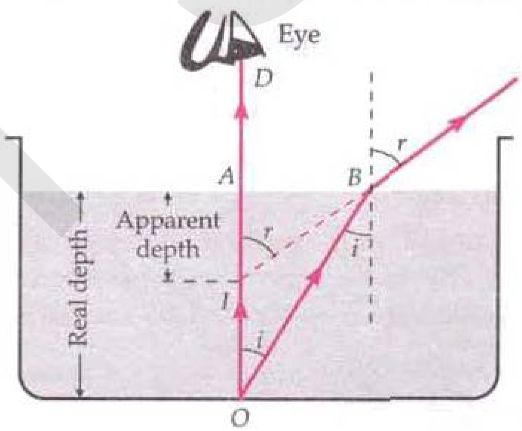


Fig. 9.29 Real and apparent depths.

So *I* is the virtual image of *O*. Clearly, the apparent depth *AI* is smaller than the real depth *AO*. That is why a water tank appears shallower or an object placed at the bottom appears to be raised.

From Snell's law, we have

$$^\omega\mu_a = \frac{\sin i}{\sin r} = \frac{\sin \angle AOB}{\sin \angle AIB} = \frac{AB/BO}{AB/BI} = \frac{BI}{BO}$$

As the size of the pupil is small, the ray *BC* will enter the eye only if *B* is close to *A*. Then

$$BI \simeq AI \text{ and } BO \simeq AO$$

$$\therefore \qquad ^a\mu_\omega = \frac{1}{^\omega\mu_a} = \frac{AO}{AI}$$

or $$\text{Refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

or $$\text{Apparent depth} = \frac{\text{Real depth}}{\text{Refractive index}}$$

As the refractive index of any medium (other than vacuum) is greater than unity, so *the apparent depth is less than the real depth.*

Normal shift. *The height through which an object appears to be raised in a denser medium is called normal shift.* Clearly

Normal shift = Real depth − Apparent depth

or $$d = AO - AI = AO - \frac{AO}{\mu}$$

$$= AO\left(1 - \frac{1}{\mu}\right)$$

or $$d = t\left(1 - \frac{1}{\mu}\right)$$

Clearly, the normal shift in the position of an object when seen through a denser medium depends on two factors :

1. The real depth of the object or the thickness (t) of the refracting medium.
2. The refractive index of the denser medium. The higher the value of μ, greater is the apparent shift 'd'.

25. *Explain how does the refraction of light affect the length of the day ?*

Apparent shift in the position of the sun at sunrise and sunset. Due to the atmospheric refraction, the sun is visible before actual sunrise and after actual sunset.

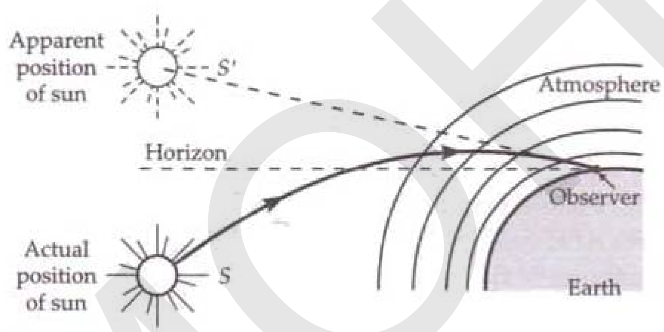


Fig. 9.30 Refraction effect at sunset and sunrise.

With altitude, the density and hence refractive index of air-layers decreases. The light rays starting from the sun S travel from rarer to denser layers. They bend more and more towards the normal.

However, an observer sees an object in the direction of the rays reaching his eyes. So to an observer standing on the earth, the sun which is actually in a position S below the horizon, appears in the position S', above the horizon. The apparent shift in the direction of the sun is by about 0.5°. Thus the sun appears to rise early by about 2 minutes and for the same reason, it appears to set late by about 2 minutes. This increases the length of the day by about 4 minutes.

26. *The sun near the horizon appears flattened at sunset and sunrise. Why ?*

Apparent flattening of the sun at sunrise and sunset. The sun near the horizon appears flattened. This is due to atmospheric refraction. The density and the refractive index of the atmosphere decrease with altitude, so the rays from the top and bottom portions of the sun on the horizon are refracted by different degrees. This causes the apparent flattening of the sun. But the rays from the sides of the sun on a horizontal plane are generally refracted by the same amount, so the sun still appears circular along its sides.

Examples based on
(i) Refraction of Light (ii) Lateral shift and (iii) Real and Apparent Depths

Formulae Used

1. Refractive index $= \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

   or $\mu = \frac{c}{v}$

2. $\mu = \frac{\text{Wavelength in vacuum}}{\text{Wavelength in medium}} = \frac{\lambda}{\lambda'}$

3. Snell's law, ${}^1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$

   or $\mu_1 \sin i = \mu_2 \sin r$

4. ${}^1\mu_2 = \frac{1}{{}^2\mu_1}$

5. ${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1$ or ${}^2\mu_3 = \frac{{}^1\mu_3}{{}^1\mu_2}$

6. Lateral shift of a ray through a rectangular slab,

   $$x = \frac{t}{\cos r} \sin(i - r)$$

   $$= t \sin i \left[1 - \frac{\cos i}{(\mu^2 - \sin^2 i)^{1/2}}\right]$$

7. $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{t}{\text{Apparent depth}}$

   Apparent depth $= \frac{t}{\mu}$

8. Apparent shift $= t\left(1 - \frac{1}{\mu}\right)$

9. Total apparent shift for compound media

   $$= t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right) + \ldots\ldots$$

Units Used

All distances are in metre, angles in degrees and refractive index μ has no units.

Example 13. *A ray of light of frequency 5×10^{14} Hz is passed through a liquid. The wavelength of light measured inside the liquid is found to be 450×10^{-9} m. Calculate the refractive index of the liquid.* [Himachal 98C]

Solution. Here $\nu = 5 \times 10^{14}$ Hz, $\lambda = 450 \times 10^{-9}$ m, $c = 3 \times 10^8$ ms^{-1}

Refractive index of the liquid,

$$\mu = \frac{c}{v} = \frac{c}{\nu\lambda}$$

$$= \frac{3 \times 10^8}{5 \times 10^{14} \times 450 \times 10^{-9}} = \mathbf{1.33.}$$

Example 14. *A light of wavelength 6000 Å in air, enters a medium with refractive index 1.5. What will be the frequency and wavelength of light in that medium ?* [IIT 97]

Solution. In air, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m, $c = 3 \times 10^8$ ms^{-1}

Refractive index of the medium, $\mu = 1.5$

When light travels from air to the refracting medium, its frequency remains unchanged.

$$\therefore \quad \nu' = \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{6 \times 10^{-7}} = \mathbf{5 \times 10^{14}\ Hz}$$

Wavelength of light in the medium,

$$\lambda' = \frac{\lambda}{\mu} = \frac{6000\ \text{Å}}{1.5} = \mathbf{4000\ Å.}$$

Example 15. *The refractive index of glass is 1.5 and that of water is 1.3. If the speed of light in water is 2.25×10^8 ms^{-1}, what is the speed of light in glass ?* [ISCE 96]

Solution. Here $^a\mu_g = \frac{c}{v_g} = 1.5$

and $$^a\mu_w = \frac{c}{v_\omega} = 1.3$$

$$\therefore \quad \frac{c}{v_\omega} \times \frac{v_g}{c} = \frac{1.3}{1.5}$$

or $$v_g = \frac{1.3}{1.5} \times v_\omega = \frac{1.3}{1.5} \times 2.25 \times 10^8$$

$$= \mathbf{1.95 \times 10^8\ ms^{-1}}.$$

Example 16. *A ray of light passes through a plane boundary separating two media whose refractive indices are $\mu_1 = 3/2$ and $\mu_2 = 4/3$. (i) If the ray travels from medium 1 to medium 2 at an angle of incidence of 30°, what is the angle of refraction ? (ii) If the ray travels from medium 2 to medium 1 at the same angle of incidence, what is the angle of refraction ?*

Solution. Here $\mu_1 = \frac{3}{2}$, $\mu_2 = \frac{4}{3}$, $i = 30°$

(*i*) When the ray travels from medium 1 to medium 2,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

or $$\frac{\sin 30°}{\sin r} = \frac{4/3}{3/2} = \frac{8}{9}$$

or $$\sin r = \frac{9}{8} \times \sin 30° = \frac{9}{8} \times \frac{1}{2} = 0.5625$$

$$\therefore \quad r = \mathbf{34°14'}.$$

(*ii*) When the ray travels from medium 2 to medium 1,

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

or $$\frac{\sin 30°}{\sin r} = \frac{3/2}{4/3} = \frac{9}{8}$$

or $$\sin r = \frac{8}{9} \times \sin 30° = \frac{8}{9} \times \frac{1}{2} = 0.4445$$

$$\therefore \quad r = \mathbf{26°24'}.$$

Example 17. *A rectangular glass slab rests in the bottom of a trough of water. A ray of light incident on water surface at an angle of 50° passes through water into glass. Calculate the angle of refraction in glass. Given that μ for water is 4/3 and that for glass is 3/2.*

Solution. Here $^a\mu_\omega = \frac{4}{3}$, $^a\mu_g = \frac{3}{2}$

$$\therefore \quad ^\omega\mu_g = \frac{^a\mu_g}{^a\mu_\omega} = \frac{3/2}{4/3} = \frac{9}{8}$$

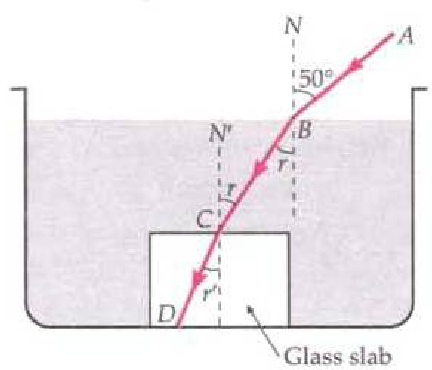


Fig. 9.31

Angle of incidence on water surface, $i = 50°$

$$\therefore \quad \frac{\sin 50°}{\sin r} = \frac{4}{3}$$

$$\therefore \quad \sin r = \frac{3}{4} \sin 50°$$

$$= \frac{3}{4} \times 0.766 = 0.5745$$

$\therefore$ Angle of refraction, $r = 35.06°$

For refraction at water-glass interface, we have

$$\frac{\sin 35.06^\circ}{\sin r'} = \frac{9}{8}$$

or $$\sin r' = \frac{8}{9} \times 0.5745 = 0.5107$$

$\therefore$ $$r' = \mathbf{30.7^\circ}.$$

Example 18. *A ray of light is incident at an angle of 60° on one face of a rectangular glass slab of thickness 0.1 m and refractive index 1.5. Calculate the lateral shift produced.*

Solution. Here $i = 60^\circ$, $\mu = 1.5$, $t = 0.1$ m

By Snell's law, $\mu = \dfrac{\sin i}{\sin r}$

$\therefore$ $$\sin r = \frac{\sin i}{\mu} = \frac{\sin 60^\circ}{1.5} = \frac{0.866}{1.5} = 0.5773$$

or $$r = 35^\circ 16'$$

Lateral shift produced,

$$x = \frac{t}{\cos r} \sin (i - r)$$

$$= \frac{0.1}{\cos 35^\circ 16'} \sin (60^\circ - 35^\circ 16')$$

$$= \frac{0.1}{\cos 35^\circ 16'} \times \sin 24^\circ 44'$$

$$= \frac{0.1 \times 0.4184}{0.8164} \text{ m}$$

$$= \mathbf{0.0513 \ m}.$$

Example 19. *The apparent depth of an object at the bottom of tank filled with a liquid of refractive index 1.3 is 7.7 cm. What is the actual depth of the liquid in the tank ?* [CBSE D 91]

Solution. Refractive index,

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$\therefore$ $$1.3 = \frac{\text{Real depth}}{7.7}$$

Hence, Real depth = 1.3 × 7.7 cm = **10.01 cm.**

Example 20. *The velocity of light in glass is $2 \times 10^8 ms^{-1}$ and that in air is $3 \times 10^8 ms^{-1}$. By how much would an ink dot appear to be raised, when covered by a glass plate 6.0 cm thick ?* [Punjab 99]

Solution. Here $v = 2 \times 10^8$ ms^{-1}, $c = 3 \times 10^8$ ms^{-1}

Refractive index of glass,

$$\mu = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

Real depth = 6.0 cm

$\therefore$ Apparent depth

$$= \frac{\text{Real depth}}{\mu} = \frac{6.0}{1.5} = 4.0 \text{ cm}$$

Distance through which the ink dot appears to be raised

$$= 6.0 - 4.0 = \mathbf{2.0 \ cm}.$$

Example 21. *A mark is made on the bottom of a beaker and a microscope is focussed on it. The microscope is raised through 1.5 cm. To what height water must be poured into the beaker to bring the mark again into focus ? Given that μ for water is 4/3.*

Solution. Here apparent shift, $d = 1.5$ cm

Let t be the height through which water must be poured into the beaker. Then

$$d = t\left(1 - \frac{1}{\mu}\right)$$

$\therefore$ $$1.5 = t\left(1 - \frac{1}{4/3}\right)$$

or $$t = 1.5 \times 4 = \mathbf{6.0 \ cm}.$$

Example 22. *The bottom of a container is a 4.0 cm thick glass ($\mu = 1.5$) slab. The container contains two immiscible liquids A and B of depths 6.0 cm and 8.0 cm respectively. What is the apparent position of a scratch on the outer surface of the bottom of the glass slab when viewed through the container ? Refractive indices of A and B are 1.4 and 1.3 respectively.*

Solution. The total apparent shift in the position of the image due to all the three media is given by

$$d = t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right) + t_3\left(1 - \frac{1}{\mu_3}\right)$$

Given $t_1 = 4.0$ cm, $t_2 = 6.0$ cm, $t_3 = 8.0$ cm

$\mu_1 = 1.5$, $\mu_2 = 1.4$, $\mu_3 = 1.3$

$\therefore$ $$d = 4.0\left(1 - \frac{1}{1.5}\right) + 6.0\left(1 - \frac{1}{1.4}\right) + 8.0\left(1 - \frac{1}{1.3}\right)$$

$$= 1.33 + 1.71 + 1.85 = \mathbf{4.89 \ cm}.$$

Example 23. *A transparent cube of side 210 mm contains a small air bubble. Its apparent distance, when viewed through one face of the cube is 100 mm and when viewed through the opposite face is 40 mm. What is the actual distance of the bubble from the second face and what is the refractive index of the material of the cube ?*

Solution. The situation is shown in Fig. 9.32. Let O be the air bubble inside the transparent cube at distance x from face I, then its distance from face II will be $(210 - x)$ mm. Let I_1 and I_2 be its images as seen from the two faces respectively.

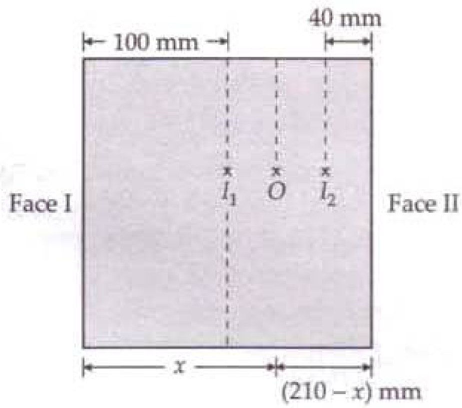


Fig. 9.32

For face I :

Real depth = x mm, apparent depth = 100 mm

$$\therefore \quad \mu = \frac{x}{100} \quad ...(i)$$

For face II :

Real depth = $(210 - x)$ mm,

apparent depth = 40 mm

$$\therefore \quad \mu = \frac{210 - x}{40} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{x}{100} = \frac{210 - x}{40} \quad \text{or} \quad x = 150 \text{ mm}$$

$\therefore$ Actual distance of the bubble from face II

$$= 210 - 150 = \mathbf{60 \text{ mm}}$$

Also, $\mu = \dfrac{x}{100} = \dfrac{150}{100} = \mathbf{1.50.}$

Example 24. *A cylindrical vessel of diameter 12 cm contains 800 π cm*^{3} *of water. A cylindrical glass piece of diameter 8.0 cm and height 8.0 cm is placed in the vessel. If the bottom of the vessel under the glass piece is seen by the paraxial rays (Fig. 9.33), locate its image. The index of refraction of glass is 1.50 and that of water is 1.33.*

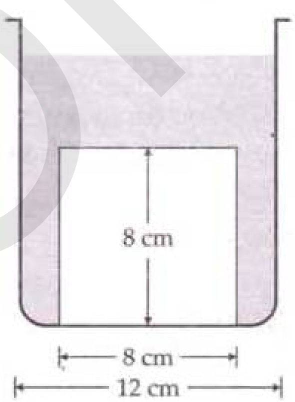


Fig. 9.33

Solution. Volume of water = $800\,\pi$ cm^3

Volume of cylindrical glass piece

$$= \pi\left(\frac{8}{2}\right)^2 \times 8 = 128\,\pi \text{ cm}^3$$

Total volume of water and glass piece

$$= 800\,\pi + 128\,\pi = 928\,\pi \text{ cm}^3$$

Height of water level from the bottom

$$= \frac{\text{Volume}}{\pi r^2} = \frac{928\,\pi}{\pi \times (6)^2} = 25.78 \text{ cm}$$

$\therefore$ Depth of water above the glass piece

$$= 25.78 - 8.0 = 17.78 \text{ cm}$$

Total apparent shift of the bottom

$$= t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right)$$

$$= 17.78\left(1 - \frac{1}{1.33}\right) + 8.0\left(1 - \frac{1}{1.50}\right)$$

$$= 4.44 + 2.66 = \mathbf{7.1 \text{ cm}}$$

Thus, the image is seen at 7.1 cm above the bottom.

Problems For Practice

1. A film of oil of refractive index 1.20, lies on water of refractive index 1.33. A light ray is incident at 30° in the oil on the oil-water boundary. Calculate the angle of refraction in water. (Ans. 27°)
2. A printed page is kept pressed by a glass cube ($\mu = 1.5$) of edge 6.0 cm. By what amount will the printed letters appear to be shifted when viewed from the top ? (Ans. 2.0 cm)
3. A travelling microscope is focussed on a mark made on a paper. When a slab of 1.47 cm thickness is placed on the mark, the microscope has to be raised through 0.49 cm to focus the mark again. Calculate the refractive index of glass. (Ans. 1.5)
4. The velocity of light in a transparent medium is 1.8×10^8 ms^{-1}, while that in vacuum is 3×10^8 ms^{-1}. Find by how much the bottom of the vessel containing the liquid appears to be raised if the depth of the liquid is 0.25 m. (Ans. 0.1 m)
5. Calculate the index of refraction of a liquid from the following into glass : (*a*) Reading for the bottom of an empty beaker : 11.324 cm (*b*) Reading for the bottom of the beaker, when partially filled with the liquid : 11.802 cm (*c*) Reading for the upper level of the liquid in the beaker : 12.895 cm (Ans. 1.437)
6. While determining the refractive index of a liquid experimentally, the microscope was focussed at the bottom of a beaker, when its reading was 3.965 cm. On pouring liquid upto a height 2.537 cm inside the beaker, the reading of the refocussed microscope was 3.348 cm. Find the refractive index of the liquid. (Ans. 1.321)
7. A vessel contains water upto a height of 20 cm and above it an oil upto another 20 cm. The refractive indices of water and oil are 1.33 and 1.30 respectively. Find the apparent depth of vessel when viewed from above. (Ans. 30.4 cm)

HINTS

1. Refractive index of water relative to oil is

$$\therefore \; {}^o\mu_w = \frac{{}^a\mu_w}{{}^a\mu_o} = \frac{1.33}{1.20} = 1.11$$

From Snell's law,

$${}^o\mu_w = \frac{\sin i}{\sin r} \quad \therefore \quad 1.11 = \frac{\sin 30^\circ}{\sin r}$$

or $\sin r = \dfrac{1}{2 \times 1.11} = 0.45$

or $r \approx 27^\circ$

2. Normal shift,

$$d = t\left(1 - \frac{1}{\mu}\right) = 6.0\left(1 - \frac{1}{1.5}\right) = \mathbf{2.0\ cm}.$$

3. As $d = t\left(1 - \dfrac{1}{\mu}\right) \quad \therefore \quad 0.49 = 1.47\left(1 - \dfrac{1}{\mu}\right)$

or $\dfrac{1}{\mu} = 1 - \dfrac{1}{3} = \dfrac{2}{3}$ or $\mu = \mathbf{1.5}$.

4. Here $v = 1.8 \times 10^8\ \text{ms}^{-1}$, $c = 3 \times 10^8\ \text{ms}^{-1}$

$\therefore$ Refractive index, $\mu = \dfrac{c}{v} = \dfrac{3 \times 10^8}{1.8 \times 10^8} = \dfrac{5}{3}$

Real depth = 0.25 m

$\therefore$ Apparent depth $= \dfrac{\text{Real depth}}{\mu} = \dfrac{0.25}{5/3} = 0.15$ m

Distance through which bottom appears to be raised

$= 0.25 - 0.15 = \mathbf{0.1\ m}.$

5. Real depth

= Reading from the upper level of the liquid – Reading from the bottom of the empty beaker

$= 12.895 - 11.324 = 1.571$ cm

Apparent depth

= Reading from the upper level of the liquid – Reading from the bottom of the beaker when partially filled with liquid

$= 12.895 - 11.802 = 1.093$ cm

$$\mu = \frac{1.571}{1.093} = \mathbf{1.437}.$$

6. Real depth = 2.537 cm

Apparent shift in the position of bottom of the beaker $= 3.965 - 3.348 = 0.617$ cm

Apparent depth $= 2.537 - 0.617 = 1.920$ cm

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{2.537}{1.920} = \mathbf{1.321}$$

7. Total apparent shift $= t_1\left(1 - \dfrac{1}{\mu_1}\right) + t_2\left(1 - \dfrac{1}{\mu_2}\right)$.

9.15 TOTAL INTERNAL REFLECTION

27. *Explain the phenomenon of total internal reflection. Under what conditions does it take place ? Derive the relation connecting the refractive index and critical angle for a given pair of media.*

Total internal reflection. If light passes from an optically denser medium to a rarer medium, then at the interface, the light is partly reflected back into the denser medium and partly refracted to the rarer medium. This reflection is called *internal reflection*. Under certain conditions, the whole of the incident light can be made to be reflected back into the denser medium. This gives rise to an interesting phenomenon called total internal reflection.

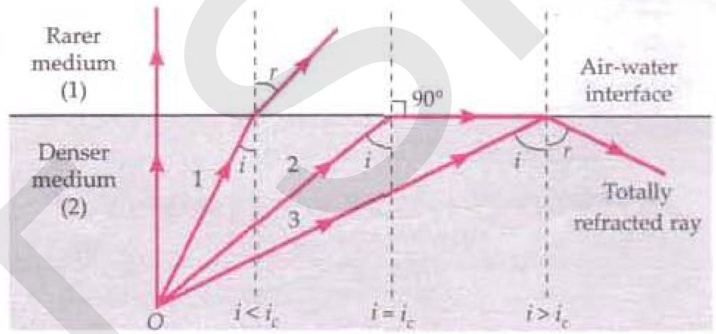


Fig. 9.34 Total internal reflection.

As shown in Fig. 9.34, when a ray of light (ray 1) travels at a small angle of incidence from a denser medium to a rarer medium, say from water to air, the refracted ray bends away from the normal so that the angle of refraction is greater than the angle of incidence. As the angle of incidence increases, the corresponding angle of refraction also increases. Then for a certain angle of incidence (ray 2), the angle of refraction becomes 90°, *i.e.*, the refracted ray goes along the surface of separation.

The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° is called ***critical angle*** *of the denser medium and is denoted by* i_c.

If the angle of incidence is increased beyond i_c (ray 3), no light is refracted into the rarer medium (since the angle of refraction cannot be greater than 90°), but whole of it is reflected back into the denser medium in accordance with the laws of reflection. This phenomenon is known as *total internal reflection*.

The phenomenon in which a ray of light travelling at an angle of incidence greater than the critical angle from denser to a rarer medium is totally reflected back into the denser medium is called ***total internal reflection****.*

Necessary conditions for total internal reflection :

1. *Light must travel from an optically denser to an optically rarer medium.*
2. *The angle of incidence in the denser medium must be greater than the critical angle for the two media.*

Relation between critical angle and refractive index. From Snell's law,

$$\frac{\sin i}{\sin r} = {}^2\mu_1 = \frac{1}{{}^1\mu_2}$$

When $i = i_c$, $r = 90°$. Therefore,

$$\frac{\sin i_c}{\sin 90°} = \frac{1}{{}^1\mu_2} \quad \text{or} \quad {}^1\mu_2 = \frac{1}{\sin i_c}$$

If the rarer medium is air, then $\mu_1 = 1$ and $\mu_2 = \mu$ (say) and we get

$$\mu = \frac{1}{\sin i_c}$$

Thus the refractive index of any medium is equal to the reciprocal of the sine of its critical angle.

Table 9.1 Critical angles of some transparent media

Substance	*Refractive index*	*Critical angle*
Water	1.33	48.75°
Crown glass	1.52	41.14°
Dense flint glass	1.65	37.31°
Diamond	2.42	24.41°

9.16 APPLICATIONS OF TOTAL INTERNAL REFLECTION

28. *Discuss the important applications of the phenomenon of total internal reflection.*

Applications of total internal reflection. The phenomenon of total internal reflection can be used to explain some effects observed in daily life and also it finds use in some optical devices as explained below :

1. Sparkling of diamond. The brilliancy of diamonds is due to total internal reflection. As the refractive index of diamond is very large, its critical angle is very small, about 24.4°. The faces of diamond are so cut that the light entering the crystal suffers total internal reflections repeatedly, and hence gets collected inside but it comes out through only a few faces. Hence the diamond sparkles when seen in the direction of emerging light.

2. Mirage. *It is an optical illusion observed in deserts or over hot extended surfaces like a coal-tarred road, due to which a traveller sees a shimmering pond of water some distance ahead of him and in which the surrounding objects like trees, etc, appear inverted.*

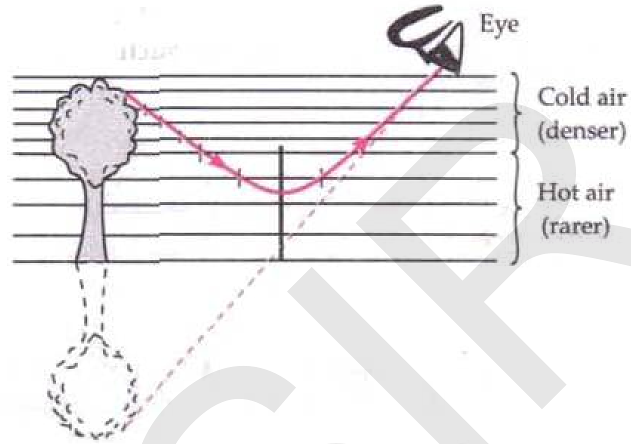


Fig. 9.35 Formation of mirage.

On a hot summer day, the surface of the earth becomes very hot. The layers of air near the earth are more heated than the higher ones. Hence the density and refractive index of air layers increase as we move high up. As the rays of light from a distant object like a tree travel towards the earth through layers of decreasing refractive index, they bend more and more away from the normal. A stage is reached when the angle of incidence becomes greater than the critical angle, the rays are totally reflected. These rays then move up through layers of increasing refractive index, and therefore undergo refraction in a direction opposite to that in the first case. These rays reach the observer's eyes and he sees an inverted image of the object, as if formed in a pond of water.

3. Totally reflecting prisms. Refer to Section 9.17.

4. Optical fibres. Refer to Section 9.18.

9.17 TOTALLY REFLECTING PRISMS

29. *What is a totally reflecting prism ? How can it be used to (i) deviate a ray through 90° (ii) deviate a ray through 180° and (iii) invert an image without the deviation of the rays.*

Totally reflecting prism. *A right-angled isosceles prism, i.e., a 45°-90°-45° prism is called a totally reflecting prism.* Whenever a ray falls normally on any face of such a prism, it is incident on the inside face at 45°, that is at an angle greater than the critical angle of glass (about 42°); hence this ray is always totally internally reflected.

These prisms may be used in *three* ways :

(*i*) **To deviate a ray through 90°.** As shown in Fig. 9.36(*a*), as the light is incident normally on one of the faces containing right angle, it enters the prism without deviation. It is incident on the hypotenuse face at an angle of 45°, greater than the critical angle. The light is totally internally reflected. Having been deviated through 90°, the light passes through third face without any further deviation. Such prisms are used in *periscopes*.

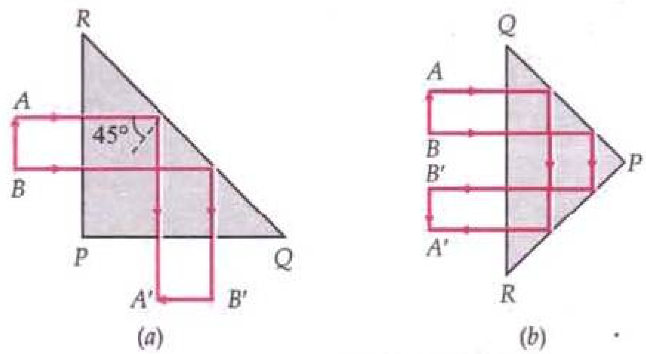


Fig. 9.36 (*a*) To deviate a ray through 90°
(*b*) To invert an image with deviation of rays through 180°.

(*ii*) **To invert an image with deviation of rays through 180°.** As shown in Fig. 9.36(*b*), the light is incident normally on the hypotenuse face, it first suffers total internal reflection from one shorter face and then from the other shorter face. The final beam emerges through the hypotenuse face, parallel to the incident beam. The deviation is 180°. Such a prism is called a *porroprism*.

(*iii*) **To invert an image without deviation of rays. (Erecting prism).** As shown in Fig. 9.37, the light enters at one shorter face at an angle. After refraction, it is totally reflected from the hypotenuse face and then refracted out of the other shorter face to become parallel to the incident beam. The rays do not suffer any deviation, only their order is reversed. The incident ray, which is on the top, emerges from the bottom of the prism. Such prisms are called *erecting prisms* and are used in binoculars and in projection lanterns.

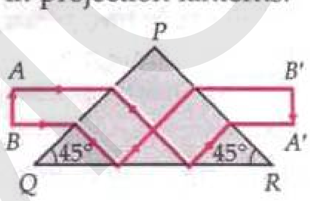


Fig. 9.37 To invert an image without deviation of rays.

30. *State the advantages of totally reflecting prisms over plane mirrors.*

Advantages of totally reflecting prisms over plane mirrors. The totally reflecting prisms have many advantages over plane mirrors as reflectors :

1. In prisms, the light is *totally* reflected, while there is always some loss of intensity in case of plane mirrors.
2. The reflecting properties of prisms are permanent, while these are affected by tarnishing in case of plane mirrors.
3. No multiple images are formed in prisms, while a plane mirror forms a number of faint images in addition to a prominent image.

9.18 OPTICAL FIBRES

31. *What are optical fibres ? On which principle do they work ? How does light propagate through an optical fibre ? What is a light pipe ?*

Optical fibres. These days we find in the market some decorative lamps provided with fine plastic fibres. At their one ends, the fibres are fixed over an electric lamp while their free ends form a fountain like structure. When the lamp is switched on, the light travels from the bottom of each fibre and appears at the tip of free end as a bright dot of light. The plastic fibres in these lamps are optical fibres. *The working of optical fibres is based on the phenomenon of total internal reflection.*

An optical fibre is a hair-thin long strand of quality glass or quartz surrounded by a glass coating of slightly lower reflective index. It is used as a guided medium for transmitting an optical signal from one place to another.

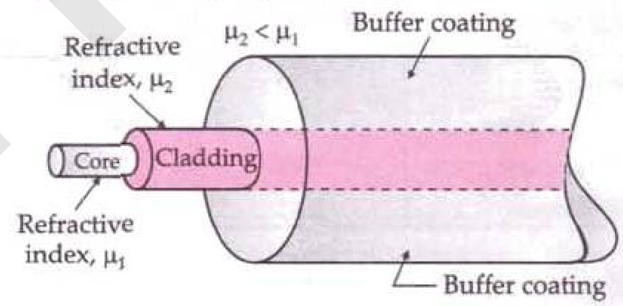


Fig. 9.38 Optical fibre structure.

Construction. An optical fibre consists of *three* main parts :

(*i*) **Core.** The central cylindrical core is made of high quality glass/silica/plastic of refractive index μ_1 and has a diameter about 10 to 100 μm.

(*ii*) **Cladding.** The core is surrounded by a glass / plastic jacket of refractive index $\mu_2 < \mu_1$. In a typical optical fibre, the refractive indices of core and cladding may be 1.52 and 1.48 respectively.

(*iii*) **Buffer coating.** For providing safety and strength, the core cladding of optical fibres is enclosed in a plastic jacket.

Propagation of light through an optical fibre. As shown in Fig. 9.39(*a*), when light is incident on one end of the fibre at a small angle, it goes inside and suffers repeated total internal reflections because the angle of incidence is greater than the critical angle of the fibre material with respect to its outer coating. As there is no loss of intensity in total internal reflection, the outcoming beam is of as much intensity as the incident beam. Even if the fibre is bent, light easily travels through along the fibre.

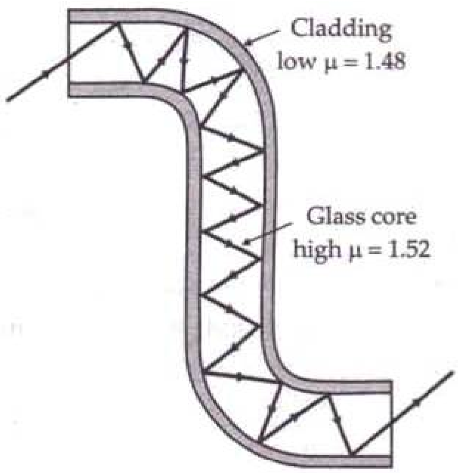


Fig. 9.39 (*a*) Propagation of light through an optical fibre.

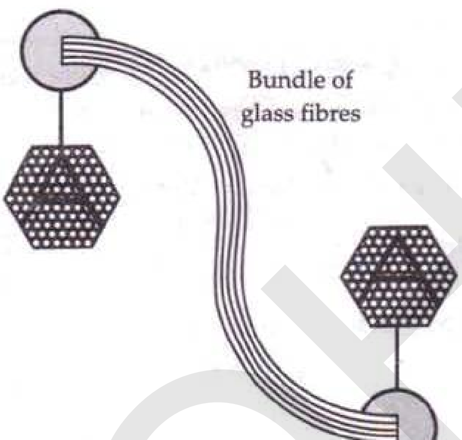


Fig. 9.39 (*b*) A light pipe.

A bundle of optical fibres is called a light pipe. A single fibre cannot be used to see the complete image of an object. But, if the image is broken into a large number of fine dots and each portion of the image is seen through a separate fibre, the complete image can be seen. A light pipe can be used to transmit such an image accurately.

32. *Mention some important applications of optical fibres.*

Applications of optical fibres. Some of the important applications are as follows :

1. *As a light pipe, optical fibres are used in medical and optical examination.* A light pipe is inserted into the stomach through the mouth. Light transmitted through the outer layers of the light pipe is scattered by the various parts of stomach into the central portion of the light pipe to produce a final image with excellent details. The technique is called endoscopy.

2. *They are used in transmitting and receiving electrical signals in telecommunication.* The electrical signals are first converted to light by suitable transducers. Each fibre can transmit about 2000 telephone conversations without much loss of intensity.

3. They are used for transmitting optical signals and two dimensional pictures.

4. In the form of *photometric sensors,* they are used for measuring the blood flow in the heart.

5. In the form of *refractometers,* they are used to measure refractive indices of liquids.

Examples based on Total Internal Reflection

Formulae Used

1. Critical angle, i_c = Angle of incidence in denser medium for which angle of refraction is 90° in rarer medium.
2. Refractive index of denser medium, $\mu = \dfrac{1}{\sin i_c}$
3. Total internal reflection occurs when $i > i_c$.

Units Used

Angle i_c is in degrees and μ has no units.

Example 25. *Find the value of critical angle for a material of refractive index* $\sqrt{3}$. [CBSE F 94]

Solution. Here $\mu = \sqrt{3}$

$$\sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0.5773$$

$\therefore$ Critical angle, $i_c \simeq 35.3°$.

Example 26. *Calculate the speed of light in a medium, whose critical angle is 30°.* [CBSE OD 10]

Solution. Here $i_c = 30°$

As $\quad \mu = \dfrac{c}{v} = \dfrac{1}{\sin i_c}$

$$\therefore \quad v = c \sin i_c = 3 \times 10^8 \times \sin 30°$$
$$= 3 \times 10^8 \times 0.5$$
$$= \mathbf{1.5 \times 10^8\ ms^{-1}}.$$

Example 27. *The velocity of light in a liquid is* $1.5 \times 10^8 ms^{-1}$ *and in air, it is* $3 \times 10^8 ms^{-1}$. *If a ray of light passes from this liquid into air, calculate the value of critical angle.* [Punjab 99]

Solution. Here $v = 1.5 \times 10^8 ms^{-1}$,

$$c = 3 \times 10^8 ms^{-1}$$

Refractive index of the liquid,

$$\mu = \frac{c}{v} = \frac{3\times10^8}{1.5\times10^8} = 2 \quad \text{or} \quad \sin i_c = \frac{1}{\mu} = \frac{1}{2}$$

$\therefore$ Critical angle, $i_c = \mathbf{30°}$.

Example 28. *Determine the direction in which a fish under water sees the setting sun. Refractive index of water is 1.33.*

Solution. Fig 9.40 shows the setting sun in the direction of water surface. A ray of light starting from the sun enters the eye of the fish.

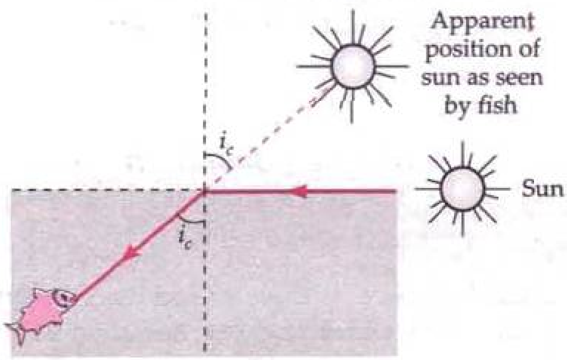


Fig. 9.40

The apparent position of the sun makes an angle i_c with the vertical.

From Snell's law

$$\frac{\sin 90°}{\sin i_c} = 1.33 \quad \text{or} \quad \sin i_c = \frac{1}{1.33} = 0.7518$$

$$\therefore \qquad i_c = \sin^{-1}(0.7518) = 48.7°$$

Angle between the apparent position of the sun and the horizontal $= 90 - 48.7 = \mathbf{41.3°}$.

Example 29. *Determine the critical angle for a glass-air surface, if a ray of light which is incident in air on the surface is deviated through 15°, when its angle of incidence is 40°.* [ISCE 93]

Solution. The path of the refracted ray is shown in Fig. 9.41.

Clearly, $r + 15 = 40°$ or $r = 25°$

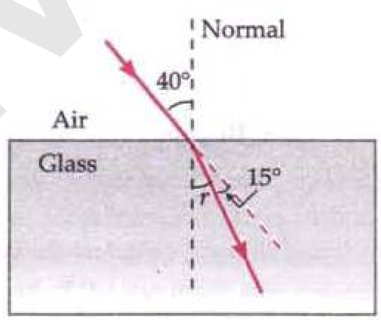


Fig. 9.41

Refractive index of glass,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 40°}{\sin 25°}$$

$$= \frac{0.6428}{0.4226} = 1.52$$

$$\therefore \qquad \sin i_c = \frac{1}{\mu} = \frac{1}{1.52} = 0.6579$$

Critical angle, $i_c = \mathbf{41.14°}$.

Example 30. *A glass slab is immersed in water. Find the critical angle at glass-water interface. Given ${}^a\mu_g = 1.5$ and ${}^a\mu_w = 1.33$.* [Punjab 96]

Or

A ray of light passes from glass ($\mu_g = 3/2$) to water ($\mu_w = 4/3$). What is the critical angle of incidence ?

Solution. From Snell's law of refraction,

$$\frac{\sin i}{\sin r} = \frac{\mu_\omega}{\mu_g}$$

$$\therefore \qquad \frac{\sin i_c}{\sin 90°} = \frac{1.33}{1.5}$$

or $\qquad \sin i_c = 0.8867$

$$\therefore \qquad i_c = \sin^{-1}(0.8867) = \mathbf{62°28'}.$$

Example 31. *The critical angle of incidence in a glass slab placed in air is 45°. What will be the critical angle when it is immersed in water of refractive index 1.33 ?*

Solution. $${}^a\mu_g = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2} = 1.414$$

Refractive index of glass w.r.t. water will be

$${}^\omega\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1.414}{1.33}$$

When glass slab is immersed in water, the critical angle i'_c is given by

$$\sin i'_c = \frac{1}{{}^\omega\mu_g}$$

$$= \frac{1}{\frac{1.414}{1.33}} = \frac{1.33}{1.414} = 0.9432$$

$$\therefore \qquad i'_c = \mathbf{70°36'}.$$

Example 32. *A ray of light incident on the horizontal surface of a glass slab at 70° just grazes the adjacent vertical surface after refraction. Calculate the critical angle and refractive index of the glass.*

Solution. As shown in Fig. 9.42, the refracted ray will graze the vertical surface *BC* only when the ray *QR* is incident at critical angle i_c.

Clearly, $r + i_c = 90°$ or $r = 90° - i_c$

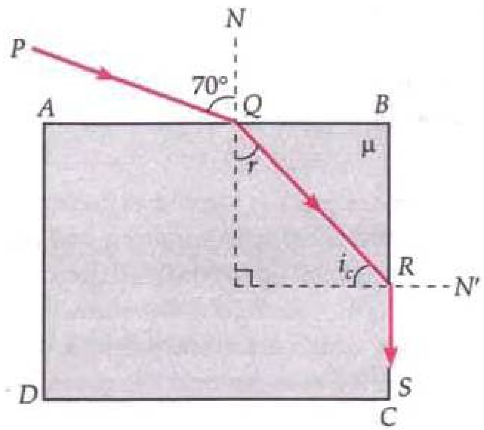


Fig. 9.42

Using Snell's law for refraction at face *AB*, we get

$$\frac{\sin 70°}{\sin r} = \mu$$

or $$\sin r = \frac{\sin 70°}{\mu}$$

or $$\sin (90° - i_c) = \frac{\sin 70°}{\mu}$$

or $$\cos i_c = \frac{\sin 70°}{\mu}$$

For refraction at face *BC*, we have

$$\sin i_c = \frac{1}{\mu}$$

$\therefore$ $$\tan i_c = \frac{\sin i_c}{\cos i_c} = \frac{1/\mu}{\sin 70°/\mu}$$

$$= \frac{1}{\sin 70°} = \frac{1}{0.9397} = 1.0642$$

or $$i_c = 46°47'$$

Hence $$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 46°47'}$$

$$= \frac{1}{0.7288} = \mathbf{1.372.}$$

Example 33. *For a situation shown in Fig. 9.43, find the maximum angle i for which the light suffers total internal reflection at the vertical surface.* [AIIMS 2015]

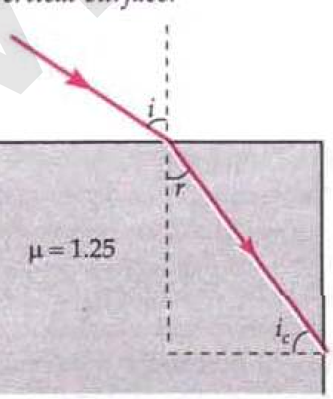


Fig. 9.43

Solution. The critical angle i_c is given by

$$\sin i_c = \frac{1}{\mu} = \frac{1}{1.25} = \frac{4}{5}$$

As $i_c + r = 90°$, therefore

$$\sin r = \sin (90° - i_c) = \cos i_c = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

From Snell's law,

$$\frac{\sin i}{\sin r} = 1.25$$

or $$\sin i = 1.25 \times \sin r = 1.25 \times \frac{3}{5} = 0.75$$

or $$i = 48.6°$$

If the angle of incidence at vertical surface is greater than i_c, then i will be less than 48.6°. Hence the maximum value of i, for which total internal reflection occurs at the vertical surface, is 48.6°.

Example 34. *A point source of light S is placed at the bottom of a vessel containing a liquid of refractive index 5/3. A person is viewing the source from above the surface. There is an opaque disc of radius 1 cm floating on the surface. The centre O of the disc lies vertically above the source S. The liquid from the vessel is gradually drained out through a tap. What is the maximum height of the liquid for which the source cannot be seen at all ?* [IIT]

Solution. As shown in Fig. 9.44, suppose the height $OS = h$ is such that $\angle OSA = i_c$. Then any other ray from S will be totally internally reflected because then the angle of incidence would be greater than i_c.

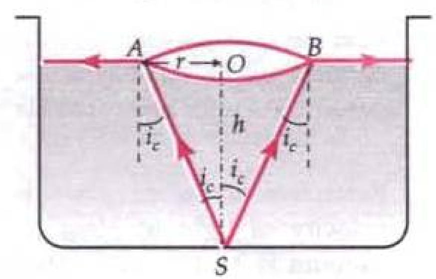


Fig. 9.44

In ΔOSA, $$\sin i_c = \frac{OA}{AS} = \frac{r}{\sqrt{r^2 + h^2}}$$

Also $$\sin i_c = \frac{1}{\mu}$$

$\therefore$ $$\frac{1}{\mu} = \frac{r}{\sqrt{r^2 + h^2}} \quad \text{or} \quad \frac{1}{\mu^2} = \frac{r^2}{r^2 + h^2}$$

$$r^2 + h^2 = \mu^2 r^2 \quad \text{or} \quad h^2 = r^2 (\mu^2 - 1)$$

$\therefore$ $$h = r\sqrt{\mu^2 - 1} = 1\sqrt{\left(\frac{5}{3}\right)^2 - 1}$$

$$= \frac{4}{3} = \mathbf{1.33\ cm.}$$

Example 35. *The refractive index of water is 4/3. Obtain the value of the semivertical angle of the cone within which the entire outside view would be confined for a fish under water. Draw an appropriate ray diagram.*

[CBSE Sample Paper 03]

Solution. Clearly, the fish can see the outside view of the cone with semivertical angle,

$$\frac{\theta}{2} = i_c$$

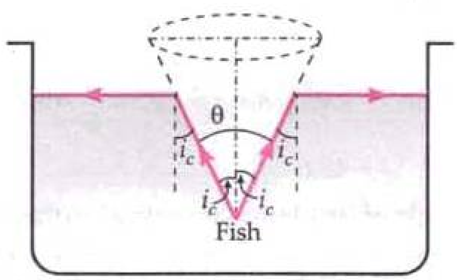


Fig. 9.45

But $\mu = \frac{1}{\sin i_c}$

or $\frac{4}{3} = \frac{1}{\sin i_c}$

or $\sin i_c = \frac{3}{4} = 0.75$

$\therefore \theta/2 = i_c = \sin^{-1}(0.75)$

$= 48.6°$

Problems For Practice

1. Find the critical angle for a ray of light going from paraffin oil to air. Given the refractive index of paraffin oil with respect to air is 1.44. **[Haryana 02]**

(Ans. 43.98°)

2. Refractive index of glass is 1.5. Calculate the velocity of light in glass if velocity of light in vacuum is 3×10^6 ms^{-1}. Also calculate the critical angle for glass-air interface. **[Haryana 98 ; ISCE 95]**

(Ans. 2×10^8 ms^{-1}, 41°49′)

3. An optical fibre (μ = 1.72) is surrounded by a glass coating (μ = 1.50). Find the critical angle for total internal reflection at the fibre-glass interface.

(Ans. 60.7°)

4. What is the small index of refraction of the material of a right-angled prism with equal sides for which a ray of light entering one of the sides normally will be totally reflected ? (Ans. 1.414)

5. Find the maximum angle of refraction when a ray of light is refracted from glass (μ = 1.5) to air.

(Ans. 90°)

6. Calculate the critical angle for glass-air surface, if a ray of light which is incident in air on the glass surface is deviated through 15°, when angle of incidence is 45°. **[CBSE OD 03]**

(Ans. 45°)

7. A luminous object *S* is located at the bottom of a big pool of liquid of refractive index μ and depth h. The object *S* emits rays upwards in all directions, so that a circle of light is formed at the surface of the liquid by the rays which are refracted into the air. What happens to the rays beyond the circle ? Determine the radius and the area of the circle.

(Ans. Rays beyond the circle are totally reflected into the liquid, Radius = $\frac{h}{\sqrt{\mu^2 - 1}}$, Area = $\frac{\pi h^2}{\mu^2 - 1}$).

8. A liquid of refractive index 1.5 is poured into a cylindrical jar of radius 20 cm upto a height of 20 cm. A small bulb is lighted at the centre of the bottom of the jar. Find the area of the liquid surface through which the light of the bulb passes into air. **[ISCE 98]**

(Ans. 1004.8 cm^2)

9. A point source of monochromatic light '*S*' is kept at the centre of the bottom of a cylinder of radius 15.0 cm. The cylinder contains water (refractive index 4/3) to a height of 7.0 cm. Draw the ray diagram and calculate the area of water surface through which the light emerges in air. **[CBSE D 15C]**

(Ans. 197.82 cm^2)

HINTS

1. $\sin i_c = \frac{1}{\mu} = \frac{1}{1.44} = 0.6944$

$\therefore$ i_c = **43.98°**.

2. $v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5}$

$= \mathbf{2 \times 10^8\ ms^{-1}}$.

Also, $\sin i_c = \frac{1}{\mu} = \frac{1}{1.5} = \frac{2}{3} = 0.6667$

$\therefore$ i_c = **41°49′**.

3. $\sin i_c = \frac{1.50}{1.72} = 0.8721$

$\therefore$ i_c = **60.7°**.

4. As shown in Fig. 9.46, the ray *PQ* enters through the side *AB* normally and is incident on *AC* at an angle of 45°. It will be totally reflected along *QR* if the critical angle for the material of the prism is 45°.

$$\therefore \quad \mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45^\circ} = \sqrt{2} = \mathbf{1.414}$$

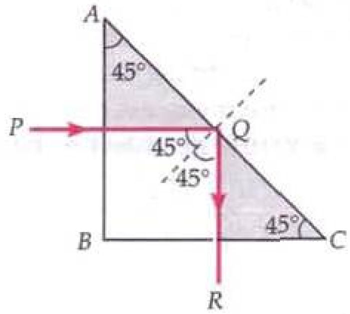


Fig. 9.46

5. When the ray of light is incident at the critical angle i_c, the angle of refraction is maximum and is equal to 90°

6. Here $i = 45^\circ$, $\quad r = 45^\circ - 15^\circ = 30^\circ$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1}{\sqrt{2}} \times \frac{2}{1} = \sqrt{2}$$

$$\therefore \ \sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{2}} \qquad \text{or} \qquad i_c = \mathbf{45^\circ}.$$

7. Refer to Fig. 9.47. The light rays emerge through a circle of radius r.

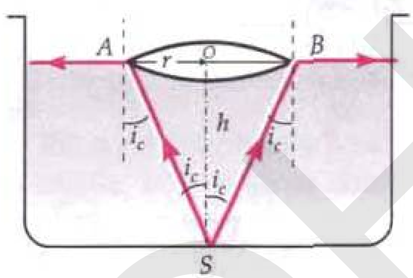


Fig. 9.47

$$\text{Radius,} \quad r = h \tan i_c = h \cdot \frac{\sin i_c}{\cos i_c}$$

$$= h \cdot \frac{\frac{1}{\mu}}{\sqrt{1 - \frac{1}{\mu^2}}} = \frac{h}{\sqrt{\mu^2 - 1}}$$

$$\text{Area of patch} = \pi r^2 = \frac{\pi h^2}{\mu^2 - 1}.$$

8. Area of the liquid surface through which light passes into air

$$= \frac{\pi h^2}{\mu^2 - 1}$$

$$= \frac{3.14 \times (20)^2}{(1.5)^2 - 1} = \frac{3.14 \times 400}{1.25} \text{ cm}^2$$

$$= \mathbf{1004.8 \ cm^2}$$

9. Refer to Fig. 9.47. The light rays emerge through a circle of radius r.

Radius, $r = h \tan i_c$

$$\sin i_c = \frac{1}{\mu} = \frac{3}{4}$$

$$\cos i_c = \sqrt{1 - \left(\frac{3}{4}\right)^2} = \frac{\sqrt{7}}{4}$$

$$\therefore \quad \tan i_c = \frac{3}{4} \times \frac{4}{\sqrt{7}} = \frac{3}{\sqrt{7}}$$

Area of the patch through which light emerges

$$= \pi r^2 = \pi h^2 \tan^2 i_c$$

$$= \pi \times (7)^2 \times \frac{9}{7} = 63\pi \text{cm}^2 = \mathbf{197.82 \ cm^2}.$$

9.19 SPHERICAL LENSES

33. *What is a lens ? Mention the different types of spherical lenses.*

Spherical lenses. Most of us are familiar with lenses. As magnifying glasses, lenses have been in use for centuries. Lenses used in spectacles help us to read with comfort. Various optical instruments like camera, projector, microscope, telescopes, etc., cannot function without lenses.

*A **lens** is a piece of a refracting medium bounded by two surfaces, at least one of which is a curved surface.*

The commonly used lenses are the spherical lenses. These lenses have either both surfaces spherical or one spherical and the other a plane one. Lenses can be divided into *two* categories :

(*i*) *Convex or converging lenses, and*

(*ii*) *Concave or diverging lenses.*

(*i*) **Convex or converging lens.** It is thicker at the centre than at the edges. It converges a parallel beam of light on refraction through it. It has a real focus.

Types of convex lenses :

(*a*) *Double convex or biconvex lens.* In this lens, both surfaces are convex.

(*b*) *Planoconcave lens.* In this lens, one side is convex and the other is plane.

(*c*) *Concavoconvex.* In this lens, one side is convex and the other is concave.

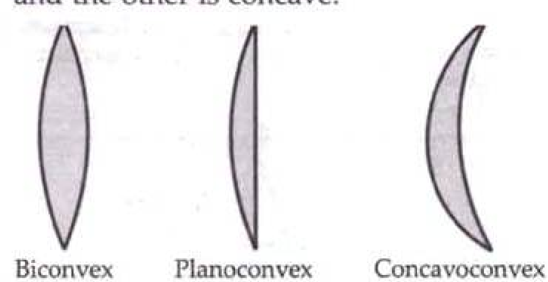


Fig. 9.48 Different types of convex lenses.

(*ii*) **Concave or diverging lens.** It is thinner at the centre than at the edges. It diverges a parallel beam of light on refraction through it. It has a virtual focus.

Types of concave lenses :

(*a*) *Double concave or biconcave lens.* In this lens, both sides are concave.

(*b*) *Planoconcave lens.* In this lens, one side is plane and the other is concave.

(*c*) *Convexoconcave lens.* In this lens, one side is convex and the other is concave.

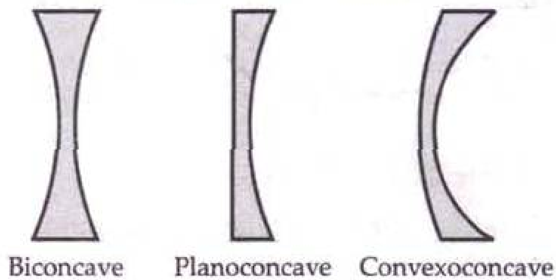


Fig. 9.49 Different types of concave lenses.

9.20 DEFINITIONS IN CONNECTION WITH SPHERICAL LENSES

34. *Define the various terms in connection with spherical lenses.*

Definitions in connection with spherical lenses :

(*i*) ***Centre of curvature* (C).** *The centre of curvature of the surface of a lens is the centre of the sphere of which it forms a part. Because a lens has two surfaces, so it has two centres of curvature.*

(*ii*) ***Radius of curvature* (R).** *The radius of curvature of the surface of a lens is the radius of the sphere of which the surface forms a part.*

(*iii*) ***Principal axis* (C_1C_2).** *It is the line passing through the two centres of curvature of the lens.*

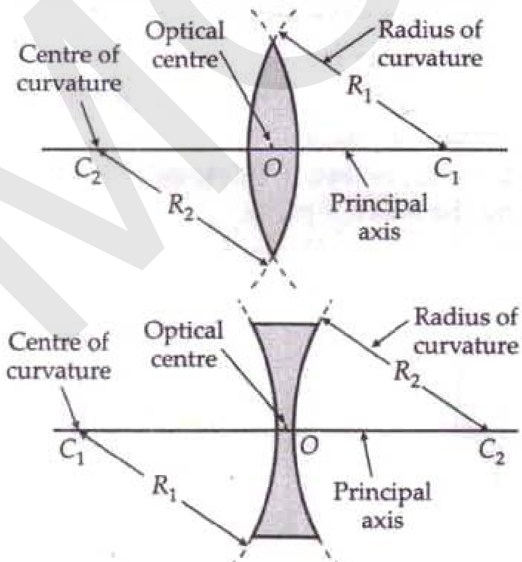


Fig. 9.50 Characteristics of convex and concave lenses.

(*iv*) ***Optical centre.*** *If a ray of light is incident on a lens such that after refraction through the lens the emergent ray is parallel to the incident ray, then the point at which the refracted ray intersects the principal axis is called the optical centre of the lens.* In Fig. 9.51(*a*), *O* is the optical centre of the lens. It divides the thickness of the lens in the ratio of the radii of curvature of its two surfaces. Thus :

$$\frac{OP_1}{OP_2} = \frac{P_1C_1}{P_2C_2} = \frac{R_1}{R_2}$$

If the radii of curvature of the two surfaces are equal, then the optical centre coincides with the geometric centre of the lens.

For the ray passing through the optical centre, the incident and emergent rays are parallel. However, the emergent ray suffers some lateral displacement relative to the incident ray. This lateral displacement decreases with the decrease in thickness of the lens. Hence a ray passing through the optical centre of a thin lens does not suffer any lateral deviation, as shown in Figs. 9.51(*b*) and (*c*).

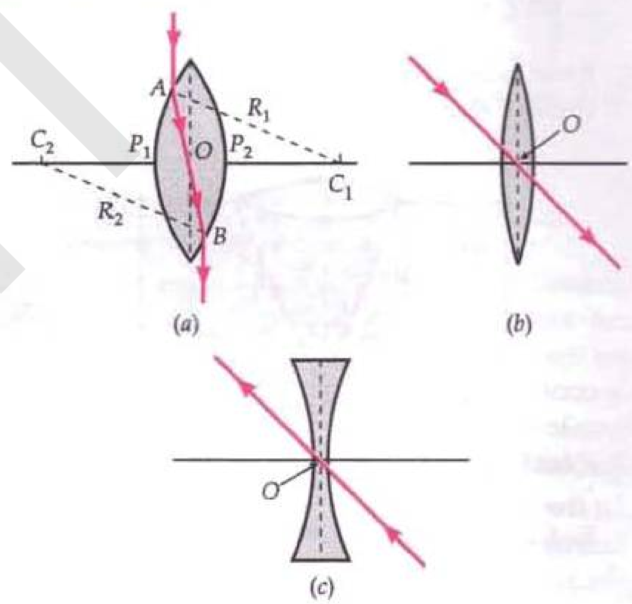


Fig. 9.51 Optical centre.

(*v*) ***Principal foci and focal length* :**

First principal focus. *It is a fixed point on the principal axis such that rays starting from this point (in convex lens)*

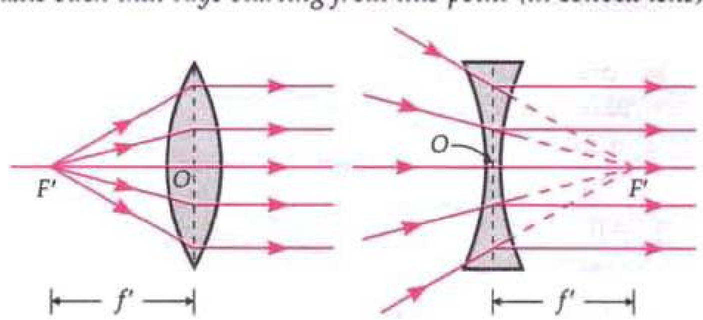


Fig. 9.52 First principal focus and first focal length.

or appearing to go towards this point (in *concave lens), after refraction through the lens, become parallel to the principal axis.* It is represented by F_1 or F'. The plane passing through this point and perpendicular to the principal axis is called the ***first focal plane.*** The distance between first principal focus and the optical centre is called the ***first focal length***. It is denoted by f_1 or f'.

Second principal focus. *It is a fixed point on the principal axis such that the light rays incident parallel to the principal axis, after refraction through the lens, either converge to this point (in convex lens) or appear to diverge from this point (in concave lens).* The plane passing through this point and perpendicular to principal axis is called the ***second focal plane***. The distance between the second principal focus and the optical centre is called the ***second focal length.*** It is denoted by f_2 or f.

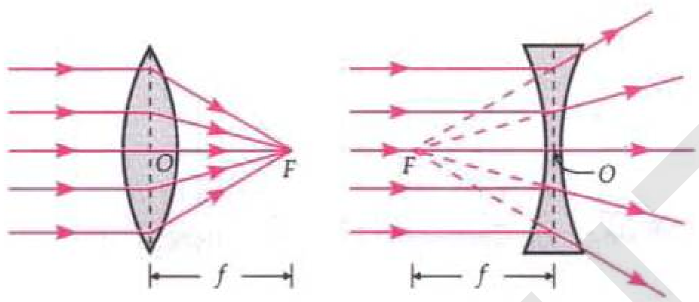


Fig. 9.53 Second principal focus and first focal length.

Generally, the focal length of a lens refers to its second focal length. It is obvious from the above figures that the foci of a convex lens are *real* and those of a concave lens are *virtual*. Thus the *focal length of a convex lens is taken positive and the focal length of a concave lens is taken negative.*

If the medium on both sides of a lens is same, then the numerical values of the first and second focal lengths are equal. Thus

$$f = f'.$$

(*vi*) ***Aperture.*** It is the diameter of the circular boundary of the lens.

9.21 NEW CARTESIAN SIGN CONVENTION FOR SPHERICAL LENSES

35. *State the new Cartesian sign convention for spherical lenses. What are the important consequences of this sign convention ?*

New Cartesian sign convention for spherical lenses.

1. All distances are measured from the *optical centre of the lens.*
2. The distances measured in the *same direction* as the incident light are taken *positive.*
3. The distances measured in the direction *opposite to the direction* of the incident light are taken *negative.*
4. Heights measured *upwards* and perpendicular to the principal axis are taken *positive.*
5. Heights measured *downwards* and perpendicular to the principal axis are taken *negative.*

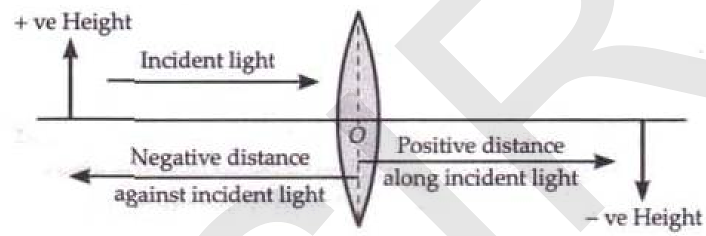


Fig. 9.54 New Cartesian sign convention for a spherical lens.

Consequences of the sign convention :

1. *The focal length of a converging lens is positive and that of a diverging lens is negative.*
2. *Object distance is always negative.*
3. The distance of *real image* is *positive* and that of *virtual image is negative.*
4. The object height h_1 is always positive. Height h_2 of virtual erect image is positive and that of real inverted image is negative.
5. The linear magnification $m = h_2 / h_1$ is positive for a virtual image and negative for a real image.

Before deriving formulae for spherical lenses, we first consider refraction by a single spherical surface.

9.22 REFRACTION AT A CONVEX SPHERICAL SURFACE

36. *By stating the sign-convention and assumptions used, derive the relation between the distance of object, distance of image and radius of curvature of a convex spherical surface, when refraction takes place*

(*i*) *from optically rarer to optically denser medium and the image formed is real,*

(*ii*) *from optically rarer to optically denser medium and the image formed is virtual,*

(*iii*) *from optically denser to optically rarer medium and the image formed is real, and*

(*iv*) *from optical denser to optically rarer medium and the image formed is virtual.*

New Cartesian sign convention for refraction at a spherical surface.

1. All distances are measured from the pole of the spherical surface.

2. The distances measured in the direction of incident light are positive.
3. The distances measured in the opposite direction of incident light are negative.

Assumptions used in the study of refraction at a spherical surface :

1. The object taken is a point object placed on the principal axis.
2. The aperture of the spherical refracting surface is small.
3. The incident and refracted rays make small angles with the principal axis so that the sines or tangents of these angles may be taken equal to the angles themselves.

Refraction at a convex spherical surface :

(*i*) ***The object lies in rarer medium and the image formed is real.*** In Fig. 9.55, *APB* is a convex refracting surface which separates a rarer medium of refractive index μ_1 from a denser medium of refractive index μ_2. Let *P* be the pole, *C* be the centre of curvature and $R = PC$ be the radius of curvature of this surface. Suppose a point object *O* is placed on the principal axis in the rarer medium. Starting from the point object *O*, a ray *ON* is incident at an angle *i*. After refraction, it bends towards the normal *CN* at an angle of refraction *r*. Another ray *OP* is incident normally on the convex surface and passes undeviated. The two refracted rays meet at point *I*. So *I* is the real image of point object *O*.

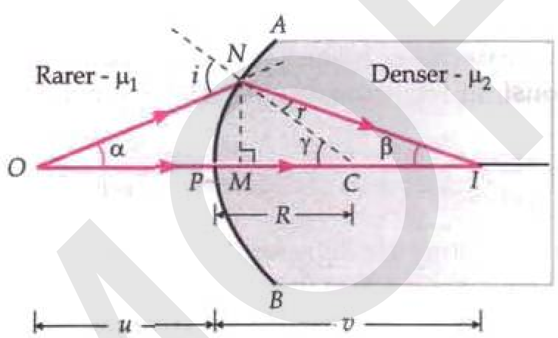


Fig. 9.55 Refraction from rarer to denser medium, when the image is real.

Draw *NM* perpendicular to the principal axis. Let α, β and γ be the angles, as shown in Fig. 9.55.

In ΔNOC, *i* is an exterior angle, therefore,

$$i = \alpha + \gamma$$

Similarly, from $\Delta\, NIC$, we have

$$\gamma = r + \beta$$

or $$r = \gamma - \beta$$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{OP} \qquad [\because\ P \text{ is close to } M]$$

$$\beta \simeq \tan\beta = \frac{NM}{MI} \simeq \frac{NM}{PI}$$

and $$\gamma \simeq \tan\gamma = \frac{NM}{MC} \simeq \frac{NM}{PC}$$

From Snell's law of refraction,

$$\mu_1 \sin i = \mu_2 \sin r$$

As *i* and *r* are small, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_1 i = \mu_2 r$$

or $$\mu_1[\alpha + \gamma] = \mu_2[\gamma - \beta]$$

or $$\mu_1\left[\frac{NM}{OP} + \frac{NM}{PC}\right] = \mu_2\left[\frac{NM}{PC} - \frac{NM}{PI}\right]$$

or $$\mu_1\left[\frac{1}{OP} + \frac{1}{PC}\right] = \mu_2\left[\frac{1}{PC} - \frac{1}{PI}\right]$$

or $$\frac{\mu_1}{OP} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Using new Cartesian sign convention, we find

Object distance, $OP = -u$

Image distance, $PI = +v$

Radius of curvature, $PC = +R$

$$\therefore \quad \frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

or $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

NOTE If first medium is air, then $\mu_1 = 1$ and $\mu_2 = \mu$, we have

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}.$$

(*ii*) ***The object lies in the rarer medium and the image formed is virtual.*** When the object *O* in the

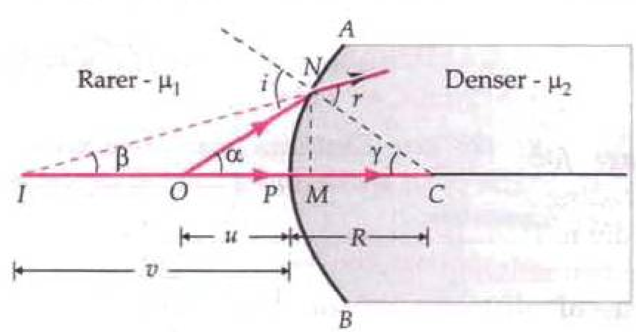


Fig. 9.56 Refraction from rarer to denser medium, when the image is virtual.

rarer medium lies close to the pole P of the convex refracting surface, the two refracted rays appear to diverge from a point I on the principal axis, as shown in Fig. 9.56. So I is the virtual image of the point object O.

From ΔNOC, $\quad i=\alpha+\gamma$

From ΔNCI, $\quad r=\beta+\gamma$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small

$$\therefore \quad \alpha \simeq \tan\alpha=\frac{NM}{OM}=\frac{NM}{OP} \qquad [\because\ M \text{ is close to } P]$$

$$\beta \simeq \tan\beta=\frac{NM}{IM}=\frac{NM}{IP}$$

and $$\gamma \simeq \tan\gamma=\frac{NM}{MC}=\frac{NM}{PC}$$

From Snell's law of refraction,

$$\mu_1 \sin i=\mu_2 \sin r$$

As i and r are small, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_1 i=\mu_2 r$$

or $$\mu_1(\alpha+\gamma)=\mu_2(\beta+\gamma)$$

or $$\mu_1\left[\frac{NM}{OP}+\frac{NM}{PC}\right]=\mu_2\left[\frac{NM}{IP}+\frac{NM}{PC}\right]$$

or $$\mu_1\left[\frac{1}{OP}+\frac{1}{PC}\right]=\mu_2\left[\frac{1}{IP}+\frac{1}{PC}\right]$$

or $$\frac{\mu_1}{OP}-\frac{\mu_2}{IP}=\frac{\mu_2-\mu_1}{PC}$$

Using new Cartesian sign convention, we find that

Object distance, $\quad OP=-u$

Image distance, $\quad IP=-v$

Radius of curvature, $\quad PC=+R$

$$\therefore \quad \frac{\mu_1}{-u}-\frac{\mu_2}{-v}=\frac{\mu_2-\mu_1}{R}$$

or $$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

(iii) ***The object lies in the denser medium and the image formed is real.*** Fig. 9.57 shows a convex refracting surface which is convex towards the rarer medium. The point object O lies in the denser medium. The two refracted rays meet at point I. So I is the real image of the point object O.

From ΔNOC, $\gamma=i+\alpha$ or $i=\gamma-\alpha$

From ΔNIC, $\quad r=\beta+\gamma$

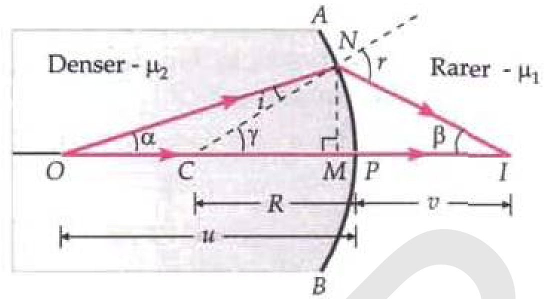


Fig. 9.57 Refraction from denser to rarer medium when the image is real.

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha=\frac{NM}{OM}=\frac{NP}{OP} \qquad [\because\ M \text{ is close to } P]$$

$$\beta \simeq \tan\beta=\frac{NM}{MI}=\frac{NM}{PI}$$

and $$\gamma \simeq \tan\gamma=\frac{NM}{CM}=\frac{NM}{CP}$$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\mu_2 \sin i=\mu_1 \sin r$$

As i and r are small angles, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_2 i=\mu_1 r$$

or $$\mu_2(\gamma-\alpha)=\mu_1(\beta+\gamma)$$

or $$\mu_2\left[\frac{NM}{CP}-\frac{NM}{OP}\right]=\mu_1\left[\frac{NM}{PI}+\frac{NM}{CP}\right]$$

or $$\mu_2\left[\frac{1}{CP}-\frac{1}{OP}\right]=\mu_1\left[\frac{1}{PI}+\frac{1}{CP}\right]$$

or $$-\frac{\mu_1}{PI}-\frac{\mu_2}{OP}=\frac{\mu_1-\mu_2}{CP}$$

Using the new Cartesian sign convention, we have

Object distance, $\quad OP=-u$

Image distance, $\quad PI=+v$

Radius of curvature, $\quad CP=-R$

$$\therefore \quad -\frac{\mu_1}{v}-\frac{\mu_2}{-u}=\frac{\mu_1-\mu_2}{-R}$$

$$\frac{\mu_1}{v}-\frac{\mu_2}{u}=\frac{\mu_1-\mu_2}{R}$$

(iv) ***The object lies in the denser medium and the image formed is virtual.*** If the point object O placed on the principal axis lies close to the pole of the refracting

surface, then the two refracted rays appear to come from the point I, as shown in Fig. 9.58. So I is the virtual image of the point object O.

From ΔNOC, $i+\gamma=\alpha$ or $i=\alpha-\gamma$

From ΔNIC, $r+\gamma=\beta$ or $r=\beta-\gamma$

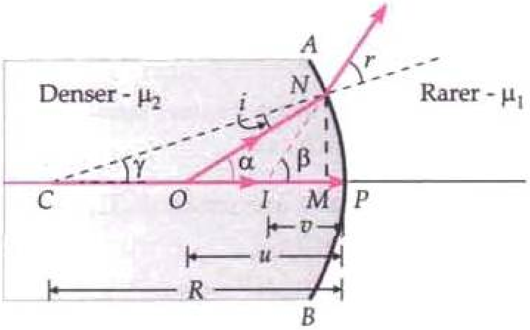


Fig. 9.58 Refraction from denser to rarer medium when the image is virtual.

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} = \frac{NM}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan\beta = \frac{NM}{IM} = \frac{NM}{IP}$$

$$\gamma \simeq \tan\gamma = \frac{NM}{CM} = \frac{NM}{CP}$$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\mu_2 \sin i = \mu_1 \sin r$$

As i and r are small angles, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_2 i = \mu_1 r$$

or $$\mu_2(\alpha-\gamma) = \mu_1(\beta-\gamma)$$

or $$\mu_2\left[\frac{NM}{OP}-\frac{NM}{CP}\right] = \mu_1\left[\frac{NM}{IP}-\frac{NM}{CP}\right]$$

or $$\mu_2\left[\frac{1}{OP}-\frac{1}{CP}\right] = \mu_1\left[\frac{1}{IP}-\frac{1}{CP}\right]$$

or $$-\frac{\mu_1}{IP}+\frac{\mu_2}{OP} = -\frac{\mu_1-\mu_2}{CP}$$

Using the new Cartesian sign convention, we have

Object distance, $OP=-u$

Image distance, $IP=-v$

Radius of curvature, $CP=-R$

$$\therefore \quad -\frac{\mu_1}{-v}+\frac{\mu_2}{-u} = -\frac{\mu_1-\mu_2}{-R}$$

or $$\frac{\mu_1}{v}-\frac{\mu_2}{u} = \frac{\mu_1-\mu_2}{R}$$

9.23 REFRACTION AT A CONCAVE SPHERICAL SURFACE

37. *By stating the sign conventions and assumptions used, derive the relation between object distance, image distance and radius of curvature of a concave spherical surface when the refraction takes place (i) from optically rarer to optically denser medium and (ii) from optically denser to optically rarer medium.*

For new Cartesian sign convention and the assumption used, refer to the answer of the previous question.

Refraction at a concave spherical surface.

(*i*) ***The object lies in the rarer medium.*** In Fig. 9.59, APB is a concave refracting surface separating two media of refractive indices μ_1 and μ_2.

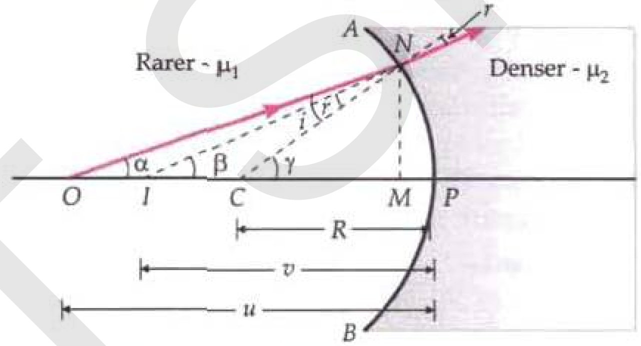


Fig. 9.59 Refraction at a concave surface when the object lies in the rarer medium.

Let

P = Pole of the concave surface APB

C = Centre of curvature of the concave surface

O = Point object placed on the principal axis

I = Virtual image of point object O

In ΔNOC, γ is an exterior angle, therefore

$$\gamma = \alpha + i \quad \text{or} \quad i = \gamma - \alpha$$

Similarly, from $\Delta\, NIC$, we have

$$\gamma = \beta + r \quad \text{or} \quad r = \gamma - \beta$$

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan\beta = \frac{NM}{IM} \simeq \frac{NM}{IP}$$

$$\gamma \simeq \tan\gamma = \frac{NM}{CM} \simeq \frac{NM}{CP}$$

From Snell's law of refraction,

$$\mu_1 \sin i = \mu_2 \sin r$$

As i and i are small angles, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_1 i = \mu_2 r$$

or $$\mu_1[\gamma-\alpha]=\mu_2[\gamma-\beta]$$

or $$\mu_1\left[\frac{NM}{CP}-\frac{NM}{OP}\right]=\mu_2\left[\frac{NM}{CP}-\frac{NM}{IP}\right]$$

or $$\mu_1\left[\frac{1}{CP}-\frac{1}{OP}\right]=\mu_2\left[\frac{1}{CP}-\frac{1}{IP}\right]$$

or $$-\frac{\mu_1}{OP}+\frac{\mu_2}{IP}=\frac{\mu_2-\mu_1}{CP}$$

Using new Cartesian sign convention, we find

Object distance, $OP=-u$

Image distance, $IP=-v$

Radius of curvature, $CP=-R$

$$\therefore \quad \frac{-\mu_1}{-u}+\frac{\mu_2}{-v}=\frac{\mu_2-\mu_1}{-R}$$

or $$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

(*ii*) ***The object lies in the denser medium.*** As shown in Fig. 9.60, when the point object O is placed in the denser medium, the refracted rays appear to diverge from a point I in the denser medium. So I is the virtual image of the point object O.

From ΔNOC, $i=\alpha+\gamma$

From ΔNIC, $r=\beta+\gamma$

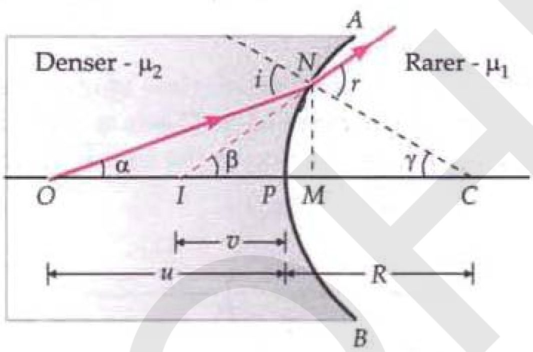


Fig. 9.60 Refraction at a concave surface when the object lies in the denser medium.

Suppose all the rays are *paraxial*. Then the angles i, r, α, β and γ will be small.

$$\therefore \quad \alpha \simeq \tan\alpha=\frac{NM}{OM}=\frac{NM}{OP} \qquad [\because M \text{ is close to } P]$$

$$\beta \simeq \tan\beta=\frac{NM}{IM}=\frac{NM}{IP}$$

$$\gamma \simeq \tan\gamma=\frac{NM}{MC}=\frac{NM}{PC}$$

From Snell's law of refraction, for refraction from denser to rarer medium, we have

$$\mu_2 \sin i=\mu_1 \sin r$$

As i and r are small angles, so

$$\sin i \simeq i \quad \text{and} \quad \sin r \simeq r$$

$$\therefore \quad \mu_2 i=\mu_1 r$$

or $$\mu_2[\alpha+\gamma]=\mu_1[\beta+\gamma]$$

or $$\mu_2\left[\frac{NM}{OP}+\frac{NM}{PC}\right]=\mu_1\left[\frac{NM}{IP}+\frac{NM}{PC}\right]$$

or $$\mu_2\left[\frac{1}{OP}+\frac{1}{PC}\right]=\mu_1\left[\frac{1}{IP}+\frac{1}{PC}\right]$$

or $$-\frac{\mu_1}{IP}+\frac{\mu_2}{OP}=\frac{\mu_1-\mu_2}{PC}$$

Using new Cartesian sign convention, we find

Object distance, $OP=-u$

Image distance, $IP=-v$

Radius of curvature, $PC=+R$

$$\therefore \quad \frac{-\mu_1}{-v}+\frac{\mu_2}{-u}=\frac{\mu_1-\mu_2}{R}$$

or $$\frac{\mu_1}{v}-\frac{\mu_2}{u}=\frac{\mu_1-\mu_2}{R}$$

For Your Knowledge

- For both convex and concave spherical surfaces, the refraction formulae are same, only proper signs of u, v and R are to be used.
- For refraction *from rarer to denser medium*, the refraction formula is
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R} \qquad ...(1)$$
- For refraction *from denser to rarer medium*, we interchange μ_1 and μ_2 and obtain the refraction formula,
$$\frac{\mu_1}{v}-\frac{\mu_2}{u}=\frac{\mu_1-\mu_2}{R} \qquad ...(2)$$
- If the rarer medium is air ($\mu_1=1$) and the denser medium has refractive index μ (*i.e.*, $\mu_2=\mu$), then for refraction from rarer to denser medium, from (1) we get the relation :
$$\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{R} \qquad ...(3)$$
For refraction from denser to rarer medium, we put $\mu_1/\mu_2=1/\mu$ in (2) and get the relation :
$$\frac{1/\mu}{v}-\frac{1}{u}=\frac{(1/\mu)-1}{R} \qquad ...(4)$$
- For an object placed in air, the refraction formula (3) is applicable, *i.e.*, $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{R}$.
As R is positive for a convex surface, v will be negative if the value of u is less than $R/(\mu-1)$. In that case, the image will be formed in air and will be virtual.
As R is negative for a concave surface, the value of v will also be negative for all negative values of u. Thus image will always be formed in air and will be virtual.
- The factor $\frac{\mu_2-\mu_1}{R}$ is called ***power factor*** of the spherical refracting surface. It gives a measure of the degree to which the refracting surface can converge or diverge the rays of light passing through it.

Examples based on Refraction through Spherical Surfaces

Formulae Used

1. For refraction from rarer to denser medium,
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$
2. For refraction from denser to rarer medium,
$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$
3. Power of a surface,
$$P = \frac{\mu_2 - \mu_1}{R} = \frac{\mu - 1}{R} \quad \text{(For air)}$$
4. First principal focal length, $f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$
5. Second principal focal length, $f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$

$$\therefore \quad \frac{f_2}{f_1} = -\frac{\mu_2}{\mu_1}$$

Units Used

Distances u, v, f and R are in metre, power P is in dioptre (D), refractive indices μ_1, μ_2 and μ have no units.

Example 36. *Light from a point source in air falls on a convex spherical glass surface ($\mu = 1.5$, radius of curvature $= 20$ cm). The distance of light source from the glass surface is 100 cm. At what position is the image formed ?* *[NCERT]*

Solution. Here $\mu_1 = 1$, $\mu_2 = 1.5$, $u = -100$ cm

$R = +20$ cm,

[R is +ve for a convex refracting surface]

As $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\therefore \quad \frac{1.5}{v} + \frac{1}{100} = \frac{1.5 - 1}{20} = \frac{1}{40}$$

or
$$\frac{3}{2v} = \frac{1}{40} - \frac{1}{100} = \frac{5 - 2}{200} = \frac{3}{200}$$

$$\therefore \quad v = +\mathbf{100\ cm}$$

Thus the image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

Example 37. *A glass dumbbell of length 30 cm and refractive index 1.5 has ends of 3 cm radius of curvature. Find the position of the image formed due to refraction at one end only, when the object is situated in air at a distance of 12 cm from the end of the dumbbell along the axis.*

Solution. Refraction occurs from air to glass at convex spherical surface APB.

Therefore,

$$u = -12 \text{ cm}, \quad R = +3 \text{ cm}, \quad \mu_1 = 1, \quad \mu_2 = 1.5$$

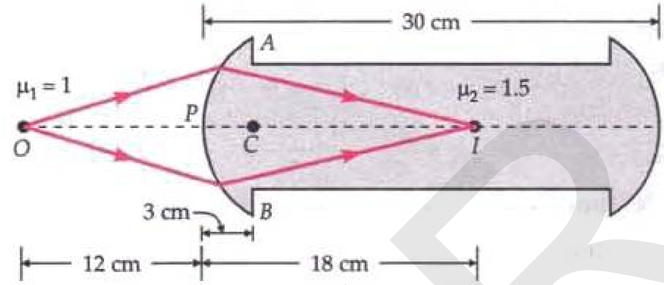


Fig. 9.61

As
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\therefore \quad \frac{1.5}{v} + \frac{1}{12} = \frac{1.5 - 1}{3} = \frac{0.5}{3}$$

or
$$\frac{1.5}{v} = \frac{0.5}{3} - \frac{1}{12} = \frac{2 - 1}{12} = \frac{1}{12}$$

or
$$v = 1.5 \times 12 = \mathbf{18\ cm}$$

As v is positive, so a real image is formed at 18 cm from the end P of the dumbbell.

Example 38. *The diameter of a glass sphere is 15 cm. A beam of light strikes the sphere, which converges at point 30 cm behind the pole of the spherical surface. Find the position of the image if $\mu = 1.5$.*

Solution. In the absence of glass sphere, the light rays will converge at point O. So O acts as virtual object for the image I for refraction at the first surface.

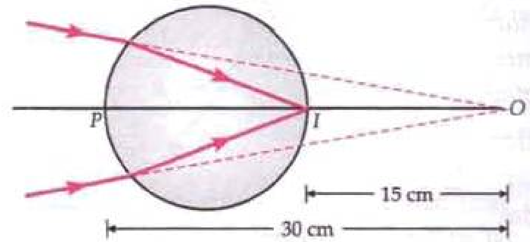


Fig. 9.62

$$\therefore \quad u = PO = +30 \text{ cm},$$
$$\mu_1 = 1, \quad \mu_2 = 1.5$$
$$R = +\frac{15}{2} = +7.5 \text{ cm}$$

As the light passes from rarer to denser medium, so
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or
$$\frac{1.5}{v} - \frac{1}{30} = \frac{1.5 - 1}{7.5} = \frac{0.5}{7.5} = \frac{1}{15}$$

or
$$\frac{1.5}{v} = \frac{1}{15} + \frac{1}{30} = \frac{1}{10}$$

or
$$v = +10 \times 1.5 = +\mathbf{15\ cm.}$$

Thus the image is formed at the other end (I) of the diameter.

Example 39. *What curvature must be given to the bounding surface of $\mu = 1.5$ for virtual image of an object in the medium of $\mu = 1$ at 10 cm to be formed at a distance of 40 cm. Also calculate power of the surface and two principal focal lengths of the surface.*

Solution. Here $u = -10$ cm, $v = -40$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$

As the object is placed in the rarer medium, so

$$\frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$$

or $$\frac{1.5 - 1}{R} = \frac{1.5}{-40} + \frac{1}{10} = \frac{2.5}{40} = \frac{1}{16}$$

or $$R = 16 \times 0.5 = \mathbf{8\ cm}$$

As R is positive, the refracting surface is convex.

Power of surface,

$$P = \frac{\mu_2 - \mu_1}{R} = \frac{1.5 - 1}{8 \text{ cm}} = \frac{0.5}{0.08 \text{ m}} = \mathbf{6.25\ D}$$

First principal focal length,

$$f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1} = \frac{1 \times 8}{0.5} = \mathbf{-16\ cm}$$

Second principal focal length,

$$f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1} = \frac{1.5 \times 8}{0.5} = \mathbf{24\ cm}$$

Example 40. *A mark placed on the surface of a glass sphere is viewed through glass from an oppositely directed position. If the diameter of the sphere is 20 cm; find the position of the image. Refractive index of glass is 1.5.*

Solution. Figure 9.63 shows a glass sphere of radius 10 cm. The mark O on its surface acts as object. The incident ray OA is in glass and refracted ray AB is in air. I is the image of O. Thus

$\mu_1 = 1$, $\mu_2 = 1.5$, $u = OP = -20$ cm

$R = -10$ cm [Minus sign taken for refraction at concave surface]

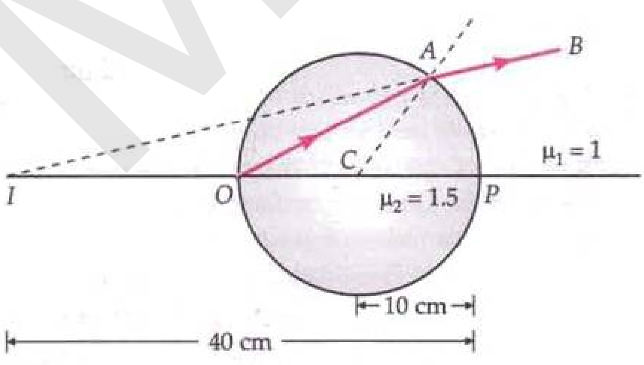


Fig. 9.63

As light passes from denser to rarer medium, so

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

or $$\frac{1}{v} + \frac{1.5}{20} = \frac{1 - 1.5}{-10}$$

or $$\frac{1}{v} = \frac{1}{20} - \frac{3}{40} = \frac{2 - 3}{40} = -\frac{1}{40}$$

or $$v = -40 \text{ cm}$$

Negative sign shows that the image is virtual. It is formed on the same side of the refracting surface as the object at a distance of 40 cm from the pole P.

Example 41. *A small air bubble in a glass sphere of radius 2 cm appears to be 1 cm from the surface when looked at, along a diameter. If the refractive index of glass is 1.5, find the true position of the air bubble.*

Solution. Here incident ray OA is in glass and refracted ray AB is in air. I is the final image of the air bubble at O.

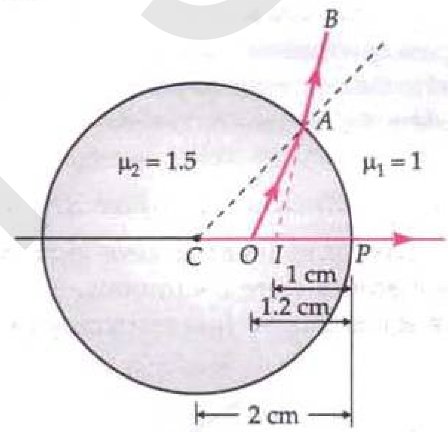


Fig. 9.64

Here $\mu_1 = 1$, $\mu_2 = 1.5$, $v = -1$ cm, $R = -2$ cm

As $$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

$$\therefore \quad \frac{1}{-1} - \frac{1.5}{u} = \frac{1 - 1.5}{-2} = \frac{1}{4} \quad \text{or} \quad -\frac{1.5}{u} = \frac{1}{4} + 1 = \frac{5}{4}$$

or $$u = -\frac{1.5 \times 4}{5} = \mathbf{-1.2\ cm}$$

Thus the air bubble O lies at 1.2 cm from the refracting surface within the sphere.

Example 42. *An empty spherical flask of diameter 15 cm is placed in water of refractive index $\frac{4}{3}$. A parallel beam of light strikes the flask. Where does it get focussed, when observed from within the flask ?*

Solution. Figure 9.65 shows a spherical flask placed inside water. The centre of the flask is the centre of curvature of the spherical refracting surface. A parallel beam of light falling on the flask diverges and appears to come from point I.

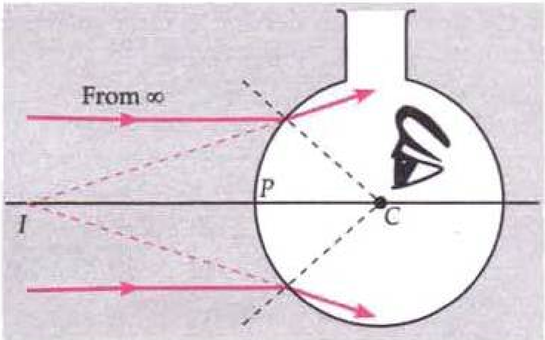


Fig. 9.65

Here $\mu_1 = 1, \quad \mu_2 = \frac{4}{3}, \quad u = -\infty, \quad R = +\frac{15}{2}$ cm

As the light travels from denser to rarer medium, so

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \quad \text{or} \quad \frac{1}{v} - \frac{4/3}{-\infty} = \frac{1 - 4/3}{15/2}$$

or $$v = \frac{-45}{2} = -22.5 \text{ cm.}$$

Example 43. *A sunshine recorder globe of 30 cm diameter is made of glass of refractive index $\mu = 1.5$. A ray enters the globe parallel to the axis. Find the position from the centre of the sphere where the ray crosses the axis.*

[CBSE Sample Paper 98]

Solution. ***For refraction at surface AP_1.*** The ray SA parallel to the axis is incident on glass globe at point A. If the glass medium were continuous, it would have met the axis at point I_1. So I_1 is real image of the object at infinity.

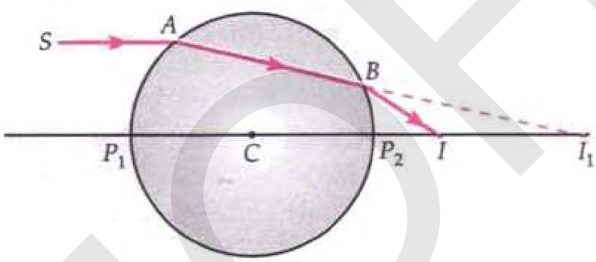


Fig. 9.66

$\therefore \mu_1 = 1, \quad \mu_2 = 1.5, \quad u = -\infty, \quad R = +\frac{30}{2} = 15$ cm

Let $P_1I_1 = v'$. As refraction takes place from rarer to denser medium, we use the relation

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v'} - \frac{1}{-\infty} = \frac{1.5 - 1}{15} \quad \text{or} \quad v' = \frac{1.5 \times 15}{0.5} = 45 \text{ cm}$$

For refraction at surface BP_2. The ray AB (before meeting point I_1) suffers another refraction at surface BP_2. The real image I_1 acts as virtual object for refraction at surface BP_2 and I is the real image.

$$\therefore \quad u = P_2I_1 = P_1I_1 - P_1P_2 = 45 - 30 = 15 \text{ cm}$$
$$R = -15 \text{ cm}$$

Let $P_2I = v$. As refraction occurs from denser to rarer medium, so we use the relation

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

$$\frac{1}{v} - \frac{1.5}{15} = \frac{1 - 1.5}{-15} \quad \text{or} \quad \frac{1}{v} = \frac{2}{15} \quad \text{or} \quad v = 7.5 \text{ cm}$$

Distance of image I from the centre of the sphere is

$$CI = CP_2 + P_2I = 15 + 7.5 = \mathbf{22.5 \text{ cm}}.$$

Problems For Practice

1. A convex refracting surface of radius of curvature 20 cm separates two media of refractive indices 4/3 and 1.60. An object is placed in the first medium ($\mu = 4/3$) at a distance of 200 cm from the refracting surface. Calculate the position of the image formed. **[Punjab 01]**

   (Ans. At 240 cm in denser medium)

2. One end of a cylindrical rod is grounded to a hemispherical surface of radius $R = 20$ mm. It is immersed in water of refractive index 1.33. If the refractive index of the rod is 1.50, find the position of the image of an object placed on the axis of the rod inside water at 10 cm from the pole.

   (Ans. Virtual image at 31.25 cm from pole and inside water)

3. A concave spherical surface of refractive index 3/2 is immersed in water of refractive index 4/3. If a point object lies in water at a distance of 10 cm from the pole of the refracting surface, calculate the position of the image. Given that radius of curvature of the spherical surface is 18 cm.

   (Ans. Virtual image at 10.52 cm from pole and inside the water)

4. A mark placed on the surface of glass sphere is viewed through glass from a position directly opposite. If the diameter of the sphere is 10 cm and refractive index of glass is 1.5, find the position of the image. (Ans. 20 cm towards mark from the surface opposite to mark)

5. A glass sphere of radius 15 cm has a small bubble 6 cm from its centre. The bubble is viewed along a diameter of the sphere from the side on which it lies. How far from the surface will it appear to be if the refractive index of glass is 1.5 ?

   (Ans. Virtual image is seen at 7.5 cm from the spherical surface)

6. An object is placed 50 cm from the surface of a glass sphere of radius 10 cm along the diameter. Where

will the final image be formed after refraction at both the surfaces ? μ of glass = 1.5.

(Ans. At 20 cm from the centre of the sphere)

7. A spherical surface of radius 30 cm separates two transparent media A and B with refractive indices 1.33 and 1.48 respectively. The medium A is on the convex side of the surface. Where should a point object be placed in medium A so that the paraxial rays become parallel after refraction at the surface ?

(Ans. At 266 cm from the pole)

8. Fig. 9.67 shows a small air bubble inside a glass sphere ($\mu = 1.5$) of radius 10 cm. The bubble is 4.0 cm below the surface and is viewed normally from the outside. Find the apparent depth of the bubble. (Ans. 3 cm below the surface)

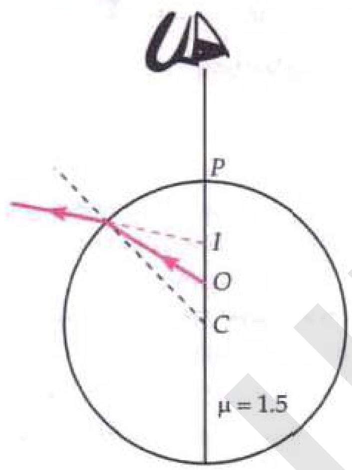


Fig. 9.67

HINTS

1. Here $\mu_1 = 4/3, \mu_2 = 1.60, u = -200$ cm, $R = +20$ cm

As $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\therefore \quad \frac{1.60}{v} - \frac{4/3}{-200} = \frac{1.60 - 4/3}{20}$$

or $v = +$ **240 cm**, in denser medium.

2. Here $\mu_1 = 1.33,\ \mu_2 = 1.5,\ u = -10$ cm,

$R = +20$ mm $= +2$ cm

As the light travels from rarer to denser medium, so

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or $$\frac{1.5}{v} + \frac{1.33}{10} = \frac{1.5 - 1.33}{2} = \frac{17}{200}$$

or $$\frac{1.5}{v} = \frac{17}{200} - \frac{133}{1000} = \frac{85 - 133}{1000} = -\frac{48}{1000}$$

or $$v = -\frac{1.5 \times 1000}{48} = -\mathbf{31.25\ cm}$$

The negative sign shows that image is virtual.

3. Here $u = -10$ cm, $\mu_1 = 4/3, \mu_2 = 3/2$, $R = -18$ cm

As $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \therefore \quad \frac{3/2}{v} - \frac{4/3}{-10} = \frac{3/2 - 4/3}{-18}$

or $v = -$**10.52 cm**, in water.

4. Here $u = -10$ cm, $R = -5$ cm, $\mu_1 = 1, \mu_2 = 1.5, v = ?$

As refraction occurs from denser to rarer medium at concave surface, so

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \quad \text{or} \quad \frac{1}{v} + \frac{1.5}{10} = \frac{1 - 1.5}{-5} = \frac{1}{10}$$

On solving, $v = -$ **20 cm.**

5. Refer to Fig. 9.68. Here $u = PO = -9$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$, $v = ?$

Use formula for refraction from denser to rarer medium.

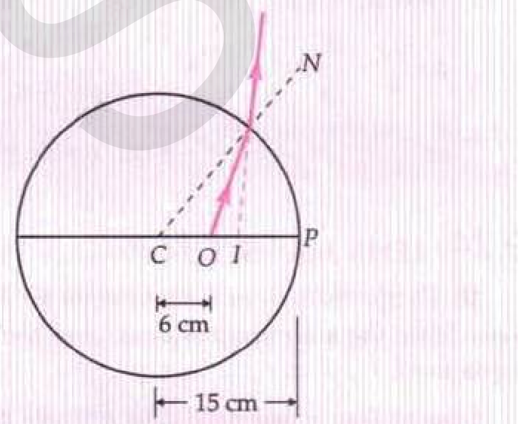


Fig. 9.68

6. Refer to Fig. 9.69. *For refraction at first face* AP_1.

$u = -50$ cm, $R = +10$ cm, $\mu_1 = 1, \mu_2 = 1.5, v' = ?$

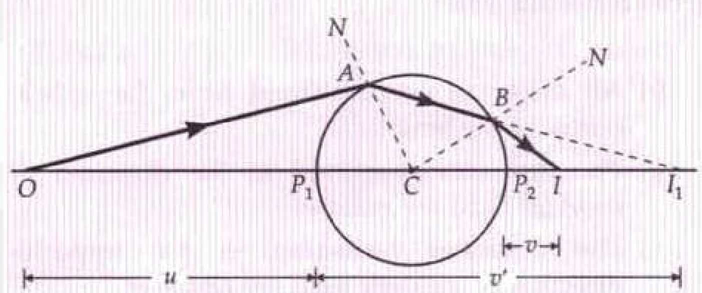


Fig. 9.69

As refraction occurs from rarer to denser medium,

so $$\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{or} \quad \frac{1.5}{v'} + \frac{1}{50} = \frac{1.5 - 1}{10} = \frac{0.5}{10}$$

On solving, $v' = 50$ cm.

For refraction at second surface BP_2. The real image I_1 acts as virtual object.

$u = I_1P_2 = v' - 2R = 50 - 20 = 30$ cm,

$v = ?,\ R = -10$ cm.

As refraction occurs from denser to rarer medium at concave surface, so

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \quad \text{or} \quad \frac{1}{v} - \frac{1.5}{30} = \frac{1-1.5}{-10}$$

On solving, $v = 10$ cm

Distance of final image from the centre of the sphere = 10 + 10 = **20 cm.**

7. Here $R = +30$ cm, $\mu_1 = 1.33$, $\mu_2 = 1.48$, $v = \infty$, $u = ?$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or $$\frac{1.48}{\infty} - \frac{1.33}{u} = \frac{1.48 - 1.33}{30}$$

or $$u = -\frac{1.33 \times 30}{0.15} = -\mathbf{266\ cm.}$$

8. Here $\mu_1 = 1, \mu_2 = 1.5$, $u = OP = -4$ cm, $R = -10$ cm

As $$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \quad \therefore \quad \frac{1}{v} - \frac{1.5}{-4} = \frac{1-1.5}{-10}$$

or $$v = -\frac{40}{13} \simeq -\mathbf{3\ cm}, \text{ inside glass sphere.}$$

9.24 LENS MAKER'S FORMULA

38. *Derive the lens maker's formula for a double convex lens. State the assumptions made and the convention of signs used.*

Lens maker's formula. *This formula relates the focal length of a lens to the refractive index of the lens material and the radii of curvature of its two surfaces.* This formula is so called because it is used by manufacturers to design lenses of required focal length from a glass of given refractive index.

New Cartesian sign convention for spherical lenses :

(*i*) All distances are measured from the optical centre of the lens.

(*ii*) The distances measured in the direction of incident light are positive.

(*iii*) The distances measured in the opposite direction of incident light are negative.

Assumptions made in the derivation of lens maker's formula :

(*i*) The lens used is thin so that the distances measured from its surfaces may be taken equal to those measured from its optical centre.

(*ii*) The object is a point object placed on the principal axis.

(*iii*) The aperture of the lens is small.

(*iv*) All the rays are *paraxial, i.e.,* they make very small angles with the normals to the lens faces and with the principal axis.

Lens maker's formula for a double convex lens. As shown in Fig. 9.70, consider a thin double convex lens of refractive index μ_2 placed in a medium of refractive index μ_1. Here $\mu_1 < \mu_2$. Let B and D be the poles, C_1 and C_2 be the centres of curvature, and R_1 and R_2 be the radii of curvature of the two lens surfaces ABC and ADC, respectively.

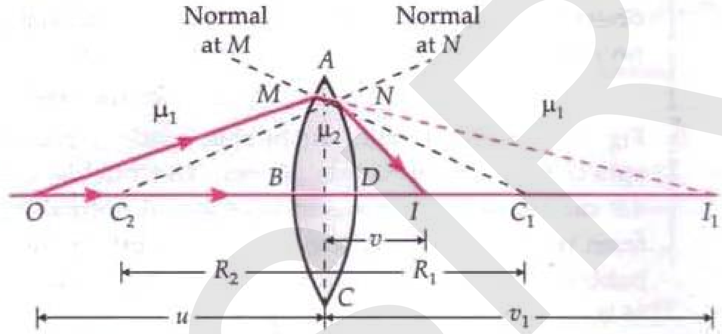


Fig. 9.70 Refraction through a double convex lens.

Suppose a point object O is placed on the principal axis in the rarer medium of refractive index μ_1. The ray OM is incident on the first surface ABC. It is refracted along MN, bending towards the normal at this surface. If the second surface ADC were absent, the ray MN would have met the principal axis at I_1. So we can treat I_1 as the real image formed by first surface ABC in the medium of refractive index μ_2.

For refraction at surface ABC, we can write the relation between the object distance u, image distance v_1 and radius of curvature R_1 as

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad ...(1)$$

But actually the ray MN suffers another refraction at surface ADC, bending away from the normal at point N. The emergent ray meets the principal axis at point I which is the final image of O formed by the lens. For refraction at second surface, I_1 acts as a virtual object placed in the medium of refractive index μ_2 and I is the real image formed in the medium of refractive index μ_1. Therefore, the relation between the object distance v_1, image distance v and radius of curvature R_2 can be written as

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \qquad ...(2)$$

Adding equations (1) and (2), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{v} - \frac{1}{u} = \left[\frac{\mu_2 - \mu_1}{\mu_1}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad ...(3)$$

If the object is placed at infinity ($u=\infty$), the image will be formed at the focus, *i.e.*, $v=f$. Therefore,

$$\frac{1}{f}=\left[\frac{\mu_2-\mu_1}{\mu_1}\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad ...(4)$$

This is ***lens maker's formula***.

When the lens is placed in *air*, $\mu_1=1$, and $\mu_2=\mu$. The lens maker's formula takes the form :

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

From equations (3) and (4), we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

This is the *thin lens formula* which gives relationship between u, v and f of a lens.

39. *Derive the lens maker's formula for a double concave lens.*

Lens maker's formula for a double concave lens. As shown in Fig. 9.71, consider a thin double concave lens of refractive index μ_2 placed in a medium of refractive μ_1. Here $\mu_1<\mu_2$. Let B and E be the poles, and R_1 and R_2 be the radii of curvature of the two lens surfaces ABC and DEF, respectively.

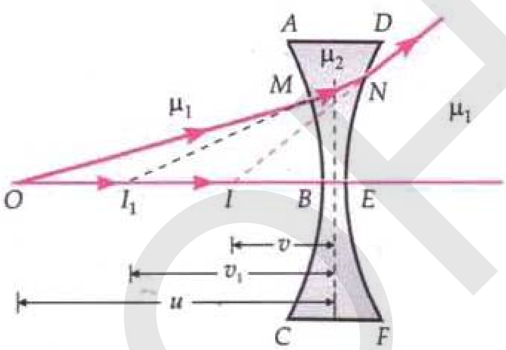


Fig. 9.71 Refraction through a double concave lens.

Suppose a point object O is placed on the principal axis in the rarer medium of refractive index μ_1. First the spherical surface ABC forms its virtual image I_1. As refraction occurs from rarer to denser medium, so we can write the relation between object distance u, image distance v_1 and radius of curvature R_1 as

$$\frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \quad ...(1)$$

But the lens material is not continuous. The ray MN suffers another refraction at N and emerges along IN. So I is the final virtual image of the point object O. The image I_1 acts as an object for refraction at surface DEF from denser to rarer medium. So the relation between object distance v_1, image distance v and radius of curvature R_2 can be written as

$$\frac{\mu_1}{v}-\frac{\mu_2}{v_1}=\frac{\mu_1-\mu_2}{R_2} \quad ...(2)$$

Adding equations (1) and (2), we get

$$\frac{\mu_1}{v}-\frac{\mu_1}{u}=(\mu_2-\mu_1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

or

$$\frac{1}{v}-\frac{1}{u}=\left[\frac{\mu_2-\mu_1}{\mu_1}\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

If an object is placed at infinity, then the image is formed at the focus *i.e.*, $v=f$, so

$$\frac{1}{f}=\left[\frac{\mu_2-\mu_1}{\mu_1}\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

This is *lens* ***maker's formula***.

When the lens is placed in air, $\mu_1=1$ and $\mu_2=\mu$. The lens maker's formula takes the form :

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Examples based on

Lens Maker's Formula

Formulae Used

1. For the lens of material of refractive index μ_2 placed in a medium of refractive index μ_1,

$$\frac{1}{f}=\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right].$$

2. When the lens is placed in air,

$\mu_1=1$ and $\mu_2=\mu$.

$$\therefore \quad \frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right].$$

3. f and R are positive for convex surfaces and negative for concave surfaces.

Units Used

Focal length f and radii of curvature R_1 and R_2 are in metre, refractive indices μ_1, μ_2 and μ have no units.

Example 44. *The radius of curvature of each face of biconcave lens, made of glass of refractive index 1.5 is 30 cm. Calculate the focal length of the lens in air.* [CBSE OD 96]

Solution. Here $\mu=1.5$, $R_1=-30$ cm, $R_2=+30$ cm

Using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= (1.5 - 1)\left[\frac{1}{-30} - \frac{1}{30}\right] = -0.5 \times \frac{2}{30} = -\frac{1}{30}$$

$\therefore \quad f = \mathbf{-30\ cm}$.

Example 45. *The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. If focal length is 12 cm, what is the refractive index of glass ?* [NCERT]

Solution. Here $f = +12$ cm, $R_1 = +10$ cm,

$$R_2 = -15 \text{ cm}, \ \mu = ?$$

As $$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$\therefore$ $$\frac{1}{12} = (\mu - 1)\left[\frac{1}{10} + \frac{1}{15}\right] = (\mu - 1) \times \frac{5}{30}$$

or $$\mu - 1 = \frac{6}{12} = 0.5 \quad \therefore \quad \mu = \mathbf{1.5}.$$

Example 46. *A biconvex lens has a focal length 2/3 times the radius of curvature of either surface. Calculate the refractive index of lens material.* [CBSE D 10]

Solution. Here $f = \frac{2}{3}R, \quad R_1 = R, \quad R_2 = -R$

As $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ $$\frac{3}{2R} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\frac{3}{4} = \mu - 1 \quad \text{or} \quad \mu = 1 + 0.75 = \mathbf{1.75}$$

Example 47. *The radii of curvature of a double convex lens of glass ($\mu = 1.5$) are in the ratio 1 : 2. This lens renders the rays parallel coming from an illuminated filament at a distance of 6 cm. Calculate the radii of curvature of its surfaces.*

Solution. Here $f = +6$ cm, $\mu = 1.5$, $R_1 = +R$,

$$R_2 = -2R$$

As $$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$\therefore$ $$\frac{1}{6} = (1.5 - 1)\left[\frac{1}{R} + \frac{1}{2R}\right]$$

or $$\frac{1}{6} = 0.5 \times \frac{3}{2R}$$

or $$R = \frac{0.5 \times 3 \times 6}{2} = 4.5 \text{ cm}$$

$\therefore$ $$R_1 = +R = \mathbf{+4.5\ cm}$$

and $$R_2 = -2R = \mathbf{-9.0\ cm.}$$

Example 48. *Find the radius of curvature of the convex surface of a plano-convex lens, whose focal length is 0.3 m and the refractive index of the material of the lens is 1.5.* [ISCE 98 ; CBSE D 10]

Solution. Here $\mu = 1.5$, $f = +0.3$ m, $R_1 = \infty$, $R_2 = -R$

Using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{+0.3} = (1.5 - 1)\left[\frac{1}{\infty} + \frac{1}{R}\right]$$

or $$\frac{1}{0.3} = 0.5 \times \frac{1}{R} \quad \text{or} \quad R = \mathbf{0.15\ m}.$$

Example 49. *A convex lens of focal length 0.2 m and made of glass ($\mu = 1.50$) is immersed in water ($\mu = 1.33$). Find the change in the focal length of the lens.* [Punjab 91 ; CBSE OD 91]

Solution. For glass lens in air, ${}^a\mu_g = 1.5$, $f_a = 0.2$ m

As $$\frac{1}{f_a} = ({}^a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$\therefore$ $$\frac{1}{0.2} = (1.5 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{R_1} - \frac{1}{R_2} = 10$$

For the same lens in water, ${}^a\mu_w = 1.33$

$$\frac{1}{f_w} = ({}^w\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{1.5}{1.33} - 1\right) \times 10 = \frac{0.17 \times 10}{1.33}$$

or $$f_w = \frac{133}{170} = 0.78 \text{ m}$$

$\therefore$ Change in focal length

$$= f_w - f_a = 0.78 - 0.20 = \mathbf{0.58\ m.}$$

Example 50. *The radii of curvature of a double convex lens are 10 cm and 20 cm respectively. Calculate its focal length when it is immersed in a liquid of refractive index 1.65. State the nature of the lens in the liquid. The refractive index of glass is 1.5.*

Solution. Here $R_1 = +10$ cm, $R_2 = -20$ cm,

$${}^a\mu_l = 1.65, \ {}^a\mu_g = 1.5$$

$${}^l\mu_g = \frac{{}^a\mu_g}{{}^a\mu_l} = \frac{1.5}{1.65} = \frac{10}{11}$$

Now $$\frac{1}{f}=({}^l\mu_g-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$
$$=\left(\frac{10}{11}-1\right)\left[\frac{1}{10}+\frac{1}{20}\right]$$
$$=-\frac{1}{11}\times\frac{3}{20}=-\frac{3}{220}$$

or $$f=-\mathbf{73.33\ cm}$$

The negative value of f indicates that the lens becomes diverging when immersed in the given liquid.

Example 51. *If the refractive index from air to glass is 3/2 and that from air to water is 4/3, find the ratio of focal lengths of a glass lens in water and in air.*

Solution. Here ${}^a\mu_g=\frac{3}{2}$, ${}^a\mu_w=\frac{4}{3}$,

$$ {}^w\mu_g=\frac{{}^a\mu_g}{{}^a\mu_w}=\frac{3/2}{4/3}=\frac{9}{8}$$

Let f_w and f_a be the focal lengths of glass lens in water and air respectively. Then

$$\frac{1}{f_w}=({}^w\mu_g-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad ...(i)$$

$$\frac{1}{f_a}=({}^a\mu_g-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad ...(ii)$$

Dividing (*ii*) and (*i*), we get

$$\frac{f_w}{f_a}=\frac{{}^a\mu_g-1}{{}^w\mu_g-1}=\frac{\frac{3}{2}-1}{\frac{9}{8}-1}=\mathbf{4:1.}$$

Example 52. *A double convex lens has a focal length of 25 cm in air. When it is dipped into a liquid of refractive index 4/3, its focal length is increased to 100 cm. Find the refractive index of the lens material.*

Solution. ***For the lens in air :*** $f_a=25$ cm.

Let μ = refractive index of lens material relative to air

Then $$\frac{1}{f_a}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

or $$\frac{1}{25}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad ...(i)$$

For the lens in liquid : $f_l=100$ cm, $\mu_1=\frac{4}{3}$, $\mu_2=\mu$

$$\therefore \quad \frac{1}{f_l}=\left(\frac{\mu_2}{\mu_1}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

or $$\frac{1}{100}=\left(\frac{\mu}{4/3}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{100}{25}=\frac{\mu-1}{\frac{3\mu}{4}-1} \quad \text{or} \quad 3\mu-4=\mu-1$$

or $$\mu=\frac{3}{2}=\mathbf{1.5.}$$

Example 53. *An equiconvex lens of focal length 15 cm is cut into two equal halves as shown in Fig. 9.72. What is the focal length of each half ?* [CBSE Sample Paper 98]

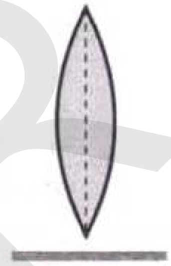


Fig. 9.72

Solution. For the equiconvex lens, let

$$R_1=+R, \quad R_2=-R$$

Then from lens maker's formula,

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R}+\frac{1}{R}\right]=\frac{2(\mu-1)}{R} \quad ...(i)$$

For each half lens, $R_1=R$, $R_2=-\infty$.

$$\therefore \quad \frac{1}{f'}=(\mu-1)\left[\frac{1}{R}-\frac{1}{-\infty}\right]=\frac{\mu-1}{R} \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get $\frac{f'}{f}=2$

or $$f'=2f=2\times 15=\mathbf{30\ cm.}$$

Problems For Practice

1. The radii of curvature of a double convex lens are 15 cm and 30 cm and its refractive index is 1.5. Calculate its focal length. **[Haryana 99]**
(Ans. 20 cm)
2. A double convex lens has radii 20 cm. The index of refraction of glass is 1.5. Compute the focal length of this lens in air and when immersed in carbon disulphide of refractive index 1.63. (Ans. 20 cm, – 125.4 cm)
3. A biconvex lens has a focal length half the radius of curvature of either surface. What is the refractive index of lens material ? (Ans. $\mu=2$)
4. Fig. 9.73 shows a thin lens with centres of curvature C_1 and C_2. Find its focal length. Take $\mu=1.5$.
(Ans. + 40 cm)

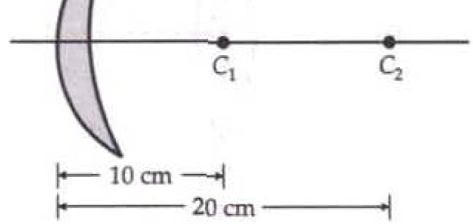


Fig. 9.73

5. A converging lens has a focal length of 20 cm in air. It is made of a material of refractive index 1.6. If it is immersed in a liquid of refractive index 1.3, what will be its new focal length ? **[CBSE D 06C ; OD 11]**
(Ans. 52 cm)

6. A plano-convex lens ($\mu = 1.5$) has a curved surface of radius 15 cm. What is its focal length ?
(Ans. 30 cm)

7. A plano-convex lens $\mu = 1.5$ has focal length of 18 cm in air. Calculate the radius of curvature of the spherical surface. **[CBSE F 94]**
(Ans. 9.0 cm)

8. The focal length of a concavo-convex lens of radii of curvature 5 cm and 10 cm is 20 cm. What will be its focal length in water ? Given ${}^a\mu_w = 4/3$.
(Ans. – 80 cm)

9. A convex lens of focal length f and refractive index 1.5 is immersed in a liquid of refractive index (*i*) 1.6 (*ii*) 1.3 and (*iii*) 1.5. What changes happen to the focal length of the lens in the three cases ? **[Kerala 92]**
[Ans. (*i*) $-8\,f$ (*ii*) $+3.25\,f$ (*iii*) ∞]

10. The focal length of a plano-convex lens is 20 cm in air. Refractive index of glass is 1.5. Calculate (*i*) the radius of curvature of lens surface and (*ii*) its focal length when immersed in liquid of refractive index 1.6. [Ans. (*i*) – 10 cm (*ii*) – 160 cm]

11. The focal length of a glass convex lens in air is 15 cm. Calculate its focal length, when it is totally immersed in water. Given ${}^a\mu_w = 4/3$ and ${}^a\mu_g = 1.5$. **[Punjab 01]**
(Ans. 60 cm)

12. The radius of curvature of each surface of a convex lens is 20 cm and the refractive index of the material of the lens is 3/2. (*i*) Calculate its focal length. (*ii*) If this is cut along the plane *AB*, what will be the focal length of the each of the two halves so formed ? (*iii*) What happens if the lens is cut along *CD* ?
[Ans. (*i*) 20 cm (*ii*) 40 cm (*iii*) f remains same but intensity of the image decreases]

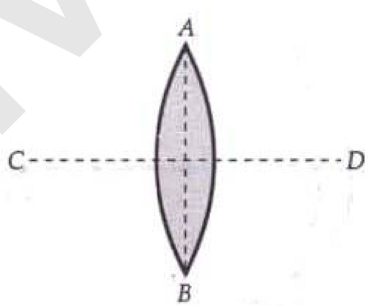


Fig. 9.74

13. A converging lens of refractive index 1.5 and of focal length 15 cm in air has the same radii of curvature for both sides. If it is immersed in a liquid of refractive index 1.7, calculate the focal length of the lens in the liquid. **[CBSE OD 08]**
(Ans. – 63.75 cm)

HINTS

1. Here $R_1 = +15$ cm, $R_2 = -30$ cm, $\mu = 1.5$

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= (1.5 - 1)\left[\frac{1}{15} - \frac{1}{-30}\right] = \frac{1}{20}$$

or $f = $ **+ 20 cm.**

2. $$\frac{1}{f_a} = ({}^a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= (1.5 - 1)\left[\frac{1}{20} - \frac{1}{-20}\right] = \frac{1}{20}$$

$\therefore$ $f_a = $ **20 cm.**

$${}^c\mu_g = \frac{{}^a\mu_g}{{}^a\mu_c} = \frac{1.5}{1.63} = \frac{150}{163}$$

$$\frac{1}{f_c} = \left(\frac{150}{163} - 1\right)\left[\frac{1}{20} - \frac{1}{-20}\right] = -\frac{13}{163} \times \frac{1}{10}$$

or $f_c = -\dfrac{1630}{13} = $ **– 125.4 cm.**

3. Here $f = R/2,\ R_1 = R,\ R_2 = -R$

$$\frac{1}{f} = \frac{2}{R} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right) \quad \text{or} \quad \mu = 2$$

4. Here both the radii of curvature are positive.

$R_1 = +10$ cm, $R_2 = +20$ cm

$$\frac{1}{f} = (1.5 - 1)\left[\frac{1}{10} - \frac{1}{20}\right] = \frac{1}{40}$$

or $f = $ **+ 40 cm.**

5. Using lens maker's formula when the lens is placed in air,

$$\frac{1}{f_a} = \left[\frac{\mu_g}{\mu_a} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{20} = \left[\frac{1.6}{1} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{20 \times 0.6} = \frac{1}{12}$$

When the lens is placed in the liquid,

$$\frac{1}{f_l} = \left[\frac{\mu_g}{\mu_l} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left[\frac{1.6}{1.3} - 1\right] \times \frac{1}{12} = \frac{0.3}{1.3 \times 12}$$

or $\quad f_l = \dfrac{1.3 \times 12}{0.3} = $ **52 cm.**

6. Here $\mu = 1.5, \quad R_1 = \infty, \quad R_2 = -15\,\text{cm}$

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.5 - 1)\left[\frac{1}{\infty} + \frac{1}{15}\right]$$

$$= 0.5 \times \frac{1}{15} = \frac{1}{30} \quad \therefore \quad f = \mathbf{30\ cm.}$$

7. Here $\mu = 1.5, \quad f = +18\,\text{cm}, \quad R_1 = \infty, \quad R_2 = -R$

Using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{18} = (1.5 - 1)\left[\frac{1}{\infty} + \frac{1}{R}\right] \quad \text{or} \quad \frac{1}{18} = 0.5 \times \frac{1}{R}$$

or $\quad R = 0.5 \times 18 = $ **9.0 cm.**

8. Here $f_a = -20\,\text{cm}, \; R_1 = -5\,\text{cm}, \; R_2 = -10\,\text{cm}$

As $\quad \dfrac{1}{f_a} = ({}^a\mu_g - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

$$\therefore \quad \frac{1}{-20} = ({}^a\mu_g - 1)\left[\frac{1}{-5} - \frac{1}{-10}\right]$$

$$\therefore \quad {}^a\mu_g = 3/2$$

When the lens is placed in water,

$$\frac{1}{f_a} = \left[\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{3/2}{4/3} - 1\right)\left[-\frac{1}{5} + \frac{1}{10}\right] = -\frac{1}{80}$$

or $\quad f_w = $ **− 80 cm.**

9. $\dfrac{1}{f} = ({}^a\mu_g - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

$$= (1.5 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $\quad \dfrac{1}{R_1} - \dfrac{1}{R_2} = \dfrac{2}{f}$

(*i*) $\dfrac{1}{f_l} = \left(\dfrac{{}^a\mu_g}{{}^a\mu_l} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right] = \left(\dfrac{1.5}{1.6} - 1\right) \times \dfrac{2}{f}$

or $\quad f_l = -8f.$

(*ii*) $\quad \dfrac{1}{f_l} = \left(\dfrac{{}^a\mu_g}{{}^a\mu_l} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right] = \left(\dfrac{1.5}{1.3} - 1\right)\dfrac{2}{f}$

or $\quad f_l = \mathbf{+3.25}\,f.$

(*iii*) $\dfrac{1}{f_l} = \left(\dfrac{1.5}{1.5} - 1\right)\dfrac{2}{f} \quad \text{or} \quad f_l = \infty.$

10. (*i*) *For the lens in air:*

$$f_a = 20\,\text{cm}, \;\; \mu = 1.5, \;\; R_1 = \infty, \;\; R_2 = ?$$

$$\frac{1}{f_a} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $\quad \dfrac{1}{20} = (1.5 - 1)\left[\dfrac{1}{\infty} - \dfrac{1}{R_2}\right]$

$\therefore \quad R_2 = $ **− 10 cm.**

(*ii*) *For the lens in liquid:*

$$\mu_2 = 1.5, \; \mu = 1.6, \; R_1 = \infty, \; R_2 = -10\,\text{cm}$$

$$\frac{1}{f_l} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{1.5}{1.6} - 1\right)\left[\frac{1}{\infty} + \frac{1}{10}\right] = -\frac{1}{160}$$

$\therefore \quad f_e = $ **−160 cm.**

11. $\dfrac{1}{f_a} = ({}^a\mu_g - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

or $\quad \dfrac{1}{15} = (1.5 - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

or $\quad \dfrac{1}{R_1} - \dfrac{1}{R_2} = \dfrac{2}{15}$

Now $\dfrac{1}{f_w} = \left(\dfrac{\mu_g}{\mu_w} - 1\right)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right] = \left(\dfrac{1.5}{4/3} - 1\right) \times \dfrac{2}{15}$

$$= \frac{1}{8} \times \frac{2}{15} = \frac{1}{60} \quad \text{or} \quad f_w = \mathbf{60\ cm.}$$

13. Here $f_a = 15\,\text{cm}, \; \mu_g = 1.5, \; \mu_l = 1.7, \; f_l = ?$

For the lens place in air :

$$\frac{1}{f_a} = (\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $\quad \dfrac{1}{15} = (1.5 - 1)\left[\dfrac{1}{R} + \dfrac{1}{R}\right] = \dfrac{1}{R}$

For the lens immersed in liquid :

$$\frac{1}{f_l} = \left(\frac{\mu_g}{\mu_l} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{1.5}{1.7} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = -\frac{0.2}{1.7} \times \frac{2}{R}$$

$$= -\frac{0.2}{1.7} \times \frac{2}{15} \quad \text{or} \quad f_l = \mathbf{-\,63.75\ cm.}$$

9.25 RULES FOR DRAWING IMAGES FORMED BY SPHERICAL LENSES

40. *State the rules used for drawing images formed by spherical lenses.*

Rules for drawing images formed by spherical lenses. The position of the image formed by any spherical lens can be found by considering any two of the following rays of light coming from a point on the object.

(*i*) A ray from the object parallel to the principal axis after refraction passes through the second principal focus F_2 [in a convex lens, as shown in Fig. 9.75(*a*)] or appears to diverge [in a concave lens, as shown in Fig. 9.75(*b*)] from the first principal focus F_1.

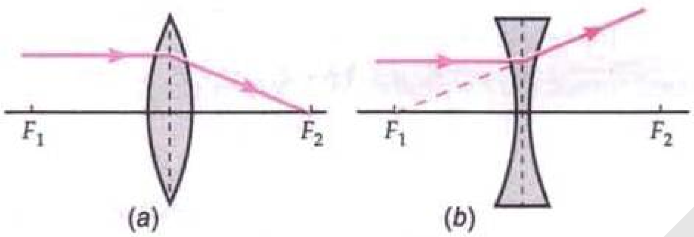


Fig. 9.75 Path of ray incident parallel to the principal axis of (*a*) convex lens (*b*) concave lens.

(*ii*) A ray of light passing through the first principal focus [in a convex lens, as shown in Fig. 9.76(*a*)] or appearing to meet at it [in a concave lens, as shown in Fig. 9.76(*b*)] emerges parallel to the principal axis after refraction.

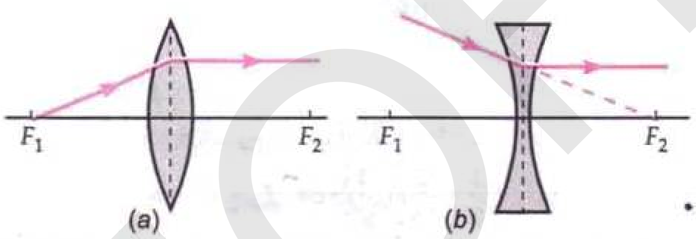


Fig. 9.76 Path of a ray passing through focus of (*a*) convex lens (*b*) concave lens.

(*iii*) A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction, as shown in Figs. 9.77(*a*) and (*b*).

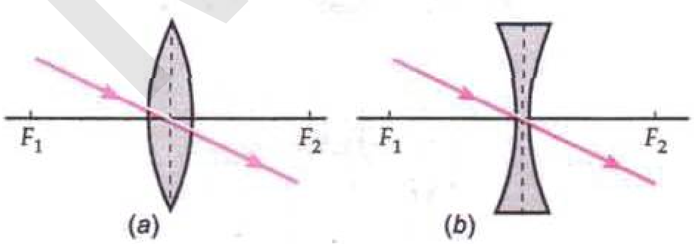
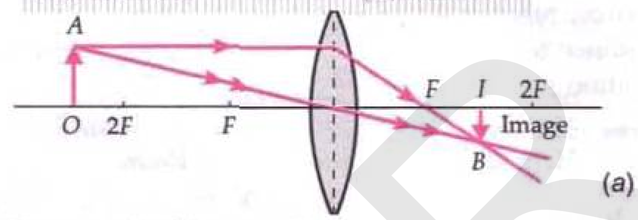


Fig. 9.77 Path of a ray passing through the optical centre (*a*) convex lens (*b*) concave lens.

Formation of images by spherical lenses :

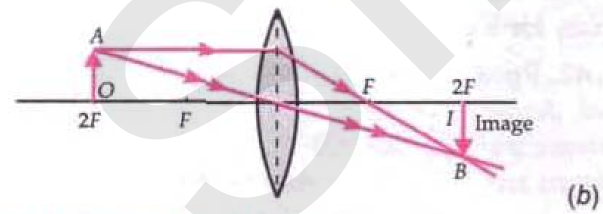
(*a*) Object beyond 2*F*. The image is

(*i*) between *F* and 2*F*	(*ii*) real
(*iii*) inverted	(*iv*) smaller



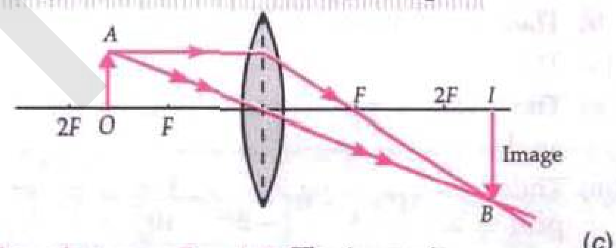
(*b*) Object at 2*F*. The image is

(*i*) at 2*F*	(*ii*) real
(*iii*) inverted	(*iv*) same size



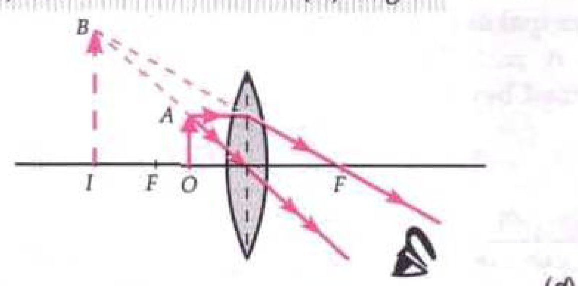
(*c*) Object between 2*F* and *F*. The image is

(*i*) beyond 2*F*	(*ii*) real
(*iii*) inverted	(*iv*) larger



(*d*) Object between *F* and *O*. The image is

(*i*) behind object	(*ii*) virtual
(*iii*) erect	(*iv*) larger



(*e*) Object in any position. The image is

(*i*) in front of object	(*ii*) virtual
(*iii*) erect	(*iv*) smaller

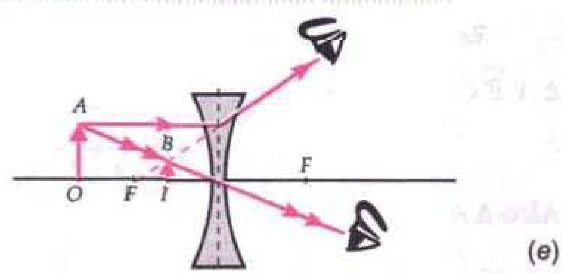


Fig. 9.78 Formation of images by spherical lenses.

9.26 THIN LENS FORMULA

41. *State the lens formula. Is the same formula applicable to both convex and concave lenses ?*

Thin lens formula. Thin lens formula is a mathematical relation between the object distance u, image distance v and focal length f of a spherical lens. This relation is :

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

In words, we can say that

$$\frac{1}{\text{Image distance}}-\frac{1}{\text{Object distance}}=\frac{1}{\text{Focal length}}$$

This formula is valid for both convex and concave lenses for both real and virtual images.

42. *By stating the sign-convention and assumptions used, derive the relation between object distance u, image distance v and focal length f for a thin convex lens, when it forms real image of an object of finite size.*

New cartesian sign convention for spherical lenses. Refer answer to Q. 35 on page 9.31.

Assumptions used in the derivation of lens formula :

(*i*) The lens used is thin.

(*ii*) The aperture of the lens is small.

(*iii*) The incident and refracted rays make small angles with the principal axis.

(*iv*) The object is a small object placed on the principal axis.

Derivation of thin lens formula for a convex lens when it forms a real image. As shown in Fig. 9.79, consider an object AB placed perpendicular to the principal axis of a thin convex lens between its F' and C'. A real, inverted and magnified image $A'B'$ is formed beyond C on the other side of the lens.

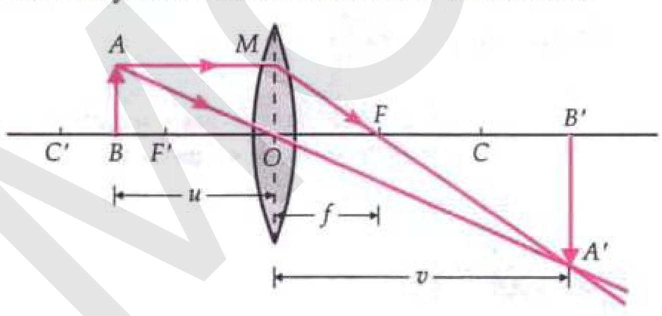


Fig. 9.79 Real image formed by a convex lens.

$\Delta A'B'O$ and ΔABO are similar,

$$\therefore \quad \frac{A'B'}{AB}=\frac{OB'}{BO} \qquad ...(1)$$

Also $\Delta A'B'F$ and ΔMOF are similar,

$$\therefore \quad \frac{A'B'}{MO}=\frac{FB'}{OF}$$

But $\quad MO = AB,$

$$\therefore \quad \frac{A'B'}{AB}=\frac{FB'}{OF} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{OB'}{BO}=\frac{FB'}{OF}=\frac{OB'-OF}{OF}$$

Using new Cartesian sign convention, we get

Object distance, $\quad BO = -u$

Image distance, $\quad OB' = +v$

Focal length, $\quad OF = +f$

$$\therefore \quad \frac{v}{-u}=\frac{v-f}{f}$$

or $\quad vf = -uv + uf \quad$ or $\quad uv = uf - vf$

Dividing both sides by uvf, we get

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

This proves the lens formula for a convex lens when it forms a real image.

43. *Derive thin lens formula for a convex lens when it forms a virtual image.*

Derivation of thin lens formula for a convex lens when it forms a virtual image. As shown in Fig. 9.80, when an object AB is placed between the optical centre O and the focus F of a convex lens, the image $A'B'$ formed by the convex lens is virtual, erect and magnified.

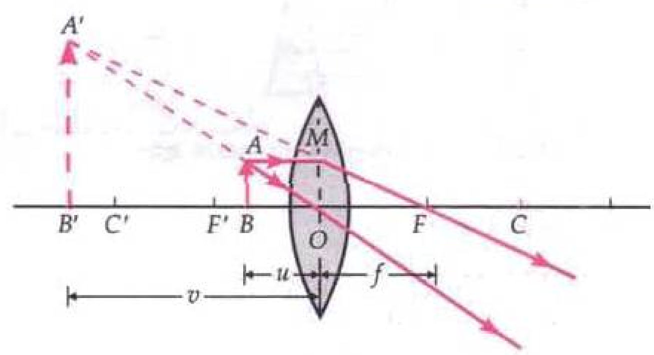


Fig. 9.80 Virtual image formed by a convex lens.

Triangles $A'B'O$ and ABO are similar.

$$\therefore \quad \frac{A'B'}{AB}=\frac{B'O}{BO} \qquad ...(1)$$

Also, triangles $A'B'F$ and MOF are similar.

$$\therefore \quad \frac{A'B'}{MO}=\frac{B'F}{OF}$$

But $MO = AB$, therefore

$$\frac{A'B'}{AB}=\frac{B'F}{OF} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{B'O}{BO} = \frac{B'F}{OF} = \frac{B'O + OF}{OF}$$

Using new cartesian sign convention,

$$BO = -u, \quad B'O = -v, \quad OF = +f$$

$$\therefore \quad \frac{-v}{-u} = \frac{-v+f}{f}$$

or $\quad -vf = uv - uf$

or $\quad uv = uf - vf$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This proves the thin lens formula for a convex lens when it forms a virtual image.

44. *Derive the thin lens formula for a concave lens.*

Derivation of thin lens formula for a concave lens. As shown in Fig. 9.81, suppose O be the optical centre and F be the principal focus of concave lens of focal length f. AB is an object placed perpendicular to its principal axis. A virtual, erect and diminished image $A'B'$ is formed due to refraction through the lens.

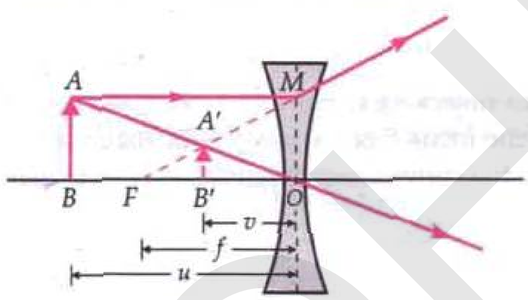


Fig. 9.81 Virtual image formed by a concave lens.

As $\quad \Delta A'B'O \sim \Delta ABO$

$$\therefore \quad \frac{A'B'}{AB} = \frac{B'O}{BO} \qquad ...(1)$$

Also, $\quad \Delta A'B'F \sim \Delta MOF$

$$\therefore \quad \frac{A'B'}{MO} = \frac{FB'}{FO}$$

But $MO = AB$, therefore

$$\frac{A'B'}{AB} = \frac{FB'}{FO} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{B'O}{BO} = \frac{FB'}{FO} = \frac{FO - B'O}{FO}$$

Using new Cartesian sign convention, we get

$$BO = -u, \quad B'O = -v, \quad FO = -f$$

$$\therefore \quad \frac{-v}{-u} = \frac{-f+v}{-f}$$

or $\quad vf = uf - uv \quad$ or $\quad uv = uf - vf$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This proves the thin lens formula for a concave lens.

9.27 LINEAR MAGNIFICATION

45. *Define linear magnification produced by a lens. Derive expressions for the magnification produced by convex and concave lenses.*

Linear magnification. *The linear magnification produced by a lens is defined as the ratio of the size of the image formed by the lens to the size of the object.* It is denoted by m. Thus

$$m = \frac{\text{Size of image}}{\text{Size of object}} = \frac{h_2}{h_1}$$

Convex lens. Earlier Fig. 9.79 shows a ray diagram for the formation of image $A'B'$ of a finite object AB by a convex lens.

Now $\quad \Delta AOB \sim \Delta A'OB'$

$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB}$$

Applying the new cartesian sign convention, we get

$A'B' = -h_2$ *(Downward image height)*

$AB = +h_1$ *(Upward object height)*

$OB = -u$ *(Image distance on left)*

$OB' = +v$ *(Image distance on right)*

$$\therefore \quad \frac{-h_2}{+h_1} = \frac{+v}{-u} \quad \text{or} \quad \frac{h_2}{h_1} = \frac{v}{u}$$

$$\therefore \text{ Magnification, } m = \frac{h_2}{h_1} = \frac{v}{u}.$$

Concave lens. Fig. 9.81 shows the formation of a virtual image $A'B'$ of a finite object AB by a concave lens.

Now $\Delta AOB \sim \Delta A'OB'$

$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB}$$

Applying the new cartesian sign convention, we get

$$A'B' = +h_2, \quad AB = +h_1$$

$$OB' = -v, \quad OB = -u$$

$$\therefore \quad \frac{+h_2}{+h_1} = \frac{-v}{-u}$$

$$\therefore \text{ Magnification, } m = \frac{h_2}{h_1} = \frac{v}{u}.$$

Linear magnification in terms of u and f. The thin lens formula is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Multiplying both sides by u, we get

$$\frac{u}{v}-1=\frac{u}{f} \quad \text{or} \quad \frac{u}{v}=1+\frac{u}{f}=\frac{f+u}{f}$$

$$\therefore \quad m=\frac{v}{u}=\frac{f}{f+u}$$

Linear magnification in terms of v and f. The thin lens formula is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Multiplying both sides by v, we get

$$1-\frac{v}{u}=\frac{v}{f} \quad \therefore \quad m=\frac{v}{u}=1-\frac{v}{f}=\frac{f-v}{f}$$

Hence $$m=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$$

For Your Knowledge

- The same thin lens formula is valid for both convex and convace lenses and for both real and virtual images.
- When $|m|>1$, the image is *magnified*.
- When $|m|<1$, the image is *diminished*.
- When $|m|=1$, the image is of the same size as the object.
- When m is *positive* (or v is negative), the image is *virtual* and *erect*.
- When m is *negative* (or v is positive), the image is *real* and *inverted*.

Examples based on
Thin Lens Formula and Linear Magnification

Formulae Used

1. Focal length of any lens is given by the thin lens formula, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$.
2. Magnification, $m=\frac{h_2}{h_1}=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$
3. In Cartesian sign convention, u is taken negative
4. In case of convex lens, v is positive for real image and negative for virtual image and f is positive.
5. In case of concave lens u, v and f are all negative.
6. Magnification m is positive for virtual image and negative for real image.

Units Used

Distances u, v, and f are in cm or m.

Example 54. *A lens forms a real image of an object. The distance of the object to the lens is u cm and the distance of the image from the lens is v cm. The given graph shows the variation of v with u. (i) What is the nature of the lens ? (ii) Using this graph, find the focal length of this lens.* [CBSE F 04]

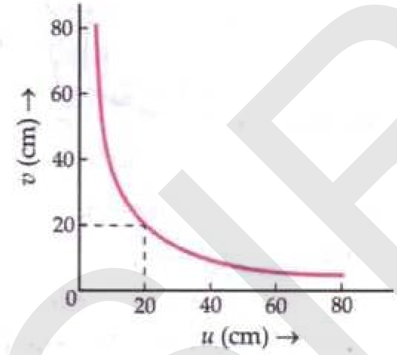


Fig. 9.82

Solution. (*i*) As the lens forms a real image, it must be a convex lens.

(*ii*) From the graph, when $u=20$ cm, we have $v=20$ cm.

For the convex lens forming a real image, u is negative and v and f are positive.

$\therefore \quad u=-20$ cm, $\quad v=+20$ cm

Using thin lens formula,

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{20}-\frac{1}{-20}=\frac{1}{10}$$

or $\quad$ $f=+$ **10 cm.**

Example 55. *A needle placed 45 cm from a lens forms an image on a screen placed 90 cm on the other side of the lens. Identify the type of the lens and determine its focal length. What is the size of image if the size of the needle is 5.0 cm ?* [Punjab 2000 ; NCERT]

Solution. Here $u=-45$ cm, $\quad v=+90$ cm

Using thin lens formula,

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{90}+\frac{1}{45}=\frac{1+2}{90}$$

$\therefore \quad$ $f=+$ **30 cm**

Positive value of f indicates that the lens is converging.

Magnification,

$$m=\frac{h_2}{h_1}=\frac{v}{u} \quad \text{or} \quad \frac{h_2}{5}=\frac{90}{-45}$$

[$\because h_1=5$ cm]

$\therefore$ Size of image, $h_2=-$ **10 cm**

Negative sign indicates that the image is real and inverted.

Example 56. *Converging light rays are falling on a convex lens as shown in Fig. 9.83. If the focal length of the lens is 30 cm, then find the position of the image.*

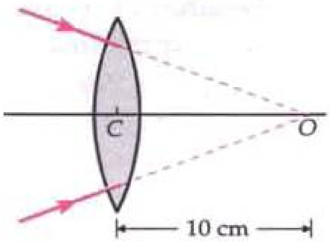


Fig. 9.83

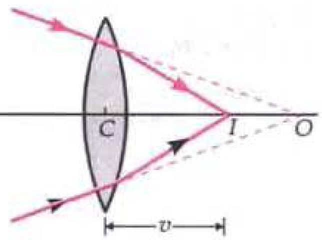


Fig. 9.84

Solution. The incident light rays appear to converge at point O. So O acts as virtual object for the lens and I is its real image as shown in Fig. 9.84. Here $u = +10$ cm, $f = +30$ cm.

From thin lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} \quad \text{or} \quad v = +\mathbf{7.5\ cm}$$

Thus the image is formed at 7.5 cm from the lens on the same side as the virtual object O.

Example 57. *A convergent beam of light passes through a diverging lens of focal length 0.2 m and comes to focus at distance 0.3 m behind the lens. Find the position of the point at which the beam would converge in the absence of the lens.* [CBSE Sample Paper 98]

Solution. As the focal length of a diverging lens is negative and the distance measured in the direction of incident ray is positive,

$$f = -0.2 \text{ m}, \quad v = +0.3 \text{ m}$$

As $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\quad\therefore\quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{0.3} + \frac{1}{0.2} = \frac{50}{6}$

or $\quad u = +\mathbf{0.12\ m}$

So in the absence of the lens the beam would converge at a point 0.12 m from the position of the lens.

Example 58. *A needle 10 cm long is placed along the axis of a convex lens of focal length 10 cm such that the middle point of the needle is at a distance of 20 cm from the lens. Find the length of the image of the needle.*

Solution. Fig. 9.85 shows a needle AB of length 10 cm, placed on the axis of a convex lens.

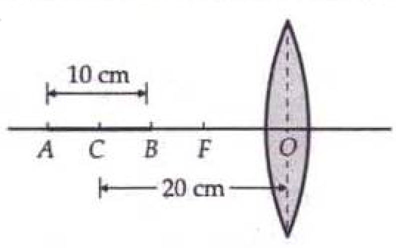


Fig. 9.85

Here $CO = 20$ cm, $AO = 20 + 5 = 25$ cm and $BO = 20 - 5 = 15$ cm.

For the image of end A of the needle.

$$u_1 = AO = -25 \text{ cm}, \quad f = +10 \text{ cm}$$

Using thin lens formula,

$$\frac{1}{v_1} = \frac{1}{f} + \frac{1}{u_1} = \frac{1}{+10} + \frac{1}{-25} = \frac{3}{50}$$

or $\quad v_1 = 50/3 = 16.67$ cm

For the image of end B of the needle.

$$u_2 = BO = -15 \text{ cm}, \quad f = +10 \text{ cm}$$

$$\therefore \frac{1}{v_2} = \frac{1}{f} + \frac{1}{u_2} = \frac{1}{+10} + \frac{1}{-15} = \frac{1}{30} \quad \text{or} \quad v_2 = 30 \text{ cm}$$

Hence the length of the image of needle AB

$$= v_2 - v_1 = 30 - 16.67 = \mathbf{13.33\ cm}.$$

Example 59. *A double convex lens made of glass of refractive index 1.5 has its both surfaces of equal radii of curvature of 20 cm each. An object of 5 cm height is placed at a distance of 10 cm from the lens. Find the position, nature and size of the image.* [CBSE OD 05]

Solution. Here $\mu = 1.5$, $R_1 = +20$ cm, $R_2 = -20$ cm.

Using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{20} + \frac{1}{20}\right) = 0.5 \times \frac{2}{20} = \frac{1}{20}$$

or $\quad f = +20$ cm

Now $u = -10$ cm, $f = +20$ cm

From thin lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{10} = -\frac{1}{20} \quad \text{or} \quad v = \mathbf{-20\ cm}$$

Magnification,

$$m = \frac{h_2}{h_1} = \frac{v}{u} \quad \text{or} \quad \frac{h_2}{5 \text{ cm}} = \frac{-20}{-10}$$

or $\quad h_2 = 2 \times 5 = \mathbf{10\ cm}$

Hence a virtual and erect image of height 10 cm is formed at a distance of 20 cm from the lens on the same side as the object.

Example 60. *A double convex lens has 10 cm and 15 cm as its two radii of curvatures. The image of an object placed 30 cm from the lens, is formed at 20 cm from the lens on the other side. Find the refractive index of the material of the lens. What will be the focal length ? What will be the focal length of the lens, if it is immersed in water of refractive index 1.33 cm ?* [ISCE 95]

Solution. Here $u = -30$ cm, $v = 20$ cm, $R_1 = +10$ cm, $R_2 = -15$ cm

Using thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{20} - \frac{1}{-30} = \frac{5}{60} = \frac{1}{12}$$

or $\quad f = \mathbf{12\ cm}$

Using lens maker's formula,

$$\frac{1}{f} = ({}^a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{12} = ({}^a\mu_g - 1)\left[\frac{1}{10} + \frac{1}{15}\right] = ({}^a\mu_g - 1) \times \frac{1}{6}$$

or $$ {}^a\mu_g = 1 + \frac{6}{12} = \mathbf{1.5}$$

When the lens is immersed in water,

$$\frac{1}{f_w} = \left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{10} + \frac{1}{15}\right) = \frac{0.17}{1.33} \times \frac{1}{6}$$

or $$f_w = \frac{1.33 \times 6}{0.17} = \mathbf{46.94\ cm}.$$

Example 61. *An illuminated object and a screen are placed 90 cm apart. What is the focal length and nature of the lens required to produce a clear image on the screen, twice the size of the object ?* [CBSE OD 10]

Solution. As the image is real, the lens must be a convex lens and it should be placed between the object and the screen.

Let distance between object and convex lens $= x$, then

$$u = -x, \quad v = 90 - x$$

Now $\quad m = \frac{v}{u} = -2$ [Minus sign as image is real]

or $$\frac{90 - x}{-x} = -2$$

or $\quad 90 - x = 2x \quad$ or $\quad x = \frac{90}{3} = 30$

$\therefore \quad u = -30$ cm, $\quad v = +60$ cm

Now $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{60} - \frac{1}{-30} = \frac{3}{60} = \frac{1}{20}$$

or $\quad f = \mathbf{20\ cm.}$

Example 62. *The image obtained with a convex lens is erect and its length is four times the length of the object. If the focal length of the lens is 20 cm, calculate the object and image distances.* [CBSE OD 10]

Solution. Here $f = 20$ cm, $m = +4$ for a virtual image.

To calculate u, we have

$$m = \frac{f}{u + f} \quad \text{or} \quad 4 = \frac{20}{u + 20} \quad \text{or} \quad u = \mathbf{-15\ cm}$$

To calculate v, we have

$$m = \frac{f - v}{f} \quad \text{or} \quad 4 = \frac{20 - v}{20} \quad \text{or} \quad v = \mathbf{-60\ cm.}$$

Example 63. *A luminous object and a screen are placed on an optical bench and a converging lens is placed between them to throw a sharp image of the object on the screen, the linear magnification of the image is found to be 2.5. The lens is now moved 30 cm nearer the screen and a sharp image is again formed. Calculate the focal length of the lens.*

Solution. In Fig. 9.86, let O and I be the positions of object and screen, respectively. Let L_1 and L_2 be the two conjugate positions of the lens, then

$$OL_1 = L_2 I = x \text{ (say)}$$

because the u and v values are just interchanged.

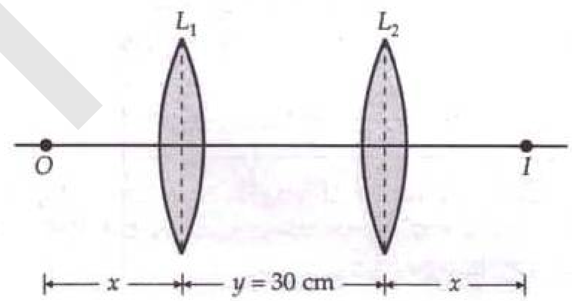


Fig. 9.86

For the lens in position L_1,

$$u = OL_1 = -x, \quad v = L_1 I = 30 + x$$

But magnification,

$$m = \frac{v}{u} = -2.5$$

(minus sign taken as image is real)

or $$\frac{30 + x}{-x} = -2.5, \quad x = 20 \text{ cm}$$

$\therefore \quad u = -20$ cm and $v = 30 + 20 = 50$ cm

As $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\therefore$ $$\frac{1}{50} + \frac{1}{20} = \frac{1}{f}$$

or $$f = \frac{50 \times 20}{50 + 20}$$

$$= \mathbf{14.3\ cm.}$$

Example 64. *In Fig. 9.87, a convex lens L is placed at a distance of 36 cm from a screen. If a point-source P is placed at 56 cm from the screen, then a circular spot of light of diameter equal to the diameter of the lens is formed. Show the image formation by a ray diagram. Calculate upto what distance the source be displaced so that its clear image can be formed on the screen.*

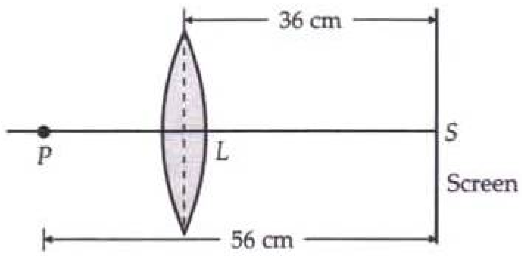


Fig. 9.87

Solution. As is clear from Fig. 9.88, the circular spot *AB* of light will be equal to the diameter of the lens if the image *I* is formed exactly in the middle of the lens and the screen.

$$\therefore \qquad u = -20 \text{ cm}, \quad v = +18 \text{ cm}$$

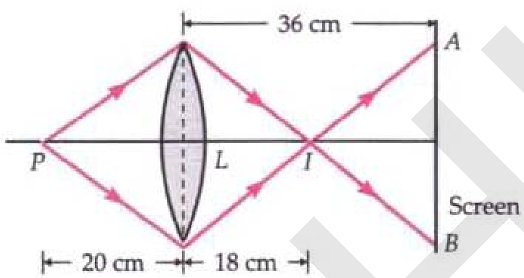


Fig. 9.88

Using thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{18} + \frac{1}{20} = +\frac{19}{180}$$

To obtain a clear image on the screen, the distance *u* of the source from the lens has to be changed. In that case, $v = +36$ cm

$$\therefore \qquad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{36} - \frac{19}{180} = -\frac{14}{180}$$

or $\qquad u = -12.86$ cm

i.e., the source *P* should be at a distance of 12.86 cm from the lens. It must be displaced by $20 - 12.86 = 7.14$ cm towards the lens.

Example 65. *In the Fig. 9.89, the direct image formed by the lens (f = 10 cm) of an object placed at O and that formed after reflection from the spherical mirror are formed at the same point O′. What is the radius of curvature of the mirror ?* [CBSE Sample Paper 08]

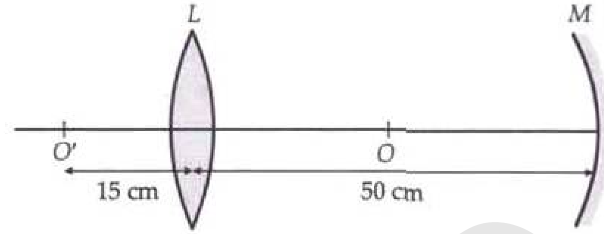


Fig. 9.89

Solution. For refraction through the convex lens, $f = +10$ cm, $v = +15$ cm

$$\therefore \qquad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{15} - \frac{1}{10} = -\frac{1}{30}$$

or $\qquad u = -30$ cm *i.e.*, $LO = 30$ cm

The image formed first by reflection from the mirror and then by refraction through the lens will be located at *O′* only if the image formed by reflection from the mirror is formed at *O* *i.e.*, if distance $OM = R$.

Hence $LO + OM = 30 \text{ cm} + R = 50$ cm

or $\qquad R = \mathbf{20}$ **cm.**

Example 66. *Calculate the distance d, so that a real image of an object at O, 15 cm in front of a convex lens of focal length 10 cm be formed at the same point O. The radius of curvature of the mirror is 20 cm. Will the image be inverted or erect ?* [CBSE Sample Paper 08]

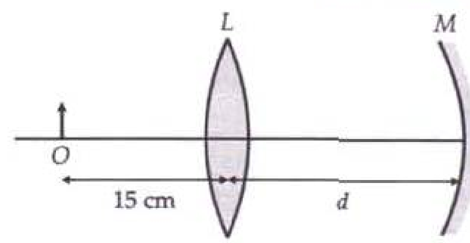


Fig. 9.90

Solution. The final image will be formed at the same point *O* if the concave mirror reverses the path of light incident on it. For this the image formed by the lens must be located at the centre of curvature of mirror *M*. Then the light will fall normally on *M* and will retrace its path after reflection.

For refraction through the convex lens,

$$u = -15 \text{ cm}, \; f = +10 \text{ cm}$$

$$\therefore \qquad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{-15} = \frac{1}{30}$$

or $\qquad v = 30$ cm

For concave mirror, $R = 20$ cm

Hence $\quad d = v + R = 30 + 20 = 50$ cm

The final image formed at *O* will be an inverted image.

Example 67. *In the following ray diagram are given the positions of an object O, image I and two lenses L_1 and L_2. The focal length of L_1 is also given. Find the focal length of L_2.*

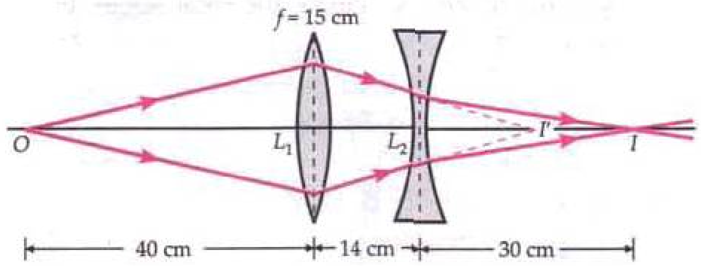


Fig. 9.91

Solution. *For the convex lens* :

$$f = +15 \text{ cm}, \quad u = -40 \text{ cm}, \quad v = ?$$

From thin lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{15} - \frac{1}{40} = \frac{5}{120} = \frac{1}{24}$$

or $\qquad v = +24$ cm

The image I' formed by the convex lens serves an object for the concave lens.

∴ *For the concave lens* :

$$u = +(24 - 14) = +10 \text{ cm}, \quad v = +30 \text{ cm}, \quad f = ?$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{30} - \frac{1}{10} = -\frac{2}{30} = -\frac{1}{15}$$

or $\qquad$ **$f = -15$ cm.**

Example 68. *From the ray diagram shown below, calculate the focal length of the concave lens.*

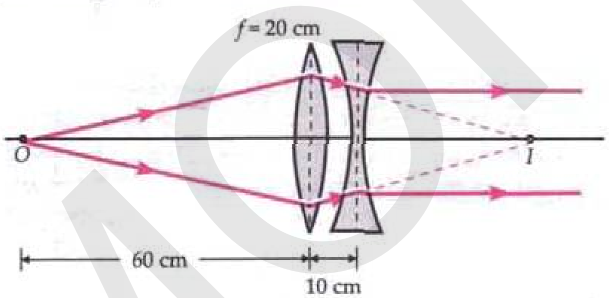


Fig. 9.92

Solution. *For the convex lens* :

$$f = +20 \text{ cm}, \quad u = -60 \text{ cm}, \quad v = ?$$

From thin lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{60} = +\frac{2}{60} = +\frac{1}{30}$$

or $\qquad v = +30$ cm

The image I' formed by the convex lens serves as an object for the concave lens. But the rays converging on the concave lens become parallel after refraction through it and form image at infinity.

∴ *For the concave lens* :

$$u = +(30 - 10) = +20 \text{ cm}, \quad v = \infty, \quad f = ?$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{\infty} - \frac{1}{20} = -\frac{1}{20}$$

or $\qquad$ **$f = -20$ cm.**

Example 69. *A convex lens of focal length 10 cm is placed coaxially 5 cm away from a concave lens of focal length 10 cm. If an object is placed 30 cm in front of the convex lens, find the position of the final image formed by the combined system.* [CBSE OD 09]

Solution. *For the convex lens* :

$$f = +10 \text{ cm}, \quad u = -30 \text{ cm}$$

From the lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}$$

or $\qquad v = +15$ cm

This image is at 10 cm from the concave lens which is placed at 5 cm from the convex lens. It will act as a virtual object.

For concave lens :

$$u = +10 \text{ cm}, \quad f = -10 \text{ cm}$$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = -\frac{1}{10} + \frac{1}{10} = 0$$

or $\qquad v = \infty$

Hence the final image is formed at infinity. The image formation is shown below.

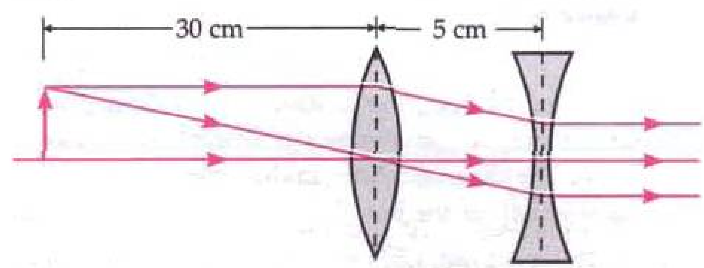


Fig. 9.93

Problems For Practice

1. Where should an object be placed from a converging lens of focal length 20 cm, so as to obtain a real image of magnification 2 ? **[CBSE OD 01]**
(Ans. 30 cm)

2. A convex lens is used to throw on a screen 10 m from the lens, a magnified image of an object. If the magnification is to be 19, find the focal length of the lens. **[CBSE OD 10]**
(Ans. 0.5 m)

3. An object is placed at 0.06 m from a convex lens of focal length 0.10 m. Calculate the position and nature of the image. **[Haryana 02]**

(Ans. 15 cm, virtual)

4. At what distance should an object be placed from a convex lens of focal length 15 cm to obtain an image three times the size of the object ? **[Punjab 04]**

(Ans. 20 cm for real image, 10 cm for virtual image)

5. A convex lens is placed on an optical bench and is moved till it gives a real image of an object at a minimum distance of 80 cm from the latter. Find the focal length of the lens. If the object is placed at a distance of 15 cm from the lens, find the position of the image. (Ans. $f = 20$ cm, virtual and erect image at 60 cm from the lens on the same side as the object)

6. A convex lens of focal length 30 cm is placed between a screen and a square plate of area 4 cm^2. The image of the plate formed on the screen is 16 cm^2. Calculate the distance between the plate and the screen. (Ans. 135 cm)

7. The image of a needle placed 45 cm from a lens is formed on a screen placed 90 cm on the other side of the lens. Find the displacement of the image, if the object is moved by 5.0 cm away from the lens. **[Punjab 2000]**

(Ans. 15 cm, towards the lens)

8. A source of light and a screen are placed 90 cm apart. Where should a convex lens of 20 cm focal length be placed in order to form a real image of the source on the screen ?

(Ans. 60 cm or 30 cm from the source)

9. An object is placed at a distance of 1.5 m from a screen and a convex lens is interposed between them. The magnification produced is 4. What is the focal length of the lens ? (Ans. 0.24 m)

10. A screen is placed 80 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations, separated by 10 cm. Calculate the focal length of the lens used. **[CBSE F 08]**

(Ans. 19.7 cm)

11. When a slide is placed 15 cm behind the lens in a projector, an image is formed at a distance of 3 m in front of the lens. (*i*) Show the image formation by a ray diagram. (*ii*) Find the focal length of the lens. (*iii*) What will be the distance between the lens and the slide in order to get an image at a distance 4 metre from the lens ? (*iv*) Determine the magnification for the case (*iii*).

[Ans. (*ii*) + 14.3 cm (*iii*) – 14.8 cm (*iv*) – 27]

12. In the diagram shown below, rays are coming from infinity and after passing through both the lenses meet on a screen placed at a distance of 30 cm from the concave lens. Calculate the focal length of the concave lens. (Ans. 15 cm)

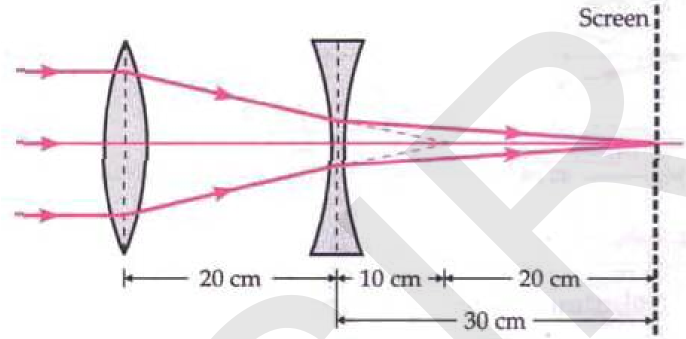


Fig. 9.94

13. In the following ray-diagram are shown the positions of the object *O*, image *I*, two lenses and a plane mirror. The focal length of one of the lenses is also given. Calculate the focal length of the other lens.

(Ans. 20 cm)

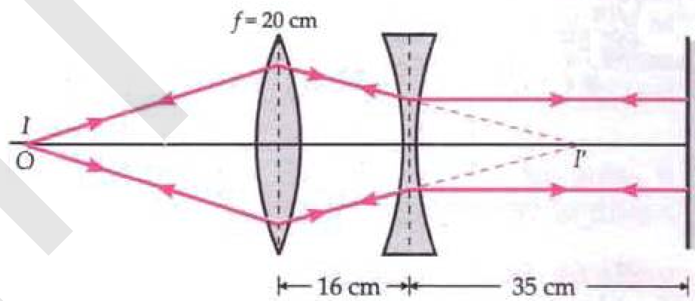


Fig. 9.95

14. In the following diagram are shown the positions of an object *O*, image *I* and two lenses. The focal length of one lens is also given. Determine the focal length of the other lens. (Ans. 12.63 cm)

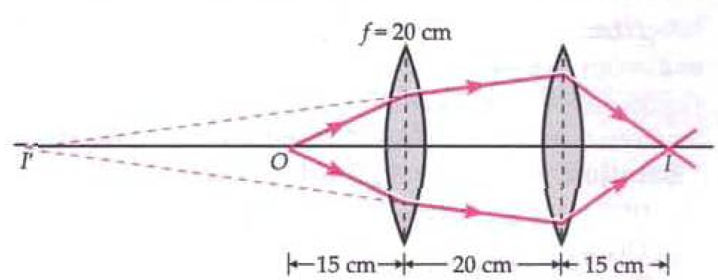


Fig. 9.96

HINTS

1. Use $m = \dfrac{f}{f+u}$.

2. $m = \dfrac{f-v}{f}$ or $-19 = \dfrac{f-10}{f}$ $\Rightarrow$ $f = 0.5$ **m.**

3. Here $u = -0.06$ m $= -6$ cm, $f = 0.10$ m $= 10$ cm

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{-6} = -\frac{1}{15}$$

or $v = $ **−15 cm**, *virtual.*

4. (*i*) *For real image* : $m = -3,\ f = +15$ cm, $u = ?$

$$m = \frac{f}{f+u} \quad \text{or} \quad -3 = \frac{15}{15+u} \quad \text{or} \quad u = -\mathbf{20\ cm.}$$

(*ii*) *For virtual image* : $m = +3,\ f = +15$ cm, $u = ?$

$$+3 = \frac{15}{15+u} \quad \text{or} \quad u = -\mathbf{10\ cm.}$$

5. The minimum distance between an object and its real image formed by a convex lens is $4f$.

$\therefore \quad 4f = 80 \quad$ or $\quad f = $ **20 cm**

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{15} = -\frac{1}{60}$$

or $\quad v = $ **− 60 cm.**

6. Areal magnification $= \dfrac{16\ \text{cm}^2}{4\ \text{cm}^2} = 4$

Linear magnification $= \pm\sqrt{4} = \pm 2$

For real image,

$$m = \frac{v}{u} = -2 \quad \text{or} \quad v = -2u$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-2u} - \frac{1}{u} = -\frac{3}{2u}$$

or $\quad u = -\dfrac{3f}{2} = -\dfrac{3 \times 30}{2} = -45$

$$v = -2u = +90\ \text{cm}$$

Distance between the plate and the screen

$$= |u| + |v| = 45 + 90 = \mathbf{135\ cm.}$$

7. Here $u = -45$ cm, $v = +90$ cm

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{90} + \frac{1}{45} = \frac{1}{30} \quad \text{or} \quad f = 30\ \text{cm}$$

When the needle is moved 5 cm away from the lens,

$$u' = -(45+5) = -50\ \text{cm},$$

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'} = \frac{1}{30} + \frac{1}{-50} = \frac{2}{150} = \frac{1}{75}$$

or $\quad v' = 75$ cm

Displacement of image $= v - v' = 90 - 75$

$= $ **15 cm**, *towards the lens.*

8. Let $u = -x$, then $v = 90 - x$, $f = +20$ cm

As $\quad f = \dfrac{uv}{u - v}$

$$\therefore \quad 20 = \frac{-x(90-x)}{-x-(90-x)} = \frac{-90x + x^2}{-90}$$

or $\quad -1800 = -90x + x^2$

or $\quad x^2 - 90x + 1800 = 0$

or $\quad (x-60)(x-30) = 0$

or $\quad x = 30, 60$

or $\quad u = $ **− 30 cm, − 60 cm.**

9. Here $m = \dfrac{v}{u} = -4 \quad$ or $\quad u = -\dfrac{v}{4}$

Also $\quad |u| + |v| = 1.5$

or $\quad \dfrac{v}{4} + v = 1.5$

or $\quad v = 1.2$ m

and $\quad u = -\dfrac{1.2}{4} = -0.3$ m

$$f = \frac{uv}{u - v} = \frac{-0.3 \times 1.2}{-0.3 - 1.2} = \mathbf{0.24\ m.}$$

10. Here $D = 80$ cm, $d = 10$ cm

$$f = \frac{D^2 - d^2}{4D} = \frac{90 \times 70}{320} = \mathbf{19.7\ cm.}$$

11. (*i*) The ray diagram is shown in Fig. 9.97.

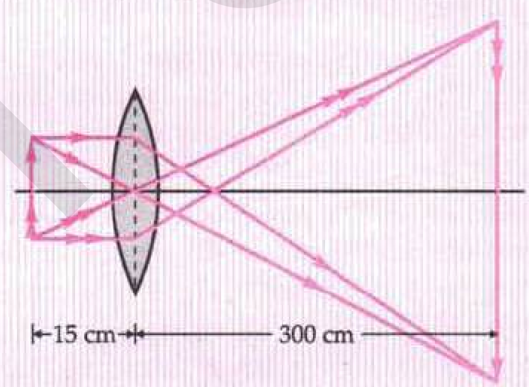


Fig. 9.97

(*ii*) Here $u = -15$ cm, $v = +3$ m $= +300$ cm

As $\quad \dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u} \quad \therefore \quad \dfrac{1}{f} = \dfrac{1}{300} - \dfrac{1}{-15} = +\dfrac{21}{300}$

or $\quad f = +\dfrac{300}{21} = $ **+ 14.3 cm.**

(*iii*) Here $\quad v = +4$ m $= +400$ cm, $f = \dfrac{300}{21}$ cm

Distance between lens and slide, $u = ?$
Using thin lens formula, we have

$$\frac{21}{300} = \frac{1}{400} - \frac{1}{u} \quad \text{or} \quad \frac{1}{u} = \frac{1}{400} - \frac{21}{300} = \frac{-81}{1200}$$

or $\quad u = -\dfrac{1200}{81} = $ **− 14.8 cm.**

(*iv*) Magnification, $m = \dfrac{v}{u} = \dfrac{400}{-1200/81} = $ **− 27.**

12. Here $u = +10$ cm, $\quad v = +30$ cm

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{30} - \frac{1}{10} = -\frac{2}{30} \quad \text{or} \quad f = \mathbf{-15\ cm.}$$

13. *For convex lens.* $u = -45$ cm, $f = +20$ cm

$$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} + \frac{1}{-45} = \frac{1}{36} \quad \text{or} \quad v = +36 \text{ cm}$$

For concave lens. $u = +(36 - 16) = 20$ cm, $v = +\infty$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{\infty} - \frac{1}{20} = -\frac{1}{20} \quad \text{or} \quad f = -\mathbf{20\ cm}.$$

14. *For first convex lens.* $u = -15$ cm, $f = +20$ cm,

$$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} + \frac{1}{-15} = -\frac{1}{60} \quad \text{or} \quad v = -60 \text{ cm}$$

For second convex lens. I' is virtual object and I is the real image. Here

$$u = -(60 + 20) = -80 \text{ cm}, \quad v = +15 \text{ cm}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{15} + \frac{1}{80} = \frac{19}{240}$$

or $f = \frac{240}{19} = \mathbf{12.63\ cm}.$

9.28 POWER OF A LENS

46. *What is meant by power of lens ? Give and define its SI unit. Which type of lens has a positive power and which one negative ? Express power of a lens in terms of its refractive index and radii of curvature.*

Power of a lens. The power of a lens is a measure of the degree of convergence or divergence of the light rays falling on it. As shown in Fig. 9.98, a convex lens of shorter focal length bends light rays towards the principal axis through a larger angle, by focussing them closer to the optical centre. Hence *smaller the focal length of a lens, more is ability to bend light rays and greater is its power.*

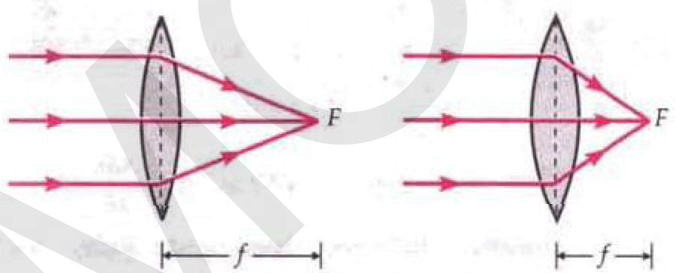


Fig. 9.98 (*a*) Large f, small bending power,
(*b*) Small f, large bending power.

The power of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from the optical centre.

In Fig. 9.99, a beam of light is incident at distance h from the optical centre O of a convex lens of focal length f. It converges the beam by angle δ.

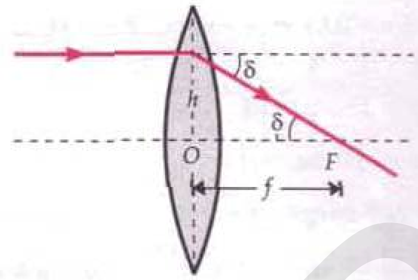


Fig. 9.99 Power of a lens.

Clearly, $\tan \delta = \frac{h}{f}$

If $h = 1$, then $\tan \delta = \frac{1}{f}$ or $P = \frac{1}{f}$

Thus the power of a lens may also be defined as the reciprocal of its focal length.

SI unit of power. The SI unit of power is ***dioptre***, denoted by D. If $f = 1$ m, then

$$P = \frac{1}{1\text{m}} = 1\text{m}^{-1} = \mathbf{1\ dioptre\ (D)}$$

*One **dioptre** is the power of a lens whose principal focal length is 1 metre.*

The focal length of a converging lens is positive and that of a diverging lens is negative. Thus, *the power of a converging lens is positive and that of a diverging lens is negative.* We can measure the power of a lens directly by a device called ***dioptremeter***. Thus, when an optician prescribes a corrective lens of power + 2.5 D, the required lens is a convex lens of focal length, $f = 1/(+2.5 \text{ D}) = +0.40 \text{ m} = +40$ cm. Similarly, a power of -4.0 D means a concave lens of focal length -25 cm.

By using lens maker's formula, the power of a lens can be expressed in terms of its refractive index μ and radii of curvature R_1 and R_2 as follows :

$$P = \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

As the power of a lens is reciprocal of its focal length, so it characterises the focal properties of the lens, such as nature, size and position of image, etc.

9.29 COMBINATION OF THIN LENSES

47. *Why do we use lens combinations in optical instruments ? Write an expression for the total magnification produced by combination of lenses.*

Lens combinations. In many optical instruments, two or more lenses are used either in contact or with a gap between them. The purpose of using a lens combination is

(*i*) To magnify an image.

(*ii*) To increase the sharpness of the final image by minimising certain defects or aberrations in it.

(*iii*) To erect the final image.

(*iv*) To increase the field of view.

Different lens combinations are used in the objectives of cameras, microscopes, telescopes and other optical instruments.

Total magnification. When lenses are used in combination, each lens magnifies the image formed by the preceding lens. Hence the total magnification m is equal to the product of the magnifications m_1, m_2 and m_3 ...., produced by the individual lenses.

$$m = m_1 \times m_2 \times m_3 \times ......$$

48. *What is an equivalent lens ? Obtain an expression for the effective focal length of two thin lenses placed in contact coaxially with each other.*

Equivalent lens. *A single lens which forms the image of an object at the same position as is formed by a combination of lenses is called an equivalent lens.*

Equivalent focal length and power of two thin lenses in contact. As shown in Fig. 9.100, let L_1 and L_2 be two thin lenses of focal length f_1 and f_2 respectively, placed coaxially in contact with one another. Let O be a point object on the principal axis of the lens system.

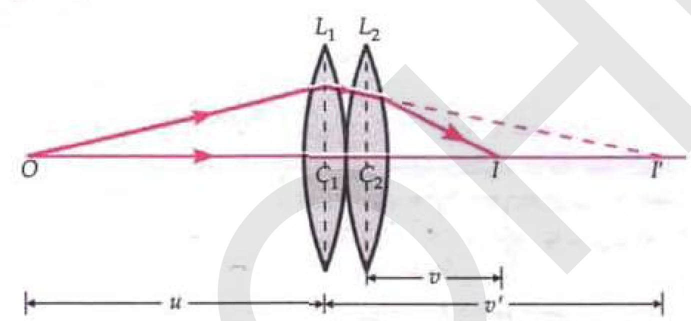


Fig. 9.100 Two thin lenses in contact.

Let $OC_1 = u$. In the absence of second lens L_2, the first lens L_1 will form a real image I' of O at distance $C_1I' = v'$. Using thin lens formula,

$$\frac{1}{f_1} = \frac{1}{v'} - \frac{1}{u} \qquad ...(1)$$

The image I' acts as a virtual object $(u = v')$ for the second lens L_2 which finally forms its real image I at distance v. Thus

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v'} \qquad ...(2)$$

Adding equations (1) and (2), we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \qquad ...(3)$$

For the combination of thin lenses in contact, if f is the equivalent focal length, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad ...(4)$$

From equations (3) and (4), we find that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

∴ Equivalent power,

$$P = P_1 + P_2$$

For n thin lenses in contact, we have

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ... + \frac{1}{f_n}$$

∴ Equivalent power,

$$P = P_1 + P_2 + P_3 + ... + P_n$$

49. *Derive an expression for the focal length of the combination of two thin lenses of focal lengths f_1 and f_2, when they are separated by distance d.*

Thin lenses separated by a small distance. As shown in Fig. 9.101, consider two thin lenses L_1 and L_2 of focal lengths f_1 and f_2, respectively. The two lenses are placed coaxially, distance 'd' apart.

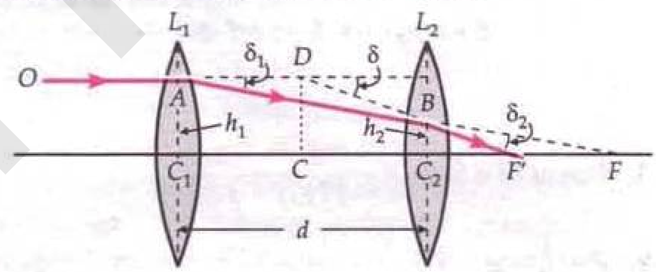


Fig. 9.101 Two thin lenses separated by a small distance.

Suppose a ray OA traversing parallel to the principal axis is incident on lens L_1. It is refracted along AF, F being the second principal focus of L_1. The deviation produced by L_1 is

$$\delta_1 \simeq \tan \delta_1 = \frac{h_1}{f_1}$$

The emergent ray is further refracted by second lens L_2 along BF'. Since the incident ray OA is parallel to the principal axis, F' should be second principal focus of the combination. The deviation produced by the second lens L_2 is

$$\delta_2 \simeq \tan \delta_2 = \frac{h_2}{f_2}$$

The final emergent ray BF', when produced backwards, meets the incident ray at point D. Obviously, δ is the final deviation produced. A single thin lens placed at C will produce the same deviation as by the combination of two lenses. Thus distance CF' is the second focal length of the combination. If f is the focal length of the combination, then

$$\delta = \frac{h_1}{f}$$

It is obvious from Fig. 9.101, that

$$\delta = \delta_1 + \delta_2$$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$$

As $\Delta AC_1F \sim \Delta BC_2F$, therefore, we have

$$\frac{AC_1}{C_1F} = \frac{BC_2}{C_2F} \quad \text{or} \quad \frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

or $$h_2 = \frac{f_1 - d}{f_1} . h_1$$

Hence $$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{f_1 - d}{f_1 f_2} . h_1$$

or $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

In terms of powers of the lenses,

$$P = P_1 + P_2 - d . P_1 . P_2$$

Examples based on

(i) Power of Lenses (ii) Combination of Lenses

Formulae Used

1. Power of a lens, $P = \frac{1}{f\,(\text{m})} = \frac{100}{f\,(\text{cm})}$
2. $P = \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$
3. For a combination of lenses, $m = m_1 \times m_2 \times m_3 \times ...$
4. For two lenses in contact, equivalent focal length F is given by

   $$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{or} \quad \text{Power, } P = P_1 + P_2$$

   For n lenses in contact, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + ... + \frac{1}{f_n}$

   Power $P = P_1 + P_2 + ... + P_n$.
5. The equivalent focal length F of two lenses separated by distance d is given by

   $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

   or Power, $P = P_1 + P_2 - d . P_1 . P_2$

Units Used

Focal lengths f, f_1, f_2, etc. and radii of curvature R_1 and R_2 are in metre or cm and powers P, P_1, P_2 etc. are in dioptre (D).

Example 70. *The radius of curvature of each surface of a convex lens of refractive index 1.5 is 40 cm. Calculate its power.* [CBSE Sample Paper 98]

Solution. Here $\mu = 1.5$, $R_1 = +40 \text{ cm} = 0.40 \text{ m}$, $R_2 = -40 \text{ cm} = -0.40 \text{ m}$

$$P = \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= (1.5 - 1)\left[\frac{1}{0.40} + \frac{1}{0.40}\right] = \mathbf{2.5\ D.}$$

Example 71. *A double convex lens of +5 D is made of glass of refractive index 1.5 with both faces of equal radii of curvature. Find the value of curvature.* [CBSE F 15]

Solution. Let $R_1 = +R$ and $R_2 = -R$. Then

$$P = \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$5 = (1.5 - 1)\left[\frac{1}{R} + \frac{1}{R}\right] = 0.5 \times \frac{2}{R}$$

$\therefore$ $$R = \frac{0.5 \times 2}{5}\text{ m} = \frac{1}{5}\text{ m} = \mathbf{20\ cm.}$$

Example 72. *A convex lens is made of glass of refractive index 1.5. If the radius of curvature of the each of the two surfaces is 20 cm, find the ratio of the powers of the lens, when placed in air to its power, when immersed in a liquid of refractive index 1.25.*

Solution. Here $R_1 = 20 \text{ cm} = 0.2 \text{ m}$,

$R_2 = -20 \text{ cm} = -0.2 \text{ m}$, $\mu_g = 1.5$, $\mu_l = 1.25$

For the lens placed in air :

$$P_1 = (\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.5 - 1)\left[\frac{1}{0.2} - \frac{1}{-0.2}\right] = 5 \text{ D}$$

For the lens placed in liquid :

$$P_2 = \left(\frac{\mu_g}{\mu_l} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \left(\frac{1.5}{1.25} - 1\right)\left[\frac{1}{0.2} - \frac{1}{-0.2}\right] = 2 \text{ D}$$

$\therefore$ $$\frac{P_1}{P_2} = \frac{5}{2} = \mathbf{5 : 2.}$$

Example 73. (*i*) *If $f = +0.5$ m, what is the power of the lens ?* (*ii*) *The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass ?* (*iii*) *A convex lens has 20 cm focal length in air. What is its focal length in water ? (Refractive index of air-water = 1.33, refractive index for air-glass = 1.5.)* [NCERT]

Solution. (*i*) Here, $f = +0.5$ m

$\therefore$ $$P = \frac{1}{f} = \frac{1}{+0.5} = \mathbf{+2\ D.}$$

(ii) Here, $R_1 = +10$ cm, $R_2 = -15$ cm, $f = +12$ cm

As $$\frac{1}{f} = (\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$\therefore$ $$\frac{1}{12} = (\mu_g - 1)\left[\frac{1}{10} - \frac{1}{-15}\right] = (\mu_g - 1)\times\frac{1}{6}$$

or $$\mu_g - 1 = 0.5$$

or $$\mu_g = \mathbf{1.5.}$$

(iii) Here, $f_a = +20$ cm, $\mu_w = 1.33$, $\mu_g = 1.5$, $f_w = ?$

For the lens placed in air :

$$\frac{1}{f_a} = (\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{20} = (1.5 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{20\times 0.5} = \frac{1}{10}$$

For the lens placed in water :

$$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{1.5}{1.33} - 1\right)\times\frac{1}{10}$$

or $$f_w = \frac{10\times 1.33}{0.17} = \mathbf{+78.2\ cm.}$$

Example 74. *Two thin lenses of focal lengths + 10 cm and − 5 cm are kept in contact. What is the (i) focal length and (ii) power of the combination ?* [CBSE D 93]

Solution. Here $f_1 = +10$ cm $= 0.10$ m,

$$f_2 = -5\ \text{cm} = -0.05\ \text{m}$$

(i) $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.10} + \frac{1}{-0.05} = -10$$

$$f = -\frac{1}{10}\ \text{m} = \mathbf{-10\ cm.}$$

(ii) $$P = P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2} = \mathbf{-10\ D.}$$

Example 75. *A converging lens of focal length 50 cm is placed coaxially in contact with another lens of unknown focal length. If the combination behaves like a diverging lens of focal length 50 cm, find the power and nature of the second lens.* [CBSE OD 04C]

Solution. Here

$f_1 = +50$ cm (converging lens)

$f = -50$ cm (diverging combination)

As $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\therefore$ $$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{-50} - \frac{1}{50} = -\frac{1}{25}$$

or $$f_2 = -25\ \text{cm} = -0.25\ \text{m}$$

The second lens is a diverging lens. Its power is

$$P_2 = \frac{1}{f_2} = \frac{1}{-0.25\ \text{m}} = \mathbf{-4\ D.}$$

Example 76. *Two lenses of powers + 15 D and − 5 D are in contact with each other forming a combination lens.*

(a) *What is the focal length of this combination ?*

(b) *An object of size 3 cm is placed at 30 cm from this combination of lenses. Calculate the position and size of the image formed.* [CBSE D 02]

Solution. (a) Power of combination,

$$P = P_1 + P_2 = +15 - 5 = 10\ \text{D}$$

$\therefore$ Focal length of combination,

$$f = \frac{1}{P} = \frac{1}{10}\ \text{m} = \mathbf{10\ cm.}$$

(b) Given $h_1 = 3$ cm, $u = -30$ cm, $f = 10$ cm

Using thin lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}$$

or $$v = +15\ \text{cm}$$

As $$m = \frac{h_2}{h_1} = \frac{v}{u}$$

$\therefore$ $$h_2 = \frac{v}{u}\times h_1 = \frac{15}{-30}\times 3 = \mathbf{-1.5\ cm}$$

Real inverted image of size 1.5 cm is formed at 15 cm on the other side of the combination.

Example 77. *A glass convex lens has a power of + 10 D. When this lens is totally immersed in a liquid, it acts as a concave lens of focal length 50 cm. Calculate the refractive index of the liquid. Given ${}^a\mu_g = 1.5$.* [Punjab 01]

Solution. Here $P = +10$ D

$$f_a = \frac{1}{P} = \frac{1}{10}\ \text{m} = 10\ \text{cm}$$

But $$\frac{1}{f_a} = ({}^a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{10} = (1.5 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{10\times 0.5} = \frac{1}{5}$$

When the lens is immersed in the liquid,

$$\frac{1}{f_l}=\left(\frac{{}^a\mu_g}{{}^a\mu_l}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

or $$\frac{1}{-50}=\left(\frac{1.5}{{}^a\mu_l}-1\right)\times\frac{1}{5}$$

or $$\frac{1.5}{{}^a\mu_l}-1=-\frac{1}{10}$$

or $$\frac{1.5}{{}^a\mu_l}=1-0.1=0.9$$

or $${}^a\mu_l=\frac{1.5}{0.9}=\mathbf{1.67}.$$

Example 78. *A real image is formed by the lens at a distance of 20 cm from the lens. The image shifts towards the combination by 10 cm when a second lens is brought in contact with the first lens. Determine the power of the second lens.*

Solution. As the image is real, so the lens is convex. Let its focal length be f_1. It forms image on the other side, so $v=+20$ cm.

Using lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we have

$$\frac{1}{20}-\frac{1}{u}=\frac{1}{f_1} \quad ...(i)$$

When the second lens is placed in contact with the first one, image shifts 10 cm closer to the combination, so the second lens is also convex. If f_2 is the focal length of second lens and f that of the combination, then

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

For the combination, $v=20-10=+10$ cm, so from lens formula,

$$\frac{1}{10}-\frac{1}{u}=\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \quad ...(ii)$$

Subtracting (*i*) from (*ii*), we get

$$\frac{1}{f_2}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

$$f_2=20 \text{ cm}=0.2 \text{ m}$$

Power of second lens,

$$P=\frac{1}{f_2}=\frac{1}{0.2}=\mathbf{0.5\ D.}$$

Example 79. *A small object is placed at a distance of 15 cm from two coaxial thin lenses in contact. The focal length of each lens is 25 cm. What will be the distance between the object and its image when both the lenses are (i) convex, (ii) concave.*

Solution. (*i*) *When both the lenses are convex* :

$$f_1=f_2=+25 \text{ cm}$$

Focal length f of the combination is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{25}+\frac{1}{25}=\frac{2}{25}$$

or $$f=12.5 \text{ cm}$$

Now $u=-15$ cm, $f=12.5$ cm

$$\therefore \quad \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{12.5}-\frac{1}{15}=\frac{1}{75}$$

or $$v=+75 \text{ cm}$$

Thus the object is at 15 cm on one side of the combination and the image is at 75 on the other side of the combination.

∴ Distance between the object and image

$$=15+75=\mathbf{90\ cm.}$$

(*ii*) *When both the lenses are concave* :

$$f_1=f_2=-25 \text{ cm.}$$

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=-\frac{1}{25}-\frac{1}{25}=-\frac{2}{25}$$

or $$f=-12.5 \text{ cm}$$

Now $u=-15$ cm, $f=-12.5$ cm

$$\therefore \quad \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=-\frac{1}{12.5}-\frac{1}{15}=-\frac{11}{75}$$

or $$v=-\frac{75}{11}=-6.8 \text{ cm}$$

Thus the image is at 6.8 cm from the combination on the same side as the object.

∴ Distance between the object and the image

$$=15-6.8=\mathbf{8.2\ cm.}$$

Example 80. *An object is situated at 20 cm on the left of a convex lens of focal length 10 cm. Another convex lens of focal length 12.5 cm is placed at a distance of 30 cm on the right of the first lens. Find the position and magnification of the final image. State also the nature of the image.*

Solution. As shown in Fig. 9.102, the lens L_1 forms image $A'B'$ of object AB. The image $A'B'$ lies within focal length of lens L_2 and acts as an object for it. $A''B''$ is the final image formed by L_2.

For the lens L_1 : $u=-20$ cm, $f=+10$ cm

From thin lens formula,

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{10}-\frac{1}{20}=+\frac{1}{20}$$

$$v=+20 \text{ cm}$$

i.e., $$L_1A'=20 \text{ cm}$$

$$A'L_2=L_1L_2-L_1A'=30-20=10 \text{ cm}$$

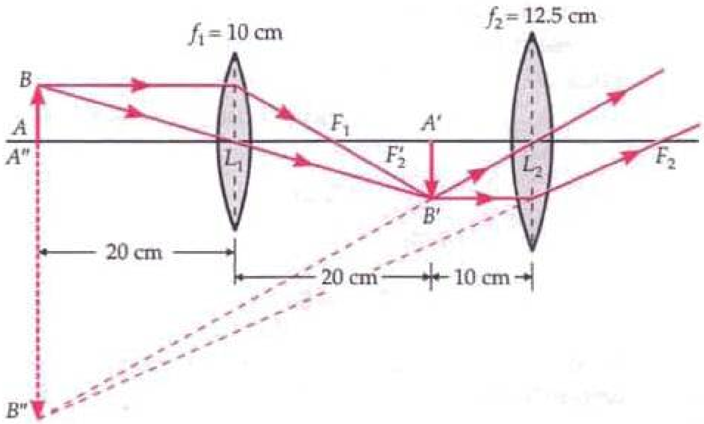


Fig. 9.102

For the lens L_2 :

$$u = -10 \text{ cm}, \; f = +12.5 \text{ cm}.$$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{12.5} - \frac{1}{10} = -\frac{1}{50}$$

or $\quad v = 50$ cm

The final image $A''B''$ is formed at 50 cm from L_2 on its left. It lies at the position of object AB and is virtual and inverted with respect to the object.

$$m_1 = \frac{A'B'}{AB} = \frac{v}{u} = \frac{20}{-20} = -1$$

$$m_2 = \frac{A''B''}{A'B'} = \frac{v}{u} = \frac{-50}{-10} = 5$$

Total magnification,

$$m = m_1 \times m_2 = -1 \times 5 = -5.$$

Example 81. *Find the position of the image formed by the lens combination given in Fig. 9.103.* **[NCERT]**

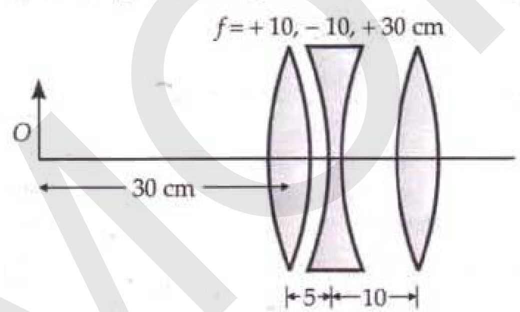


Fig. 9.103

Solution. For the image formed by the first lens,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10} \quad \text{or} \quad v_1 = 15 \text{ cm}$$

The image formed by the first lens serves as the object for the second. This is at a distance of $(15 - 5)$ cm $= 10$ cm to the right of the second lens. Though the image is real it serves as a virtual object for the second lens, which means that the rays appear to come from it. For the second lens,

$$\frac{1}{v_2} - \frac{1}{10} = \frac{1}{-10} \quad \text{or} \quad v_2 = \infty$$

The virtual image is formed at an infinite distance to the right of the second lens. This acts as an object for the third lens.

$$\frac{1}{v_3} - \frac{1}{u_3} = \frac{1}{f_3}$$

or $\quad \frac{1}{v_3} - \frac{1}{\infty} = \frac{1}{30}$ cm $\quad$ or $\quad$ $v_3 = $ **30 cm**

The final image is formed 30 cm to the right of the third lens.

Example 82. *An equiconvex lens with radii of curvature of magnitude R each, is put over a liquid layer poured on top of a plane mirror. A small needle, with its tip on the principal axis of the lens, is moved along the axis until its inverted real image coincides with the needle itself. The distance of the needle from the lens is measured to be 'a'. On removing the liquid layer and repeating the experiment the distance is found to be 'b'.*

Given that two values of distances measured represent the focal length values in the two cases, obtain a formula for the refractive index of the liquid. **[CBSE S.Paper 08]**

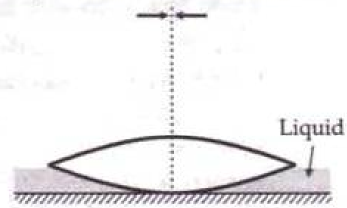


Fig. 9.104

Solution. Clearly, equivalent focal length of equiconvex lens and water lens, $f = a$

Focal length of equiconvex lens, $f_1 = b$

Focal length f_2 of water lens is given by

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{a} - \frac{1}{b} = \frac{b-a}{ab}$$

or $\quad f_2 = \frac{ab}{b-a}$

The water lens formed between the plane mirror and the equiconvex lens is a planoconcave lens.

For this lens, $\quad R_1 = -R \;$ and $\; R_2 = \infty$

Using lens maker's formula,

$$\frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $\quad \frac{b-a}{ab} = (\mu - 1)\left[\frac{1}{-R} - \frac{1}{\infty}\right]$

or $\quad \mu - 1 = \frac{(a-b)R}{ab} \quad$ or $\quad \boldsymbol{\mu = 1 + \frac{(a-b)R}{ab}}.$

Problems For Practice

1. A convex lens of focal length 25 cm is placed coaxially in contact with a concave lens of focal length 20 cm. Determine the power of the combination. Will the system be converging or diverging in nature ? **[CBSE D 13]**
(Ans. –1D, diverging)

2. A convex lens of focal length 20 cm is placed coaxially in contact with a concave lens of focal length 25 cm. Determine the power of the combination. Will the system be converging or diverging in nature ? **[CBSE D 13]**
(Ans. +1D, converging)

3. Two lenses of powers 10 D and – 5 D are placed in contact.
   (*i*) Calculate the power of the new lens.
   (*ii*) Where should an object be held from the lens, so as to obtain a virtual image of magnification 2 ? **[CBSE OD 08]**
[Ans. (*i*) 5 D (*ii*) – 10 cm]

4. At what distance must an object be placed from convex lens of power 4 D to obtain a real image three times the size of the object ? (Ans. 33.33 cm)

5. Find the focal length and power of a convex lens, which when placed in contact with a concave lens of focal length 25 cm, forms a real image 5 times the size of the object placed 20 cm from the combination. (Ans. 10 cm, 10 D)

6. The power of a thin convex lens of glass is 5 D. When it is immersed in a liquid of refractive index μ, it behaves like a divergent lens of focal length 1 m. Calculate μ of liquid, if μ of glass = 3 / 2. (Ans. 5/3)

7. A compound lens is made of two lenses having powers + 15.5 D and – 5.5 D. An object of 3 cm height is placed at a distance of 30 cm from this compound lens. Find the size of the image. (Ans. – 1.5 cm)

8. Rays coming from an object situated at infinity, fall on a convex lens and an image is formed at a distance of 16 cm from the lens. When a concave lens is kept in contact with the convex lens, the image is formed at a distance of 20 cm from the lens combination. Calculate the focal length of the concave lens. (Ans. – 80 cm)

9. An equiconvex lens, with radii of curvature of magnitude 20 cm each, is put over a liquid layer poured on top of a plane mirror. A small needle, with its tip on the principal axis of the lens, is moved along the axis until its inverted real image coincides with the needle itself. The distance of the needle, from the lens, is measured to be 30 cm.

   On removing the liquid layer, and repeating the experiment, the distance is measured to be 20 cm. Given that the two values of the distance measured represent the focal length values in the two cases, calculate the refractive index of the liquid.
[CBSE D 08C] (Ans. 1.33)

HINTS

1. $P = P_1 + P_2 = \frac{100}{f_1(\text{cm})} + \frac{100}{f_2(\text{cm})}$

$= \frac{100}{25} + \frac{100}{-20} = 4 - 5 = \mathbf{-1D}$

As the power of the combination is negative, the system will be diverging in nature.

2. Power of convex lens, $P_1 = \frac{100}{20} = +5\text{D}$

Power of concave lens, $P_2 = \frac{100}{-25} = -4\text{D}$

Power of the combination, $P = P_1 + P_2 = \mathbf{+1D}$

As the power P is positive, the combination will be converging in nature.

3. (*i*) $P = P_1 + P_2 = 10 - 5 = \mathbf{5D}$.

(*ii*) $f = \frac{1}{P} = \frac{1}{5}\text{ m} = 20\text{ cm}$

As $m = \frac{f}{f+u}$

$\therefore \quad 2 = \frac{20}{20+u}$ or $u = \mathbf{-10\ cm.}$

4. Here $m = -3$, $P = 4$ D, $f = \frac{1}{4}\text{ m} = 25\text{ cm}$

As $m = \frac{f}{f+4}$

$\therefore \quad -3 = \frac{25}{25+u}$

or $u = -\frac{100}{3} = \mathbf{-33.33\ cm.}$

5. Here $m = -5$, $u = -20$ cm

As $m = \frac{f}{f+u}$

$\therefore \quad -5 = \frac{f}{f-20}$ or $f = \frac{50}{3}$ cm

Now $\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{3}{50} - \frac{1}{-25} = \frac{3+2}{50} = \frac{1}{10}$

or $f_2 = \mathbf{+10\ cm}$ and $P = \frac{100}{10} = \mathbf{10\ D.}$

6. $P_a = (\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

or $+5 = \left(\frac{3}{2} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ...(*i*)

$$P_l = \frac{1}{f_l} = \left(\frac{\mu_g}{\mu_l} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\frac{1}{-1} = \left(\frac{3/2}{\mu_l} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(ii)$$

Divide (*i*) by (*ii*) to get the value of μ_l.

7. Equivalent power, $P = +15.5 - 5.5 = 10\text{ D}$

Equivalent focal length, $F = \frac{100}{P} = \frac{100}{10} = 10\text{ cm}$

Now $m = \frac{f}{f+u} = \frac{10}{10-30} = -\frac{1}{2}$

But $m = \frac{h_2}{h_1}$

$\therefore \quad \frac{h_2}{3} = -\frac{1}{2}$ or $h_2 = \mathbf{-1.5\text{ cm}}$.

8. Focal length of convex lens, $f_1 = +16\text{ cm}$

Combined focal length, $f = +20\text{ cm}$

As $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\therefore \quad \frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{20} - \frac{1}{16} = -\frac{1}{80}$

or $f_2 = \mathbf{-80\text{ cm}}$.

9. Proceed as in Example 82.

9.30 COMBINATION OF A LENS AND A MIRROR

Image formed by a combination of a lens and a mirror. Consider an object placed in front of a lens coaxially arranged with a mirror. The rays from the object first suffer refraction through the lens and then get reflected by the mirror. We can study the image formation by such combinations through some numerical examples given ahead.

Examples based on Image formation by a combination of a lens and a mirror

Concept Used

We first find the position of the image formed by the lens by using thin lens formula. Taking this image as real (or virtual) object for the mirror, we use mirror formula to locate the position of the final image formed by the combination.

Example 83. *A convex lens is placed in contact with a plane mirror. An axial point object, at a distance of 20 cm from this combination, has its image coinciding with itself. What is the focal length of the convex lens ?* [CBSE D 14]

Solution. Figure 9.105(*a*) shows a convex lens *L* placed in contact with plane mirror *M*. *P* is the point object, kept in front of this combination at a distance of 20 cm, from it. As the image coincides with the object itself, the rays from the object, after refraction from the lens, should fall *normally* on the mirror *M*, so that they retrace their path. For this, the rays from *P*, after refraction from the lens must form a *parallel* beam perpendicular to *M*. For clarity, *M* has been shown at a small distance from *L*, in Fig. 9.105(*b*). As the rays from *P*, form a parallel beam after refraction, *P* must be at the focus of the lens. Hence focal length of the lens is 20 cm.

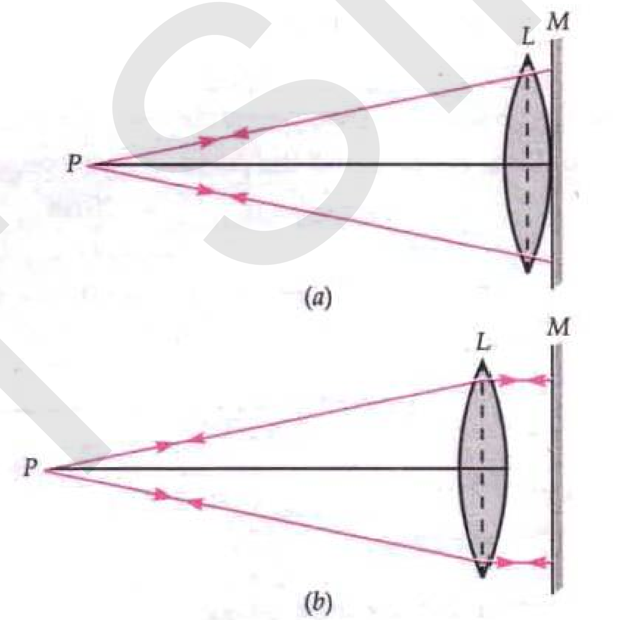


Fig. 9.105

Example 84. *A convex lens is placed over a plane mirror. A pin is now positioned so that there is no parallax between the pin and its image formed by this lens-mirror combination. How can this observation be used to find the focal length of the convex lens ? Give appropriate reasons in support of your answer.* [CBSE Sample Paper 13]

Solution. The rays must fall normally on the plane mirror so that the image of the pin coincides with itself.

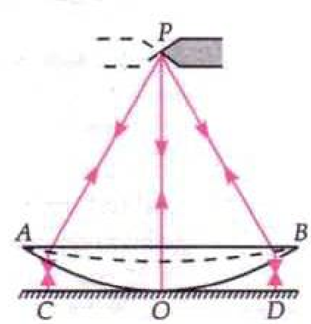


Fig. 9.106

Hence rays, like CA and DB, form a parallel beam incident on the mirror.

∴ P is the position of the focus of the lens

∴ Distance OP equals the focal length of the lens.

Example 85. *The distance between a convex lens and a plane mirror is 15 cm. The parallel rays incident on the convex lens, after reflection from the mirror, form image at the optical centre of the lens. Draw the ray-diagram and find out the focal length of the lens.*

Solution. The convex lens converges the parallel rays towards its focus F. The plane mirror reflects and converges these rays at the optical centre O. So O is the image of F formed by the plane mirror.

Focal length of convex lens

$$= OM + MF = 15 + 15 = \mathbf{30\ cm.}$$

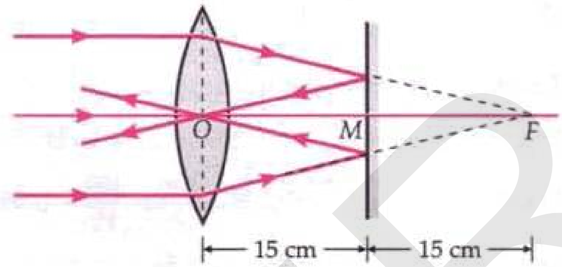


Fig. 9.107

Example 86. *A convex lens, of focal length 20 cm, has a point object on its principle axis at distance of 40 cm from it. A plane mirror is placed 30 cm behind the convex lens. Locate the position of image formed by this combination.*

Solution. We first find the position of the image formed by the lens L

$$u = -40 \text{ cm}, \qquad f = +20 \text{ cm}$$

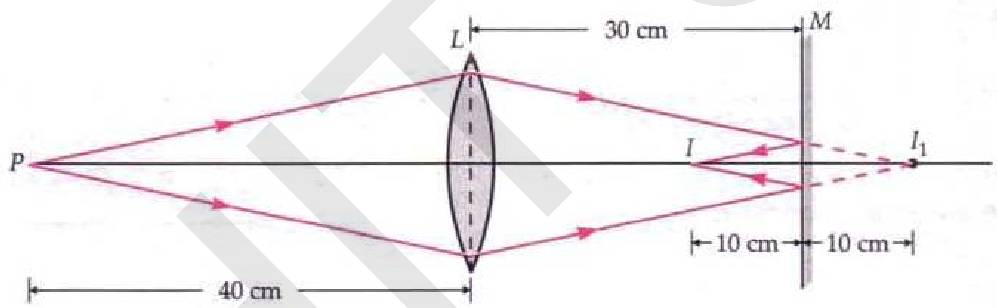


Fig. 9.108

Using thin lens formula, we get, $\frac{1}{v} - \frac{1}{(-40)} = \frac{1}{20}$ ∴ $v = +\mathbf{40\ cm}$

If there were no mirror, the lens would have formed the image at I_1. The plane mirror M is at a distance of 30 cm from the lens L. So I_1 acts as a *virtual object*, located at a distance of 10 cm behind the plane mirror M. The plane mirror, therefore, forms a *real image* (of this virtual object I_1) at I, 10 cm in front of it, as shown in Fig. 9.108.

Example 87. *A convex lens and a convex mirror, (of radius of curvature 20 cm) are placed co-axially with the convex mirror placed at a distance of 30 cm from the lens. For a point object at a distance of 25 cm from the lens, the final image, due to this combination, coincides with the object itself. What is the focal length of the convex lens ?*

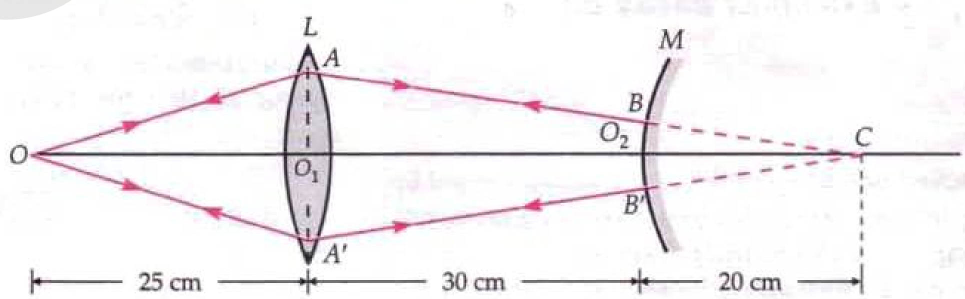


Fig. 9.109

Solution. The final image will be formed at the same point O if the convex mirror reverses the path of the light incident on it. The (refracted) rays are, therefore, falling normally on the mirror. The rays AB and $A'B'$, when produced, will meet at the centre of curvature, C of the mirror. Hence $O_2C = 20$ cm, is the radius of curvature of the mirror.

From Fig. 9.109, we then see that for the convex lens : $u = -25$ cm and $v = +(30+20)$ cm $= +50$ cm.

Using thin lens formula for the focal length f of the lens, we get

$$\frac{1}{50}-\frac{1}{-25}=\frac{1}{f}$$

$$\therefore \quad \frac{1}{f}=\frac{1+2}{50}$$

$$\therefore \quad f=\frac{50}{3}\text{ cm}=\mathbf{16.67\ cm.}$$

Example 88. *A point object O is kept at a distance of 30 cm from a convex lens of power +4 D towards its left. It is observed that when a convex mirror is kept on the right side at a distance of 50 cm from the convex lens, the image of the object O formed by the lens-mirror combination coincides with the object itself. Calculate the focal length of the convex mirror.* [CBSE Sample Paper 15]

Solution. Here the image of the combination coincides with the object itself. This implies that I is the centre of curvature of the convex mirror.

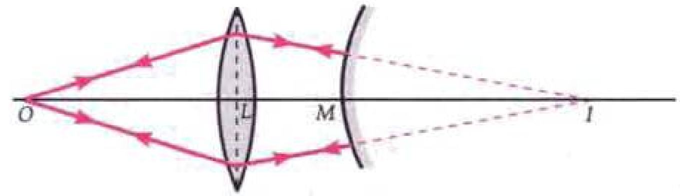


Fig. 9.110

$\therefore$ Focal length of the mirror, $f_m=\frac{MI}{2}$

For the convex lens : $u=-30$ cm, $f=+\frac{1}{4}\text{ m}=+25\text{ cm}$

As $\quad \frac{1}{f}=\frac{1}{v}-\frac{1}{u} \quad \therefore \quad \frac{1}{+25}=\frac{1}{v}-\frac{1}{-30}$

or $\quad \frac{1}{v}=\frac{1}{25}-\frac{1}{30}=\frac{1}{150}$ or $v=150\text{cm}$

$\therefore \quad MI=LI-LM=150-50=100$ cm

Hence, $\quad f_m=\frac{100}{2}=\mathbf{50\ cm.}$

Example 89. *A convex lens, of focal length 20 cm, is placed co-axially with a convex mirror of radius of curvature 20 cm. The two are kept 15 cm apart from each other. A point object is placed 60 cm in front of the convex lens. Draw a ray diagram to show the formation of the image by the combination. Determine the nature and position of the image formed.* **[CBSE OD 14]**

Solution. The ray diagram, for the image formed by the combination, is shown below in Fig. 9.111.

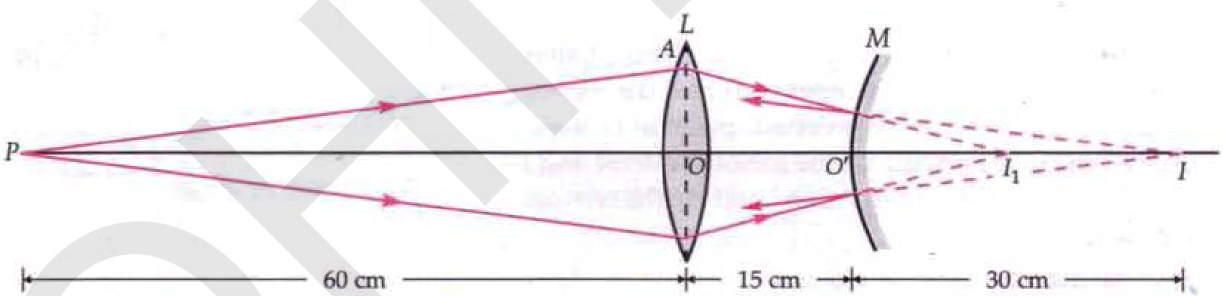


Fig. 9.111

For the convex lens : $u_1=-60$ cm and $f=+20$cm

Using thin lens formula, we get

$$\frac{1}{v_1}-\frac{1}{-60}=\frac{1}{20} \quad \text{or} \quad \frac{1}{v_1}=\frac{1}{20}-\frac{1}{60}=\frac{1}{30}$$

or $\quad v_1=30$ cm

In the absence of the mirror, the lens would have formed the image of P at I_1, which acts as a *virtual object* for the convex mirror.

$\therefore$ $O'I_1$ = distance of virtual object I_1 from convex mirror $=OI_1-OO'=(30-15)\text{cm}=15$ cm.

For the convex mirror : $u_2=+15$ cm and $R=+20$cm,

Using mirror formula, we get

$$\frac{1}{v_2}+\frac{1}{u_2}=\frac{2}{R} \quad \text{i.e.,} \quad \frac{1}{v_2}+\frac{1}{15}=\frac{2}{20}$$

or $\quad \frac{1}{v_2}=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$

$\therefore \quad v_2=+\mathbf{30\ cm}$

Hence the final image I is a virtual image formed at a distance of 30 cm behind the convex mirror.

Example 90 *A convex lens, of focal length 20 cm and a concave mirror, of focal length 10 cm, are placed co-axially 50 cm apart from each other. A beam of light coming parallel to the principal axis is incident on the convex lens. Find the position of the final image formed by this combination. Draw the ray diagram showing the formation of the image.* [CBSE OD 14]

Solution. The incident beam, on lens L, is parallel to its principal axis. Hence the lens forms an image I_1 at its focal point *i.e.*, at a distance $OI_1(=20\text{ cm})$ from the lens. This image, I_1, now acts as a real object for the concave mirror.

For the concave mirror : $u=-30$ cm and $f=-10$ cm

Using mirror formula, we get

$$\frac{1}{v}+\frac{1}{-30}=\frac{1}{-10} \quad \text{or} \quad \frac{1}{v}=\frac{1}{30}-\frac{1}{10}=-\frac{1}{15}$$

or $\qquad v=-15$ cm

The lens-mirror combination, therefore, forms a real image I at a distance of 15 cm from M.

The ray diagram is as shown in Fig. 9.112.

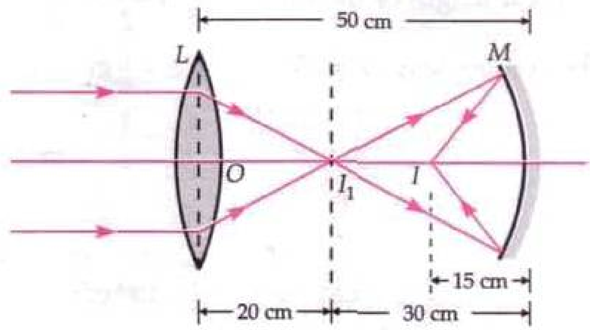


Fig. 9.112

Example 91. *A concave lens of focal length 10 cm is placed on the axis of a concave mirror of 10 cm radius at a distance of 5 cm from the mirror. An object is placed so that the light coming from it first passes through the lens, then gets reflected from the mirror, again passes through the lens to form an inverted image coincident with the object itself. Determine the position of the object.*

Solution. The rays starting from the object O after refraction from the lens and reflection from the mirror retrace their path and form inverted image at O itself. Thus the rays fall normally on the concave mirror and I should be its centre of curvature and is the virtual image of O.

For the concave lens :

$f=-10$ cm, $v=-5$ cm, $u=?$

$$\frac{1}{u}=\frac{1}{v}-\frac{1}{f}$$

$$=-\frac{1}{5}+\frac{1}{10}=-\frac{1}{10}$$

or $\quad u=$ **– 10 cm.**

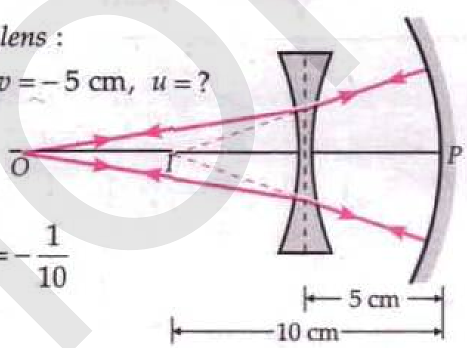


Fig. 9.113

Problems For Practice

1. A point object is placed 60 cm in front of a convex lens of focal length 30 cm. A plane mirror is placed 10 cm behind the convex lens. Where is the image formed by this system ?

(Ans. At the optical centre of the convex lens)

2. A convex lens, of focal length 15 cm and a concave mirror, of radius of curvature 20 cm, are placed co-axially 10 cm apart. An object is place in front of convex lens so that there is no parallax between the object and its image formed by the combination. Find the position of the object.

(Ans. At a distance of 30 cm from the lens)

3. Figure 9.114 shows a plane mirror M placed at a distance of 10 cm from a concave lens L. A point object is placed at a distance of 60 cm from the lens. The image formed, due to refraction by the lens and reflection by the mirror, is 30 cm behind the mirror.

What is the focal length of this lens ?

(Ans. – 30 cm)

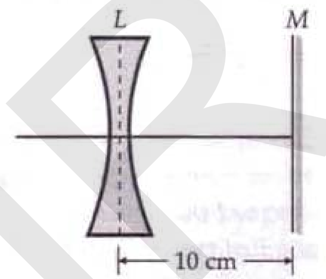


Fig. 9.114

4. An object is placed 15 cm in front of a convex lens of focal length 10 cm. Find the nature and position of the image formed. Where should a concave mirror of radius of curvature 20 cm be placed so that the final image is formed at the position of the object itself ? **[CBSE OD 15]**

HINT

4. *For the convex lens :* $u=-15$ cm, $f=+10$ cm

Using thin lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{1}{v}-\frac{1}{-15}=\frac{1}{+10}$$

or $\qquad \frac{1}{v}=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$

$\therefore \qquad v=+30$cm

$$m=\frac{v}{u}=\frac{+30}{-15}=-2$$

A real, inverted and magnified image is formed on the other side of the lens at a distance of 30 cm.

The final image formed by the concave mirror will be at the position of the object itself only if the image formed by the lens lies at the position of the centre of curvature of the mirror.

$\therefore$ Distance of the mirror from the lens

$=(30+R)$cm $=(30+20)$ cm $=$ **50 cm.**

9.31 REFRACTION THROUGH A PRISM

50. *What is a prism ? What do you mean by refracting faces, refracting edge, base, angle of prism and principal section of a prism.*

Prism. *A prism is a wedge shaped portion of a transparent refracting medium bounded by two plane faces inclined to each other at a certain angle.*

The two plane faces (ABED and ACFD) inclined to each other are called **refracting faces** *of the prism.*

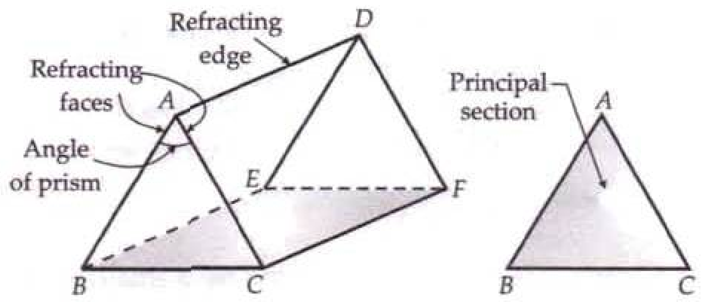


Fig. 9.115 (*a*) A glass prism. (*b*) Principal section of a prism.

The line (AD) along which the two refracting faces meet is called the **refracting edge** *of the prism.*

The third face (BCFE) of the prism opposite to the refracting edge is called the **base** *of the prism.*

The angle A included between the two refracting faces is called **angle of the prism.**

Any section of the prism cut by a plane perpendicular to the refracting edge is called **principal section** *of the prism.*

51. *Discuss the phenomenon of refraction through a prism. Derive an expression for the angle of deviation for a ray of light passing through an equilateral prism of refracting angle A.*

Refraction through a prism : Deviation produced by a prism. In Fig. 9.116, let ABC represent the principal section of prism. A ray PQ is incident on face AB. As it enters the *denser medium* (glass), it bends towards the normal along path QR. The ray QR suffers another refraction at face AC ; bending away from the normal, it emerges along RS. *The angle of deviation* δ *is the angle between the incident ray and the emergent ray.* Let i and r be the angles of incidence and refraction at the face AB, and r' and i' be angles of incidence and emergence at the face AC. Let A be the angle of the prism.

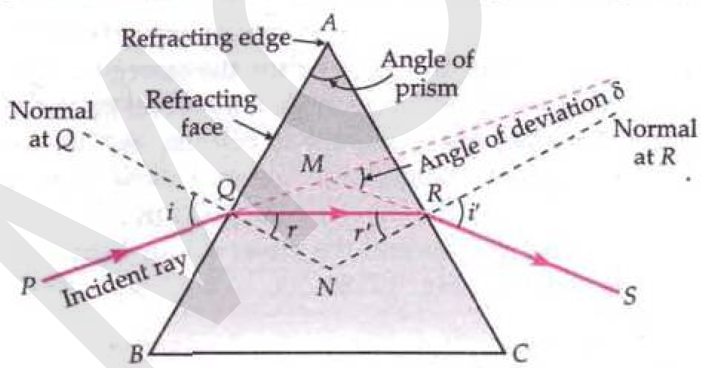


Fig. 9.116 Refraction through a prism.

From the quadrilateral $AQNR$,

$$A + \angle QNR = 180°$$

From the triangle QNR,

$$r + r' + \angle QNR = 180°$$

$$\therefore \quad A = r + r'$$

Now, from the triangle MQR, the *deviation produced* by the prism is

$$\delta = \angle MQR + \angle MRQ = (i - r) + (i' - r')$$

or δ = deviation at the first face + deviation at the second face

$$= (i + i') - (r + r')$$

or $$\delta = i + i' - A$$

or $$i + i' = A + \delta$$

i.e., Angle of incidence + Angle of emergence = Angle of prism + Angle of deviation

So when a ray of light is refracted through a prism, the sum of the angle of incidence and the angle of emergence is equal to the sum of the angle of the prism and the angle of deviation.

Factors on which the angle of deviation depends :

(*i*) The angle of incidence.

(*ii*) The material of the prism.

(*iii*) The wavelength of light used

(*iv*) The angle of the prism.

52. *Discuss the variation of the angle of deviation with that of the angle of incidence for a ray of light passing through a prism. Derive an expression for the refractive index of the material of a prism in terms of the angle of prism and the angle of minimum deviation.*

Variation of angle of deviation with angle of incidence. Fig. 9.117(*a*) shows the path of a ray of light suffering refraction through a prism of refracting angle 'A'. Fig. 9.117(*b*) shows the variation of angle of deviation δ with the angle of incidence i. For a given prism

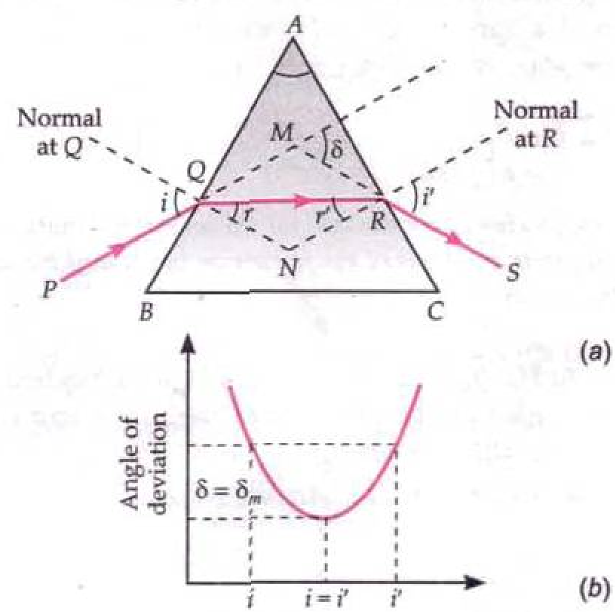


Fig. 9.117 (*a*) A ray of light passing through a prism. (*b*) Plot of variation of angle of deviation δ versus angle of incidence i for a prism.

and for a given colour of light, the angle δ depends on i only. As i increases, the angle of δ first decreases and reaches a minimum value δ_m and then increases. Clearly, any given value of δ corresponds to two angles of incidence i and i'. This fact is expected from the symmetry of i and i' in the equation : $\delta = i + i' - A$ *i.e.*, δ remains the same as i and i' are interchanged. Physically, it means that the path of the ray in Fig. 9.117(*a*) can be traced back, resulting in the same angle of deviation.

The minimum value of the angle of deviation suffered by a ray on passing through a prism is called the ***angle of minimum deviation*** and is denoted by δ_m.

Relation between refractive index and angle of minimum deviation. When a prism is in the position of minimum deviation, a ray of light passes symmetrically (parallel to the base) through the prism so that

$$i = i', \quad r = r', \quad \delta = \delta_m$$

As $\quad A + \delta = i + i'$

$\therefore \quad A + \delta_m = i + i \quad$ or $\quad i = \dfrac{A + \delta_m}{2}$

Also $\quad A = r + r' = r + r = 2r$

$\therefore \quad r = \dfrac{A}{2}$

From Snell's law, the refractive index of the material of the prism will be

$$\mu = \frac{\sin i}{\sin r} \quad \text{or} \quad \mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$

By measuring the values of A and δ_m, with the help of a spectrometer, the refractive index μ of the prism glass can be determined accurately.

9.32 DEVIATION THROUGH A PRISM OF SMALL ANGLE

53. *Derive an expression for the angle of deviation of a small prism in terms of the refractive index and the angle of the prism.*

Deviation produced by a prism of small angle. Refer to Fig. 9.117(*a*). Suppose the light is incident at a small angle i on the prism, then angles r, r' and i' will also be small.

For refraction at face AB, we have

$$\mu = \frac{\sin i}{\sin r} = \frac{i}{r} \quad \Rightarrow \quad i = \mu r$$

For refraction at face AC, we have

$$\mu = \frac{\sin i'}{\sin r'} = \frac{i'}{r'} \quad \Rightarrow \quad i' = \mu r'$$

Hence deviation produced by the prism is

$$\delta = i + i' - A = \mu r + \mu r' - A$$

$$= \mu(r + r') - A = \mu A - A$$

$[\because r + r' = A]$

or $\quad \delta = (\mu - 1)A$

Clearly, the deviation produced by a small angled prism does not depend on the angle of incidence. It depends on the angle of the prism and the refractive index of its material.

For Your Knowledge

- A ray of light suffering refraction through a prism is bent towards the base of the prism.
- The deviation produced by a prism is maximum when the angle of incidence is 90°.
  $\therefore \quad \delta_{max} = 90° + i' - A.$
- For a small angled prism, angle of deviation $\delta = (\mu - 1) A$. But for a prism with larger refracting angle, $\delta = i + i' - A$.
- There are two angles of incidence i and i' for which a ray of light passing through a prism deviates through the same angle δ.
- There is one and only one angle of incidence for which the angle of deviation is minimum.
- The deviation $[\delta = (\mu - 1) A]$ produced by a prism of small angle is independent of the angle of incidence. Moreover, this expression indicates that thin sheets of glass ($A \approx 0°$) cannot deviate light rays.

9.33 DISPERSION OF WHITE LIGHT

54. *What is dispersion of light ? Explain it with a ray diagram. Also explain the cause of dispersion of light.*

Dispersion of white light. When a narrow beam of sunlight is incident on a glass prism, the emergent light when made to fall on a screen shows several coloured bands. Broadly, the component colours are in the sequence : violet, indigo, blue, green, yellow, orange and red (given by the acronym **VIBGYOR**). The red colour bends the least and the violet colour bends the most, as shown in Fig. 9.118.

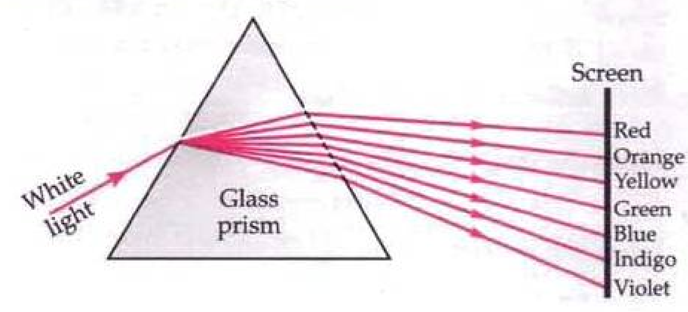


Fig. 9.118 Dispersion of white light by a glass prism.

The phenomenon of splitting of white light into its component colours on passing through a refracting medium is called ***dispersion of light****. The pattern of the coloured bands obtained on the screen is called* ***spectrum****.*

Newton's classic experiment on dispersion of white light. It can be easily seen from experiments that a prism only separates the colours already present in white light but the prism itself does not create any colour.

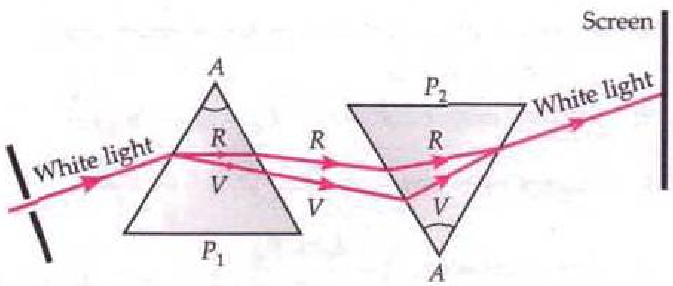


Fig. 9.119 Recombination of white light.

As shown in Fig. 9.119, take two prisms P_1, P_2 of the same glass material and of same refracting angle (A). Place the second prism P_2 upside down with respect to the first prism P_1. Allow a narrow beam of white light to fall on the prism P_1 and observe the emergent beam from prism P_2 on a screen. A patch of white light is seen on the screen. Clearly, the first prism disperses the white light into its component colours, which are then recombined by the inverted prism into white light. This proves that white light itself consists of different colours which are just dispersed by the prism.

Cause of dispersion. Each colour of light is associated with a definite wavelength. In the visible spectrum, red light is at the long wavelength end (~700 nm) while the violet light is at the short wavelength end (~400 nm). *Dispersion takes place because the refractive index of the refracting medium is different for different wavelengths.* The refractive index μ of a material for wavelength λ is given by the *Cauchy's relation* :

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$

where a, b and c are constants, the values of which depend on the nature of the material. Also, for a small-angled prism, the angle of deviation is given by

$$\delta = A(\mu - 1)$$

Now $\lambda_{red} > \lambda_{violet}$

$\therefore \quad \mu_{red} < \mu_{violet}$, and hence $\delta_{red} < \delta_{violet}$

Thus the red colour is deviated the least and the violet is deviated the most. Other colours are deviated by angles between δ_{red} and δ_{violet}. So different colours contained in white light emerge from the glass prism in different directions due to their different wavelengths, which is called dispersion.

Table 9.2 Refractive Indices for Different Wavelengths

Colour	Wavelength Å	Crown glass	Flint glass
Violet	3969	1.533	1.663
Blue	4861	1.523	1.639
Yellow	5893	1.517	1.627
Red	6563	1.515	1.622

55. *What are dispersive and non-dispersive media ? Give examples.*

Dispersive media. In some refracting media, different colours of light travel with different speeds. The variation of refractive index with wavelength is highly pronounced for such media. These media which bring about a good dispersion of white light are called dispersive media. For example, glass, quartz, etc.

Non-dispersive media. In vacuum, all colours of light travel with the same speed. So the refractive index of vacuum is independent of wavelength. White light does not undergo dispersion in vacuum. Such a medium is called a non-dispersive medium.

9.34 ANGULAR DISPERSION AND DISPERSIVE POWER

56. *Define angular dispersion and dispersive power. Write expression for these quantities in terms of refractive index.*

Angular dispersion and dispersive power. When a beam of white light passes through a prism, it gets dispersed into its constituent colours. Let δ_V, δ_R and δ be the angles of deviation for violet, red and yellow (mean) colours respectively, as shown in Fig. 9.120.

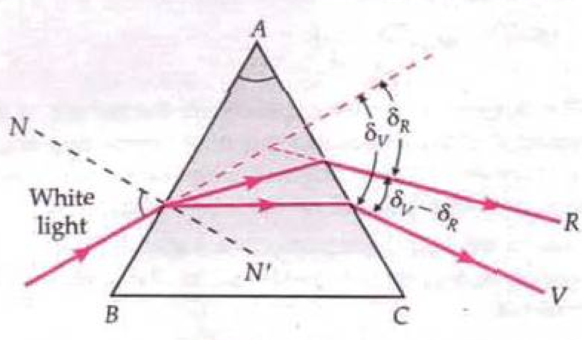


Fig. 9.120 Angular dispersion.

Then $\delta_V = (\mu_V - 1)\,A$

$\delta_R = (\mu_R - 1)\,A$

$\delta = (\mu - 1)\,A$

where μ_V, μ_R and μ are the refractive indices of the prism material for violet, red and yellow (mean) colours, respectively.

The angular separation between the two extreme colours (violet and red) in the spectrum is called the ***angular dispersion***.

$\therefore$ Angular dispersion

$$= \delta_V - \delta_R$$
$$= (\mu_V - 1)\,A - (\mu_R - 1)\,A = (\mu_V - \mu_R)\,A$$

Clearly, the angular dispersion produced by a prism depends upon (*i*) angle of the prism and (*ii*) nature of the material of the prism.

Dispersive power *is the ability of the prism material to cause dispersion*. It is defined as the ratio of the angular *dispersion to the mean deviation*.

$\therefore$ Dispersive power,

$$\omega = \frac{\text{Angular dispersion}}{\text{Mean deviation}} = \frac{\delta_V - \delta_R}{\delta}$$
$$= \frac{(\mu_V - 1)\,A - (\mu_R - 1)\,A}{(\mu - 1)\,A}$$

or $$\omega = \frac{\mu_V - \mu_R}{\mu - 1}$$

For Your Knowledge

- The refractive index (μ) of any material for yellow light is equal to the mean of the refractive indices for the violet and red lights, *i.e.*,
$$\mu = \frac{\mu_V + \mu_R}{2}$$
That is why yellow light is called ***mean light*** and its deviation is called ***mean deviation***, which is given by
$$\delta = \frac{\delta_V + \delta_R}{2}$$
- Due to its small wavelength, violet light is harmful to our eyes. So in experiments, angular dispersion and dispersive power of a material are measured for *blue* and *red* lights. Thus
$$\omega = \frac{\mu_B - \mu_R}{\mu - 1}$$
- The dispersive power depends on the nature of the material of the prism and not on its refracting angle, *A*. However, both angular dispersion and mean deviation also depend on the angle of the prism.
- Greater the dispersive power of a material, larger is the spread of a spectrum produced by the prism of the material.
- Dispersive power of flint glass is more than that of crown glass.

Examples based on
Refraction and Dispersion of Light through a Prism

Formulae Used

1. For refraction through a prism,
$$A + \delta = i + i' \quad \text{and} \quad r + r' = A$$
2. In the condition of minimum deviation,
$$i = i',\ r = r',\ \delta = \delta_m\ ;\ \mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$
3. Deviation produced by a prism of small angle,
$$\delta = (\mu - 1)\,A$$
4. Angular dispersion $= \delta_V - \delta_R = (\mu_V - \mu_R)\,A$
5. Dispersive power, $\omega = \dfrac{\delta_V - \delta_R}{\delta} = \dfrac{\mu_V - \mu_R}{\mu - 1}$
6. Mean deviation, $\delta = \dfrac{\delta_V + \delta_R}{2}$
7. Mean refractive index, $\mu = \dfrac{\mu_V + \mu_R}{2}$.

Units Used

Angles $i, i', r, r', A, \delta, \delta_m, \delta_V$ and δ_R are in degrees ; dispersive power ω and refractive indices μ_V, μ_R and μ have no units.

Example 92. *Calculate the refractive index of the material of an equilateral prism for which the angle of minimum deviation is 60°.* [CBSE Sample Paper 98]

Solution. For an equilateral prism, $A = 60°$.

Also $\delta_m = 60°$

$\therefore$ Refractive index of the prism material is

$$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{60°+60°}{2}}{\sin\frac{60°}{2}} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}.$$

Example 93. *A ray of light passes through an equilateral glass prism, such that the angle of incidence is equal to the angle of emergence. If the angle of emergence is 3/4 times the angle of the prism, calculate (i) the angle of deviation and (ii) the refractive index of the glass prism.* **[CBSE OD 13C]**

Solution. (*i*) Here $A = 60°,\ i = i' = \frac{3}{4}A = 45°,\ \mu = ?$

As $A + \delta = i + i'$

$\therefore$ $60 + \delta = 45° + 45°$ or $\delta = 90° - 60° = \mathbf{30°}$.

(*ii*) $$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{60°+30°}{2}}{\sin\frac{60°}{2}}$$
$$= \frac{\sin 45°}{\sin 30°} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2} = \mathbf{1.414}.$$

Example 94. *A ray of light incident on an equilateral glass prism shows minimum deviation of 30°. Calculate the speed of light through the prism.* [CBSE OD 01]

Solution. Here $A = 60°$, $\delta_m = 30°$

Refractive index, $\mu = \dfrac{\sin \dfrac{A+\delta_m}{2}}{\sin \dfrac{A}{2}} = \dfrac{\sin \dfrac{60°+30°}{2}}{\sin \dfrac{60°}{2}}$

$= \dfrac{\sin 45°}{\sin 30°} = \dfrac{1/\sqrt{2}}{1/2} = \sqrt{2} = 1.414$

Velocity of light in glass,

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.414} = \mathbf{2.12 \times 10^8 \, ms^{-1}}.$$

Example 95. *A ray of light passing through an equilateral triangular glass prism from air undergoes minimum deviation when angle of incidence is 3/4th of the angle of prism. Calculate the speed of light in the prism.* [CBSE D 08]

Solution. Here $i = \dfrac{3}{4} A = \dfrac{3}{4} \times 60 = 45°$

In the position of minimum derivation, $r = \dfrac{A}{2} = 30°$

$\therefore \quad \mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin 45°}{\sin 30°} = \dfrac{1}{\sqrt{2}} \times \dfrac{2}{1} = \sqrt{2}$

Hence, $v = \dfrac{c}{\mu} = \dfrac{3 \times 10^8}{\sqrt{2}} \, ms^{-1}$ $\quad [\because \mu = \dfrac{c}{v}]$

$= 0.707 \times 3 \times 10^8 ms^{-1} = \mathbf{2.12 \times 10^8 \, ms^{-1}}.$

Example 96. *Calculate the angle of minimum deviation for an equilateral triangular prism of refractive index $\sqrt{3}$.* [IPUEE 11]

Solution. Here, $A = 60°$, $\mu = \sqrt{3}$, $\delta_m = ?$

As $\quad \mu = \dfrac{\sin \dfrac{A+\delta_m}{2}}{\sin \dfrac{A}{2}}$

$\therefore \quad \sqrt{3} = \dfrac{\sin \dfrac{60°+\delta_m}{2}}{\sin 30°}$

or $\sin \dfrac{60°+\delta_m}{2} = \sqrt{3} \times \dfrac{1}{2} = \sin 60°$ or $\dfrac{\delta_m + 60°}{2} = 60°$

or $\quad \delta_m = 120 - 60 = \mathbf{60°}.$

Example 97. *A ray PQ incident on the face AB of a prism ABC, as shown in [Fig. 9.121(a)], emerges from the face AC such that AQ = AR. Draw the ray diagram showing the passage of the ray through the prism. If the angle of the prism is 60° and refractive index of the material of the prism is $\sqrt{3}$, determine the values of angle of incidence and angle of deviation.* [CBSE OD 15]

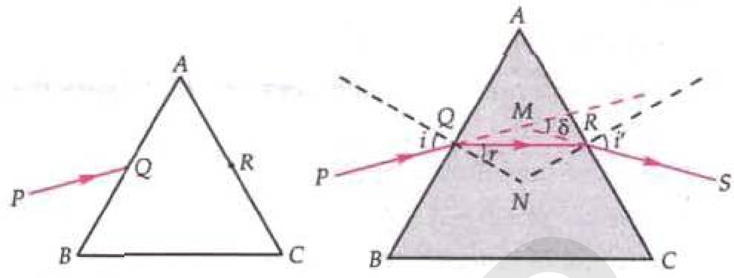


Fig. 9.121 (a) (b)

Solution. The ray QR passes parallel to the base of the prism. It is minimum deviation position of the prism.

$\therefore \quad r = \dfrac{A}{2} = \dfrac{60°}{2} = 30°$

As $\quad \mu = \dfrac{\sin i}{\sin r}$

$\therefore \quad \sqrt{3} = \dfrac{\sin i}{\sin 30°}$ or $\sin i = \sqrt{3} \times \dfrac{1}{2} = \dfrac{\sqrt{3}}{2}$

$\therefore$ Angle of incidence, $i = \mathbf{60°}$

Also, $i + i' = A + \delta$ or $60° + 60° = 60° + \delta$

$\therefore$ Angle of deviation, $\delta = \mathbf{60°}.$

Example 98. *A ray of light suffers minimum deviation, while passing through a prism of refractive index 1.5 and refracting angle 60°. Calculate the angle of deviation and angle of incidence.* [Punjab 97]

Solution. Here $\mu = 1.5$, $A = 60°$

In the position of minimum deviation,

$i = \dfrac{A+\delta_m}{2}$, $\quad r = \dfrac{A}{2}$

As $\quad \mu = \dfrac{\sin i}{\sin r} \quad \therefore \quad 1.5 = \dfrac{\sin i}{\sin 30°}$

or $\quad \sin i = 1.5 \times 0.5 = 0.75$

$i = \sin^{-1}(0.75) = \mathbf{48.6°}$

$\delta_m = 2i - A = 2 \times 48.6 - 60 = \mathbf{37.2°}.$

Example 99. *A ray of light PQ is incident at angle of 60° on the face AB of a prism of angle 30°, as shown in Fig. 9.122(a). The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face BC through which it emerges. Also calculate the refractive index of the prism material.* [CBSE D 02C]

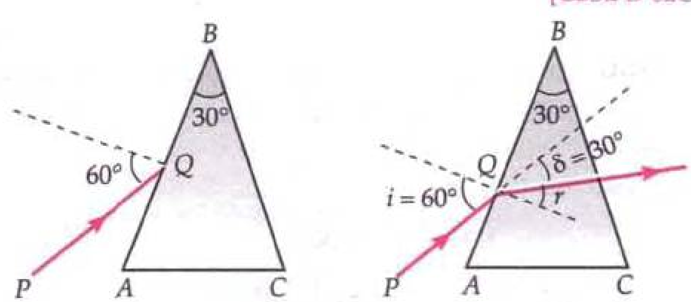


Fig. 9.122 (a) (b)

Solution. Here $i = 60°$, $A = 30°$

As the emergent ray makes an angle of 30° with the incident ray, so

angle of deviation, $\delta = 30°$

Now $i + i' = A + \delta$

$\therefore$ Angle of emergence,

$$i' = A + \delta - i = 30° + 30° - 60° = 0°$$

Thus the emergent is perpendicular to the face BC through which it emerges, as shown in Fig. 9.122(b).

When $i' = 0°$, $r' = 0°$

$\therefore$ $r = A - r' = 30° - 0° = 30°$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \mathbf{\sqrt{3}}.$$

Example 100. *A prism is found to give a minimum deviation of 51°. The same prism gives a deviation of 62°48′ for two values of the angles of incidence 40°6′ and 82°42′. Determine the refracting angle of the prism and the refractive index of the prism material.*

Solution. As the path of the rays through a prism is reversible, so out of the two given angles of incidence, one can be taken as the angle of incidence i and other as the angle of emergence i'.

$\therefore$ $i = 40°6'$, $i' = 82°42'$, $\delta = 62°48'$, $\delta_m = 51°$

As $A + \delta = i + i'$

$\therefore$ $A = i + i' - \delta = 82°42' + 40°6' - 62°48' = \mathbf{60°}$.

$$\mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{60° + 51°}{2}}{\sin \frac{60°}{2}}$$

$$= \frac{\sin 55°30'}{\sin 30°} = \frac{0.8241}{0.5000} = \mathbf{1.6482.}$$

Example 101. *A glass prism of refracting angle 60° and refractive index 1.5, is completely immersed in water of refractive index 1.33. Calculate the angle of minimum deviation of the prism in this situation.* ($\sin^{-1} 0.56 = 34.3°$)

[CBSE D 97C]

Solution. Here $^{w}\mu_{g} = \frac{\mu_g}{\mu_w} = \frac{1.5}{1.33}$

$A = 60°$, $\delta_m = ?$

Now $$^{w}\mu_{g} = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$

$$\frac{1.5}{1.33} = \frac{\sin \frac{A + \delta_m}{2}}{\sin 30°}$$

or $$\sin \frac{A + \delta_m}{2} = \frac{1.5}{1.33} \times \frac{1}{2} = 0.56$$

$\therefore$ $$\frac{A + \delta_m}{2} = \sin^{-1} 0.56 = 34.3°$$

$$\delta_m = 68.6° - 60° = \mathbf{8.6°}.$$

Example 102. *One face of a prism of refracting angle 30° and refractive index 1.414 is silvered. At what angle must a ray of light fall on the unsilvered face so that after refraction into the prism and reflection at the silvered surface it retraces its path ?*

Solution. The ray will retrace its path at the silvered face when it is incident normally on it.

$\therefore$ $i' = 0$ and so $r' = 0$

As $r + r' = A$ or $r + 0 = 30°$

$\therefore$ $r = 30°$

As $\mu = \frac{\sin i}{\sin r}$

$\therefore$ $1.414 = \frac{\sin i}{\sin 30°} = 2 \sin i$

or $\sin i = \frac{1.414}{2}$

$= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Hence $i = \mathbf{45°}$.

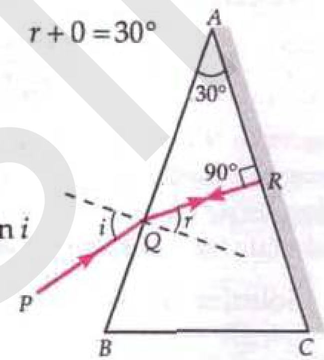


Fig. 9.123

Example 103. *An equilateral glass prism ($\mu = 1.6$) is immersed in water ($\mu = 1.33$). Calculate the angle of deviation produced for a ray of light incident at 40° on one face of the prism.*

Solution. For an equilateral glass prism, $A = 60°$,

Refractive index of glass, $^{a}\mu_{g} = 1.6$

Refractive index of water, $^{a}\mu_{w} = 1.33$

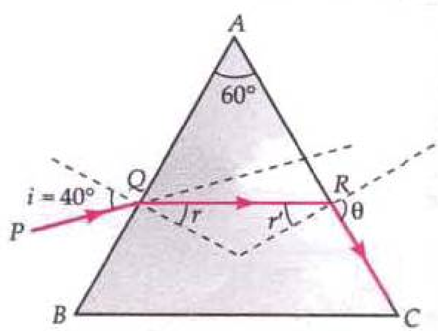


Fig. 9.124

$\therefore$ Refractive index of glass w.r.t. water is

$$^{w}\mu_{g} = \frac{^{a}\mu_{g}}{^{a}\mu_{w}} = \frac{1.6}{1.33} = 1.203$$

For refraction at face AB :

Angle of incidence, $i = 40°$

Angle of refraction $= r$, say

Then $\quad {}^{w}\mu_{g} = \dfrac{\sin i}{\sin r}$

or $\quad 1.203 = \dfrac{\sin 40^\circ}{\sin r}$

or $\quad \sin r = \dfrac{\sin 40^\circ}{1.203} = \dfrac{0.6428}{1.203} = 0.5343$

$r = 32^\circ 18'$

For refraction at face AC :

Let r' be the angle of refraction at the second face AC.

As $r + r' = A$

$\therefore \quad r' = A - r = 60^\circ - 32^\circ 18' = 27^\circ 42'.$

If i' is angle of emergence, then

$${}^{w}\mu_{g} = \frac{\sin i'}{\sin r'}$$

or $\quad \sin i' = {}^{w}\mu_{g} \times \sin r' = 1.203 \times \sin 27^\circ 42'$

$= 1.203 \times 0.4648 = 0.5592$

or $\quad i' = 34^\circ$

$\therefore$ Angle of deviation,

$$\delta = i + i' - A = 40^\circ + 34^\circ - 60^\circ = \mathbf{14^\circ}.$$

Example 104. *Find the angle of dispersion between red and violet colours produced by a flint glass prism of refracting angle 60°. Refractive indices of prism for red and violet colours are 1.622 and 1.663, respectively.* [Punjab 91]

Solution. Here

$A = 60^\circ, \quad \mu_R = 1.622, \quad \mu_V = 1.633$

Angular dispersion between red and violet colours is

$$\delta_V - \delta_R = A(\mu_V - \mu_R) = 60^\circ(1.663 - 1.622)$$
$$= \mathbf{2.46^\circ}.$$

Example 105. *A thin prism of refracting angle 2° deviates an incident ray through an angle of 1°. Find the value of refractive index of the material of the prism.*

[CBSE Sample Paper 03]

Solution. Here $A = 2^\circ$, $\delta = 1^\circ$, $\mu = 1$

Deviation through a prism of small angle is given by

$$\delta = (\mu - 1)\,A$$

$\therefore \quad \mu = \dfrac{\delta}{A} + 1 = \dfrac{1}{2} + 1 = \mathbf{1.5}.$

Example 106. *Show that the angle of minimum deviation produced by a thin prism is reduced to one-fourth (with respect to air) when it is immersed in water. Given* ${}^{a}\mu_{g} = 3/2$ *and* ${}^{a}\mu_{w} = 4/3$.

Solution. Deviation produced by thin prism,

$$\delta = (\mu - 1)\,A$$

For the prism placed in air,

$$\delta_1 = ({}^{a}\mu_{g} - 1)\,A = \left(\frac{3}{2} - 1\right) A = \frac{1}{2}\,A$$

For the prism placed in water,

$$\delta_2 = ({}^{w}\mu_{g} - 1)\,A = \left(\frac{{}^{a}\mu_{g}}{{}^{a}\mu_{w}} - 1\right) A$$
$$= \left(\frac{3/2}{4/3} - 1\right) A = \left(\frac{9}{8} - 1\right) A = \frac{1}{8}\,A$$

Hence $\quad \dfrac{\delta_2}{\delta_1} = \dfrac{1/8}{1/2} = \mathbf{\dfrac{1}{4}}.$

Example 107. *A prism with refracting angle of 60°, gives angle of minimum deviation 53°, 51° and 52° for blue, red and yellow light respectively. What is the dispersive power of the material of the prism ?* [ISCE 96]

Solution. Here $\delta_B = 53^\circ, \quad \delta_R = 51^\circ, \quad \delta_Y = 52^\circ$

Dispersive power,

$$\omega = \frac{\delta_B - \delta_R}{\delta_Y} = \frac{53 - 51}{52} = \mathbf{0.038}.$$

Example 108. *The refractive indices of flint glass for blue and red colours are 1.664 and 1.644. Calculate its dispersive power.*

Solution. Here $\quad \mu_B = 1.664, \quad \mu_R = 1.644$

$\therefore \quad \mu = \dfrac{\mu_B + \mu_R}{2} = \dfrac{1.664 + 1.644}{2} = 1.654$

Dispersive power,

$$\omega = \frac{\mu_B - \mu_R}{\mu - 1} = \frac{1.664 - 1.644}{1.654 - 1} = \mathbf{0.0305}.$$

Example 109. *A glass prism deviates the red and the blue rays through 10° and 12°, respectively. A second prism of equal angle deviates them through 8° and 10° respectively. Find the ratio of their dispersive powers.*

Solution. For the first prism, we have

$$\delta_B - \delta_R = 12^\circ - 10^\circ = 2^\circ$$

Mean deviation,

$$\delta = \frac{\delta_B + \delta_R}{2} = \frac{12^\circ + 10^\circ}{2} = 11^\circ$$

$\therefore$ Dispersive power,

$$\omega = \frac{\delta_B - \delta_R}{\delta} = \frac{2}{11}$$

Now for the second prism,

$$\delta'_B - \delta'_R = 10^\circ - 8^\circ = 2^\circ$$

and $\quad \delta' = \dfrac{\delta'_B + \delta'_R}{2} = \dfrac{10 + 8}{2} = 9^\circ$

$\therefore \quad \omega' = \dfrac{\delta'_B - \delta'_R}{2} = \dfrac{2}{9}$

Hence $\quad \dfrac{\omega}{\omega'} = \dfrac{2/11}{2/9} = \dfrac{9}{11} = \mathbf{9 : 11}.$

Example 110. *The refractive indices for a material for red, violet and yellow lights are 1.52, 1.62 and 1.59 respectively. Calculate the dispersive power of the material. If the mean deviation is 40°, then what will be the angular dispersion produced by a prism of this material ?*

Solution. Dispersive power,

$$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1} = \frac{1.62 - 1.52}{1.59 - 1} = \mathbf{0.169}$$

Also $\quad \omega = \dfrac{\delta_V - \delta_R}{\delta_Y}$

$\therefore$ Angular dispersion

$$= \delta_V - \delta_R = \omega\delta_Y = 0.169 \times 40° = \mathbf{6.76°}.$$

Example 111. *In a certain spectrum produced by a glass prism of dispersive power 0.031, it is found that the refractive index for the red ray is 1.645 and that for the blue ray is 1.665. What is the refractive index of yellow ray ?* [ISCE 95]

Solution. Here $\omega = 0.031$, $\mu_R = 1.645$, $\mu_B = 1.665$

As $\quad \omega = \dfrac{\mu_B - \mu_R}{\mu_Y - 1}$

or $\quad \mu_Y = 1 + \dfrac{\mu_B - \mu_R}{\omega}$

$$= 1 + \frac{1.665 - 1.645}{0.031}$$

$$= 1 + 0.645 = \mathbf{1.645}.$$

Example 112. *A spectrometer measures angles correct to 6′ of an arc. If an experiment gives*

$$A = 60°0' \; ; \quad \delta_m = 48°36',$$

calculate the percentage accuracy of the value of μ.

Solution. $\mu = \dfrac{\sin \dfrac{A + \delta_m}{2}}{\sin \dfrac{A}{2}}$

$$= \frac{\sin\left(\dfrac{60° + 48°36'}{2}\right)}{\sin \dfrac{60°}{2}}$$

$$= 2 \sin 54°18' = 2 \times 0.8121 = 1.6242$$

Differentiating μ with respect to δ_m, we get

$$\frac{d\mu}{d\delta_m} = \frac{1}{\sin \dfrac{A}{2}} \cos\left[\frac{A + \delta_m}{2}\right] \times \frac{1}{2}$$

$$= \frac{1}{\sin 30°} \times \frac{1}{2} \cos\left(\frac{A + \delta_m}{2}\right)$$

$$= \cos\left(\frac{A + \delta_m}{2}\right) = \cos 54°18' = 0.5835$$

or $\quad d\mu = 0.5835 \, d\delta_m$

Now the error in measuring $\delta_m = \pm 6'$

$\therefore$ Total range of error is

$$d\delta_m = 12' = \left(\frac{1}{5}\right)^\circ = \left(\frac{1}{5} \times \frac{\pi}{180}\right) \text{rad}$$

$\therefore \quad d\mu = 0.5835(d\delta_m)$

$$= 0.5835 \times \frac{1}{5} \times \frac{\pi}{180} = \frac{12.76}{6300}$$

Hence % error $= \dfrac{d\mu}{\mu} \times 100 = \dfrac{12.76}{6300} \times \dfrac{100}{1.6242}$

$$= \frac{12.76}{100.8} = \mathbf{0.12}.$$

Problems For Practice

1. A ray of light is inclined to one face of a prism at an angle of 50°. If the angle of prism be 60° and the ray be deviated through an angle of 42°, find the angle which the emergent ray makes with the second face of the prism. (Ans. 28°)
2. A ray of light strikes one face of the prism at an angle of incidence 60° and angle of refraction is 30°. If the angle of prism is 60°, find the angle of emergence. (Ans. 60°)
3. The refracting angle of a glass prism is 60°. The value of minimum deviation for light passing through the prism is 40°. Calculate the value of refractive index of the material of the prism. Given sin 50° ≈ 0.766. [CBSE D 93C] (Ans. 1.532)
4. A ray of light incident at an angle of 48° is refracted through a prism in its position of minimum deviation. The angle of prism is 60°. Calculate the refractive index of the material of the prism. (sin 48° = 0.74, sin 30° = 0.50) [CBSE 95] (Ans. 1.48)
5. A glass prism has a refracting angle of 60°. The angle of minimum deviation is 40°. If the velocity of light in vacuum is $3 \times 10^8 \text{ ms}^{-1}$, calculate the velocity of light in glass. At what angle the ray should be incident ? (Ans. $1.958 \times 10^8 \text{ ms}^{-1}$, 50°)
6. As shown in Fig. 9.125, *PQ* is the ray incident on a prism *ABC*. Show the corresponding refracted and

emergent rays. The critical angle for the material of the prism is 45°. Also find the refractive index of the material of the prism. (Ans. $\sqrt{2}$)

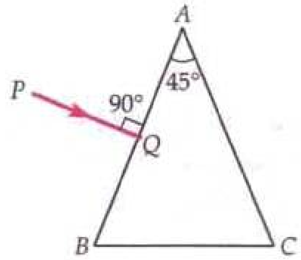


Fig. 9.125

7. A ray of light falls normally on the face of a glass prism having refractive index of 1.5. Find the angle of prism, if the ray just fails to emerge from the second face. (Ans. 41.8°)

8. A ray of light falls on a glass prism at an angle of incidence of 30°. Find the direction of the emergent ray if the angle of the prism is 60° and refractive index is 1.50. (Ans. $i' = 77°7'$)

9. A thin prism of 6° angle gives a deviation of 3°. What is the refractive index of the material of the prism ? (Ans. 1.5)

10. Once face of a prism with a refracting angle of 30° is coated with silver. A ray incident an another face at an angle of 45° is refracted and reflected from the silver coated face and retraces its path. Find the refractive index of the material of the prism. **[CBSE F 09]** (Ans. 1.414)

11. A ray of light incident on an equilateral glass prism ($\mu_{glass} = \sqrt{3}$) moves parallel to the base of the prism, inside it. What is the angle of incidence for this ray ? **[CBSE D 12]** (Ans. 60°)

12. White light is passed through a prism of 5°. If the refractive indices for red and blue rays are 1.641 and 1.659 respectively, calculate the angle of dispersion between them. **[Punjab 99C]** (Ans. 0.09°)

13. The refractive indices for lights of violet, yellow and red colours for a flint glass prism are respectively 1.632, 1.620 and 1.613. Find the dispersive power of the prism material. (Ans. 0.0306)

14. Calculate the dispersive power of crown and flint glass prisms from the following data :
For crown glass : $\mu_B = 1.522$, $\mu_R = 1.514$
For flint glass : $\mu_B = 1.662$, $\mu_R = 1.644$
(Ans. $\omega_{crown} = 0.01544$, $\omega_{flint} = 0.0276$)

15. The refractive indices of crown glass for violet and red colours are respectively 1.523 and 1.513. Determine the dispersive power of this glass. If a crown glass prism produces a mean deviation of 40°, what will be the angular dispersion ? (Ans. 0.0193, 0.772°)

16. Find the angle of flint glass prism which produces the same angular dispersion for C and F wavelengths in 10° crown glass prism.
For crown glass : $\mu_F = 1.5230$, $\mu_C = 1.5145$
For flint glass : $\mu_F = 1.6637$, $\mu_C = 1.6444$
(Ans. 4.4°)

HINTS

1. In Fig. 9.126, $i = 90° - 50° = 40°$, $A = 60°$, $\delta = 42°$
As $i + i' = A + \delta$
$\therefore \quad i' = A + \delta - i = 60° + 42° - 40° = 62°$
Angle made by emergent ray with second face $= 90° - 62° = $ **28°**.

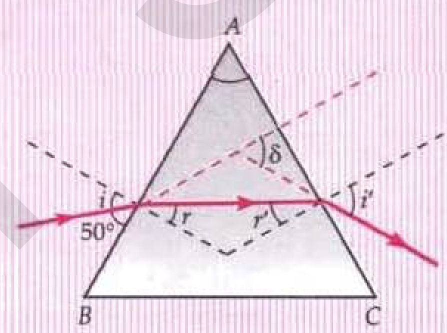


Fig. 9.126

2. Here $i = 60°$, $r = 30°$, $A = 60°$. Refer to Fig. 9.126.
As $r + r' = A$
$\therefore \quad r' = A - r = 60° - 30° = 30°$
i.e., $r = r'$ and the ray passes symmetrically through the prism
$\therefore \quad i' = i = $ **60°**.

3. $$\mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{60° + 40°}{2}}{\sin \frac{60°}{2}} = \frac{\sin 50°}{\sin 30°} = \frac{0.766}{0.5} = \mathbf{1.532}$$

4. Given $i = 48°$, $A = 60°$
In the position of minimum deviation,
$$A = r + r' = r + r \text{ or } r = \frac{A}{2}$$
$$\therefore \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin \frac{A}{2}} = \frac{\sin 48°}{\sin 30°} = \frac{0.74}{0.50} = \mathbf{1.48}$$

5. $\mu = \dfrac{c}{v} = \dfrac{\sin \dfrac{60° + 40°}{2}}{\sin 30°} = \dfrac{\sin 50°}{\sin 30°} = \dfrac{0.7660}{0.5}$

$$\therefore \quad v = \frac{0.5 \times 3 \times 10^8}{0.7660} = \mathbf{1.958 \times 10^8 \ ms^{-1}}$$

$$i = \frac{A + \delta_m}{2} = \frac{60° + 40°}{2} = \mathbf{50°}.$$

6. Refer to Fig. 9.127. As the ray *PQ* is incident normally on face *AB*, so the refracted ray *QR* goes straight and is incident on face *AC* at 45°. Now the angle of incidence is equal to critical angle ($i = i_c = 45°$), the emergent ray *QR* goes along second face *RC*.

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2}.$$

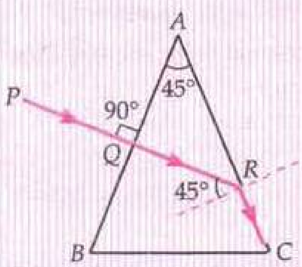


Fig. 9.127

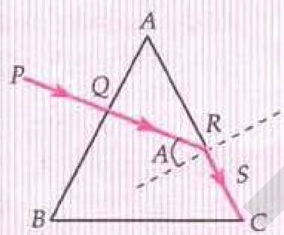


Fig. 9.128

7. Refer to Fig. 9.128. As the ray *PQ* is incident normally on face *AB*, so the refracted ray *QR* goes straight and is incident on face *AC* at an angle equal to *A*. As the ray *RS* just fails to emerge from the face *AC*, so angle *A* equals the critical angle for the prism material.

As $\quad \sin i_c = \dfrac{1}{\mu}$

or $\quad \sin A = \dfrac{1}{1.5} = 0.6667$

or $\quad A = \mathbf{41.8°}.$

8. For refraction at first face,

$$\frac{\sin i}{\sin r} = \mu \quad \text{or} \quad \frac{\sin 30°}{\sin r} = 1.5$$

or $\quad \sin r = \dfrac{\sin 30°}{1.5} = \dfrac{0.5}{1.5} = 0.3333 \quad$ or $\quad r = 19°28'$

Angle of incidence at second face,

$$r' = A - r = 60° - 19°28' = 40°32'$$

For refraction at second face, $\dfrac{\sin r'}{\sin i'} = \dfrac{1}{\mu}$

or $\quad \sin i' = \mu \times \sin r' = 1.5 \sin 40°32'$
$\quad = 1.5 \times 0.6499 = 0.9748$

$\therefore$ Angle of emergence,

$$i' = \sin^{-1}(0.9748) = \mathbf{77°7'}.$$

9. Here $A = 6°, \quad \delta = 3°, \quad \mu = ?$

For a prism of small angle, $\delta = (\mu - 1) A$

$$\therefore \quad \mu - 1 = \frac{\delta}{A} = \frac{3}{6} = 0.5$$

or $\quad \mu = 1 + 0.5 = \mathbf{1.5}.$

10. Proceed as in Example 102 on page 9.72.

12. Angular dispersion

$$= (\mu_B - \mu_R) A = (1.659 - 1.641) \times 5°$$
$$= \mathbf{0.09°}.$$

13. Proceed as in Example 108 on page 9.73.

14. *For crown glass* :

$$\mu = \frac{\mu_B + \mu_R}{2} = \frac{1.522 + 1.514}{2} = 1.518$$

$$\omega_{crown} = \frac{\mu_B - \mu_R}{\mu - 1} = \frac{1.522 - 1.514}{1.518 - 1} = \mathbf{0.01544}$$

For flint glass :

$$\mu = \frac{\mu_B - \mu_R}{2} = \frac{1.662 + 1.644}{2} = 1.653$$

$$\omega_{flint} = \frac{\mu_B - \mu_R}{\mu - 1} = \frac{1.662 - 1.644}{1.653 - 1} = \mathbf{0.0276}$$

15. $\omega = \dfrac{\mu_V - \mu_R}{\mu - 1} = \dfrac{\mu_V - \mu_R}{\dfrac{\mu_V + \mu_R}{2} - 1}$

Angular dispersion $= \delta_V - \delta_R = \omega\delta$

16. Use $(\mu_F - \mu_C) A = (\mu'_F - \mu'_C) A'$.

9.35 PURE AND IMPURE SPECTRA*

57. *Distinguish between monochromatic light and polychromatic light.*

Monochromatic and polychromatic lights. *A light of single wavelength is called* ***monochromatic light****.* The commonly used source of monochromatic light is sodium lamp which emits yellow light of two wavelengths 5890 Å and 5896 Å. As the two wavelengths are very close, so sodium lamp can be regarded as a source of monochromatic light of mean wavelength 5893 Å.

Generally, the sources of light are ***polychromatic*** *which give light of several wavelengths.* Optical filters can be used to obtain light of particular wavelength from them.

58. *What are pure and impure spectra ? Give the basic principle for the production of a pure spectrum.*

Pure and impure spectra. *A spectrum in which the component colours of the spectra of different rays overlap each other and the various colours are not distinctly seen is called* ***impure spectrum****. On the other hand, a spectrum in* *which there is no overlapping of different colours and different colours are distinctly seen is called the* ***pure spectrum.***

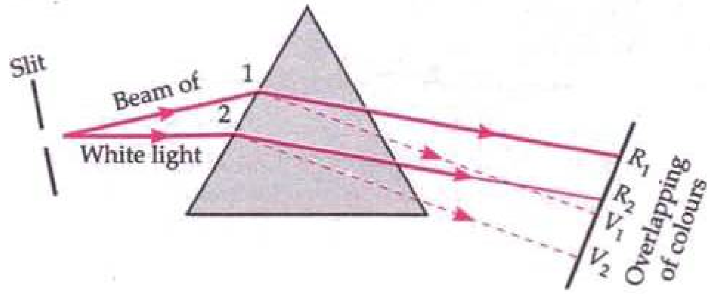


Fig. 9.129 (*a*) An impure spectrum.

Production of pure spectrum (Basic principle). As shown in Fig. 9.129(*b*), light from a bright source illuminates a *narrow slit S*. The slit is adjusted at the focus of a convex lens L_1. Parallel rays emerging from the lens fall on a prism. Rays of different colours are refracted by different amounts but the rays of same colour remain parallel to one another. The emergent rays are focussed by convex lens L_2 on a screen placed at the focus of L_2.

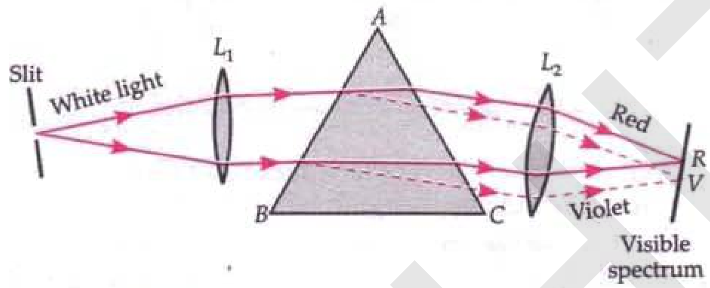


Fig. 9.129 (*b*) Production of pure spectrum.

As rays of different colours suffer different deviations, they cannot be focussed on the same screen. To overcome this difficulty, the prism should be placed in the *minimum deviation position* with respect to some mean (yellow) colour. Then the rays of different colours will suffer almost the same deviation and can be focussed on the same screen.

9.36 SPECTROMETER*

59. *What is a spectrometer ? Explain its construction. How can it be used to obtain a pure spectrum ?*

Spectrometer. *It is an optical device used for producing and studying the spectra of different light sources.*

Construction. A spectrometer has *three* main parts :

1. Collimator. It produces a parallel beam of light. It consists of two co-axial metal tubes. The outer tube is mounted horizontally and carries a convex lens L_1 at its free end. The inner tube has an adjustable vertical slit at the free end and can be slided inside the outer tube by a *rack* and *pinion* arrangement. The slit is adjusted in the focal plane of lens L_1. When a light source is kept in front of the slit, a parallel beam of light emerges from the collimator.

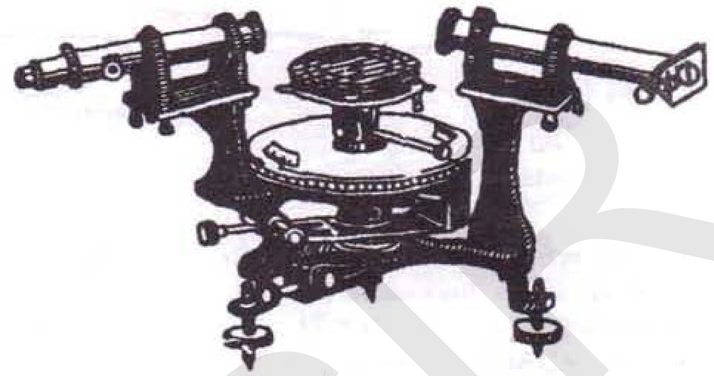


Fig. 9.130 Spectrometer.

2. Prism table. It is a circular horizontal plate on which the prism is placed. It can be adjusted at a desired height with the help of a clamping screw. It can be rotated about a vertical axis. Its position can be noted with the help of the verniers V_1 and V_2 attached to it and moving over a graduated circular scale carried by the telescope.

3. Telescope. It is used for observing the spectrum. It is an astronomical telescope having a convex lens L_2 at one end and Ramsden eyepiece at the other. It is mounted horizontally on a vertical arm attached to the main circular scale. It can be rotated about the same vertical axis about which the prism table rotates. Its position can be noted on the circular scale by the vernier's V_1 and V_2. A cross-wire is fixed at the focus of the eyepiece.

Working. *For getting a pure spectrum,* the following adjustments are made in a spectrometer :

1. Focussing the eyepiece. The eyepiece of the telescope is moved in and out so that the cross-wires are clearly visible.

2. Focussing the telescope for parallel rays. The telescope is turned towards a distant object. The distance between the eyepiece and the object is so adjusted that object becomes clearly visible. This sets the telescope for receiving the parallel rays.

3. Focussing the collimator for parallel rays. Illuminate the slit with a bright source and view it through the telescope. Adjust the distance between the slit and the collimator lens till a clear image of the slit is seen. This sets the collimator to provide a parallel beam of light.

4. Setting the prism. The prism is placed at the centre of the prism table. The prism table is rotated so that light from the collimator falls on refracting face *AB*

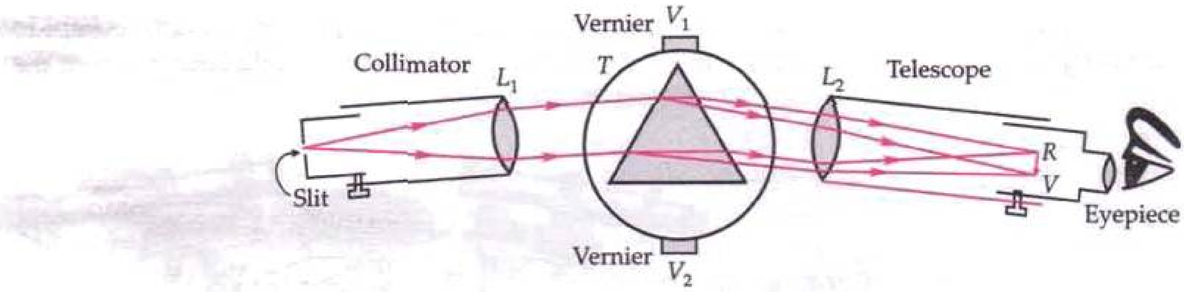


Fig. 9.131 Working of a spectrometer.

and after refraction emerges from the other face AC. The prism causes dispersion. The rays of a given colour emerge parallel to each other. They are received by the telescope. All red rays are focussed at R, all violet rays at V and rays of other colours in between and a spectrum RV is formed in the focal plane of the objective, as shown in Fig. 9.131. A magnified spectrum is viewed through the eyepiece.

5. To get rid of overlapping of colours, the prism is set in the minimum deviation position for some mean (yellow) colour. This gives a pure spectrum.

60. *State some of the important uses of a spectrometer.*

Uses of a spectrometer :

1. To measure the angle of the prism.
2. To determine the refractive index of the prism material.
3. To determine the wavelength of light.
4. To measure the dispersive power of a prism.

9.37 MEASUREMENT OF REFRACTIVE INDEX BY A SPECTROMETER*

61. *How can a spectrometer be used to determine the refractive index of the material of a prism ?*

Measurement of refractive index (μ). The refractive index μ of the material of a prism is given by

$$\mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$

To determine μ, we need to measure :

(*i*) angle of minimum deviation (δ_m) and

(*ii*) the refracting angle of the prim (A).

Measurement of δ_m. Set the prism in the minimum deviation position, as shown in Fig. 9.132. Turn the telescope so that its cross-wire coincides with mean (yellow) colour of the spectrum. Let this position be T_1. Remove the prism. Turn the telescope to position T_2 so that direct image of slit is seen. The difference between the two positions gives δ_m.

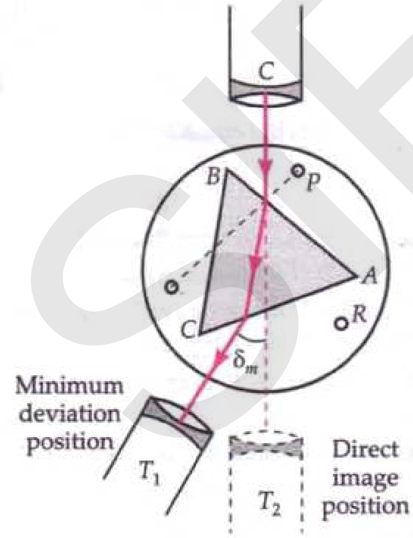


Fig. 9.132 Setting the prism in minimum deviation position.

Measurement of A. Place the prism ABC on the prism table so that light falls directly on faces AB and AC of the prism, as shown in Fig 9.133. Look for the brightest image of the slit formed by reflection of light from faces AB and AC. Set the telescope in position T_1 so that cross-wire coincides with the image of the slit from face AB. Turn the telescope to the position T_2 so as to focus the image of the slit from face AC. Let θ be the angle through which telescope turns.

Then, $A = \frac{\theta}{2}$.

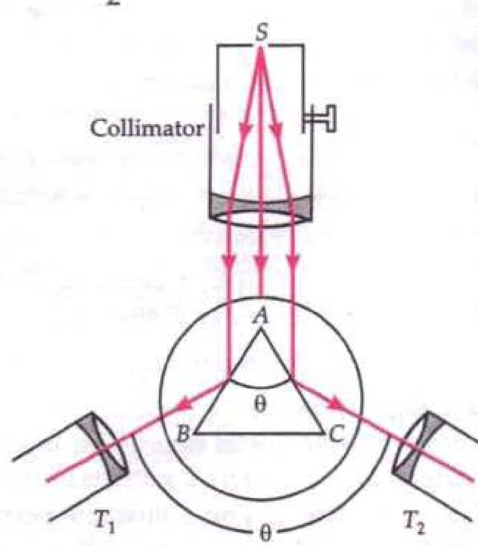


Fig. 9.133 To measure angle of the prism.

9.38 SCATTERING OF LIGHT

62. *What do you mean by scattering of light ? What are elastic and inelastic scatterings ?*

Scattering of light. *This is the phenomenon in which light is deflected from its path due its interaction with the particles of the medium through which it passes.* Basically, the scattering process involves the absorption of light by the molecules followed by its re-radiation in different directions.

Two types of scatterings :

1. ***Elastic or Rayleigh scattering.*** When the size 'a' of the scattering particles is much smaller than the wavelength 'λ' of incident light, there is no exchange of energy between the incident light and the scattering particles. Consequently, there is no change in the frequency or wavelength of the scattered light. This type of scattering is called ***elastic*** or ***Rayleigh scattering***. It obeys Rayleigh's law of scattering.

2. ***Inelastic scattering.*** When the size of the scattering particles is much greater than the wavelength of incident light *i.e.*, $a >> \lambda$, there is *interchange* of energy between incident light and the scattering particles. Consequently, the scattered light has a frequency or wavelength different from that of incident light. This type of scattering is called ***inelastic scattering***. For example, the Raman effect and Compton effect.

63. *State Rayleigh's law of scattering.*

Rayleigh's law of scattering. *According to Rayleigh's law of scattering, the intensity of light of wavelength λ present in the scattered light is inversely proportional to the fourth power of λ, provided the size of the scattering particles are much smaller than λ.* Mathematically,

$$I \propto \frac{1}{\lambda^4} \qquad [\text{For } a << \lambda]$$

Thus the scattered intensity is maximum for shorter wavelengths.

64. *Explain different phenomena of daily life which are based on scattering of light.*

Daily life phenomena based on scattering of light. Several beautiful phenomena in nature are based on scattering of light. Some of these include the blue colour of sky, white clouds, the red hues of sunrise and sunset, the rainbow, the brilliant colours of some pearls, shells, and wings of birds. We describe some of these phenomena.

1. **Blue colour of the sky.** The blue colour of the sky is due to the scattering of sunlight by the molecules of the atmosphere. As sunlight passes through atmosphere, the nitrogen and oxygen molecules of air absorb some amount of sunlight and re-emit it. The free gas molecules scatter light in all directions. But scattering is preferential. According to ***Rayleigh's law of scattering***, the intensity of scattered light,

$$I \propto \frac{1}{\lambda^4}.$$

So the light at the short wavelength (blue) end of the visible spectrum is scattered about ten times more than the light at the long wavelength (red) end. When we look at the sky, the scattered light enters our eyes and this light contains blue colour in a larger proportion. That is why the sky appears blue.

If the earth had no atmosphere, there would be no scattering of light, the sky would appear black and stars could be seen during day hours. This is what astronauts actually observe at heights 20 km above the earth where the atmosphere becomes quite thin or on the moon which has no atmosphere.

2. **Reddishness at sunset and sunrise.** When the sun is near the horizon at sunset or sunrise, the light rays have to traverse a larger thickness of the atmosphere than when the sun is overhead at noon. In accordance with Rayleigh's scattering law, the lower wavelengths in the blue region are almost completely scattered away by the air molecules. The higher wavelengths in the red region are least scattered and reach our eyes. Hence the sun appears almost reddish at sunset and sunrise.

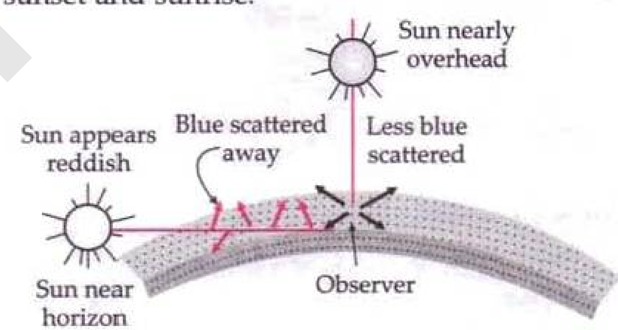


Fig. 9.134 Absorption of sunlight at sunset and sunrise.

3. **Clouds appear white.** Large particles like raindrops, dust and ice particles do not scatter light in accordance with Rayleigh's law, *i.e.*, their scattering power is not selective. They scatter light of all colours almost equally. Hence the clouds which have droplets of water with $a >> \lambda$ are generally white.

4. **Danger signals are red.** According to Rayleigh's law, the intensity of scattered light is inversely proportional to the fourth power of wavelength. In the visible spectrum, red colour has the largest wavelength, it is scattered the least. Even in foggy conditions, such a signal covers large distances without any appreciable loss of intensity due to scattering. Therefore, red coloured signals are preferred.

9.39 RAINBOW*

65. *What is a rainbow ? Explain the formation of primary and secondary rainbow.*

Rainbow. *The rainbow is nature's most spectacular display of the spectrum of light, produced by refraction, dispersion and internal reflection of sunlight by spherical rain drops.* It is observed when the sun shines on rain drops, during or after a shower. An observer standing with his back towards the sun observes it in the form of concentric circular arcs (bows) of different colours in the horizon. The inner brighter rainbow is called the *primary rainbow* and the outer fainter rainbow is called the *secondary rainbow*.

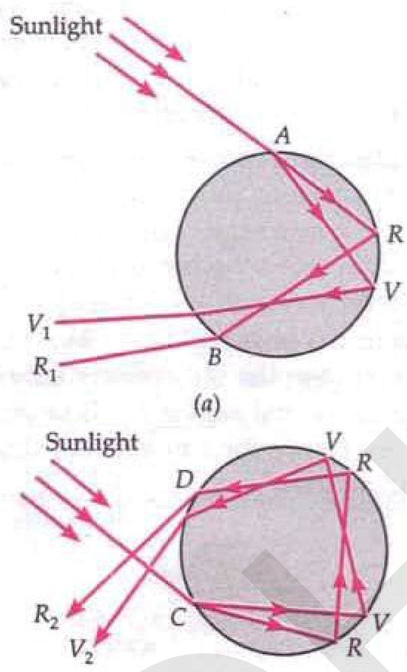


Fig. 9.135 Dispersion of sunlight of a single raindrop (*a*) primary rainbow and (*b*) secondary rainbow.

The ***primary rainbow*** is formed by rays which undergo one internal reflection and two refractions and finally emerge from the raindrops at minimum deviation. The red rays emerge from the water drops at one angle of 43° and the violet rays emerge at another angle of 41°. The parallel beam of sunlight getting dispersed at these angles produces a cone of rays at the observer's eye, as shown in Fig. 9.136. Thus the rainbow is seen as a colourful arc, with its inner edge violet and outer edge red in colour.

The ***secondary rainbow*** is formed by the rays which undergo two internal reflections and two refractions before emerging from the water drops at minimum deviation. Due to two internal reflections, the sequence of colour in secondary rainbow is opposite to that in the primary rainbow. Here the inner red rays emerge from the water drops at angle of 51° and the outer violet rays emerge at angle of 54°.

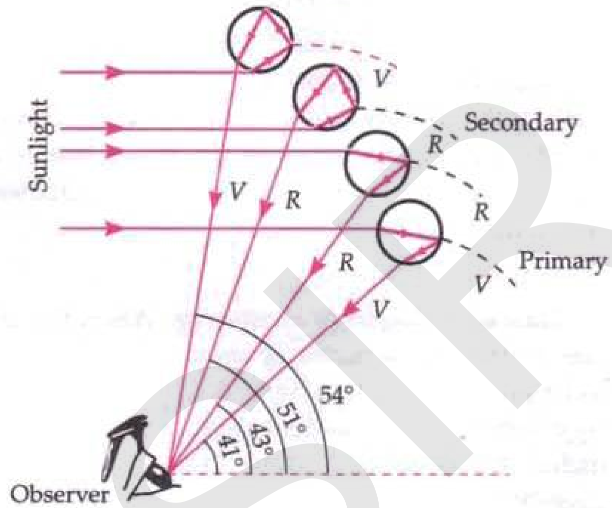


Fig. 9.136 Formation of primary and secondary rainbow.

9.40 OPTICAL INSTRUMENTS

66. *What are optical instruments ? What are the two essential features that must be possessed by optical instruments for viewing objects distinctly ?*

Optical instruments. *Optical instruments are the devices which make use of mirrors, lenses and prisms and are primarily used to extend the range of vision of human eye.* For example, microscopes are used for viewing tiny objects clearly while telescopes are used to see distant objects clearly.

Essential features of an optical instrument. The design of an optical instrument must meet the following *two* requirements :

1. ***High magnification.*** Magnification is the ratio of the size of the final image to the size of the object. An optical instrument with high magnification makes viewing more clear and comfortable, by increasing the size of the image.

2. ***Adequate resolution.*** The resolution of an optical instrument is its ability to resolve the images of two closely spaced objects so that they can be seen separately. An optical instrument with high resolution reveals the finer details of the objects.

9.41 THE HUMAN EYE*

67. *Describe the main parts of the human eye. Briefly explain its working.*

Human Eye. It is the most valuable and sensitive sense organ. It is a remarkable optical instrument.

* Not included in the latest CBSE Syllabus.

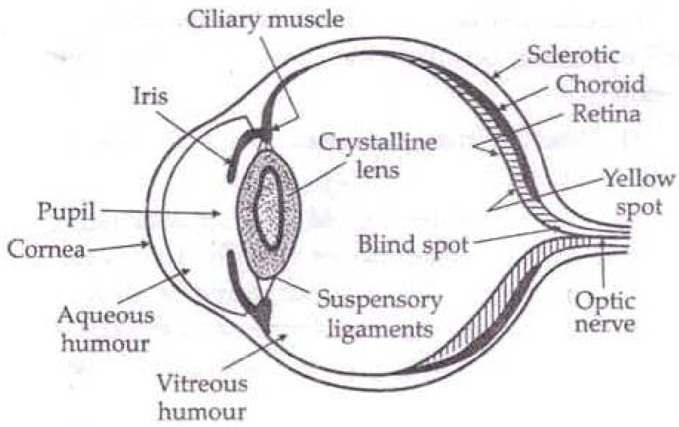


Fig. 9.137 Structure of the human eye.

Structure of the eye. As shown in Fig. 9.137, the main parts of the human eye are as follows :

1. Sclerotic. It has a tough and opaque white covering, called *sclerotic* which protects and holds the eyeball.

2. Cornea. It is the transparent membrane on the front portion of the eyeball through which light enters the eye.

3. Choroid. It is a black membrane below the sclerotic. It absorbs stray light and avoids any blurring of image due to multiple reflections in the eyeball.

4. Iris and pupil. Iris is an opaque circular diaphragm having a small central hole called the pupil. Under the muscular action of the iris, the size of the pupil becomes smaller in bright light and larger in dim light.

5. Eyelens. It is a double convex lens situated behind the iris. It is composed of a fibrous, jelly like material. The lens is held in position by *suspensory ligaments* and connected to the sclerotic by the *ciliary muscles*. By contracting or relaxing, the ciliary muscles can change the shape or curvature of the eyelens and hence change its focal length. This ability of the eyelens to change its focal length is called *accommodation*. This enables the eyelens to focus the images of objects at different distances on the retina of the eye.

6. Retina. It is a delicate membrane of nerve fibres on the inner side of the backwall of the eye. *It contains light sensitive cells called rods and cones.* Rods are sensitive to intensity of light while cones are sensitive to colours. These cells change light energy into electrical signals which send message to the brain via the *optic nerves*.

7. Blind spot and yellow spot. In the region where the optic nerve enters the eyeball, there are no rods and cones. This region is *totally insensitive* to light and is called *blind spot*. *Yellow spot* has maximum concentration of light sensitive cells. It is situated in the centre of the retina.

8. Aqueous humour and vitreous humour. Aqueous humour is a salty fluid ($\mu = 1.337$) that fills the space between the cornea and the eyelens. Vitreous humour is a jelly like fluid ($\mu = 1.437$) that fills the space between the retina and the eyelens.

Action of the eye. The transparent structures like cornea, aqueous humour, eyelens and vitreous humours together constitute a single converging lens. As the rays from an object enter the eye, they suffer refractions on passing successively through these structures and get converged. A real and inverted image is formed on the retina. The light sensitive cells of retina get activated and generate electrical signals that are sent to the brain through the optic nerves. Our brain translates the inverted image into an erect image.

68. *What do you mean by the term accommodation ? Explain, how can the eye see objects at far and near distances.*

Accommodation. *Accommodation is the ability or property of the eyelens due to which it can change its curvature or focal length so that images of objects at various distances can be formed on the same retina.* The focal length of the eyelens is automatically changed with the help of ciliary muscles as follows :

(a) ***Viewing far off objects.*** When the ciliary muscles are completely relaxed, the eyelens is thin and its focal length is maximum (equal to distance between eyelens and retina). The rays coming from the distant object are parallel to each other and they are focussed at the retina as shown in Fig. 9.138(*a*).

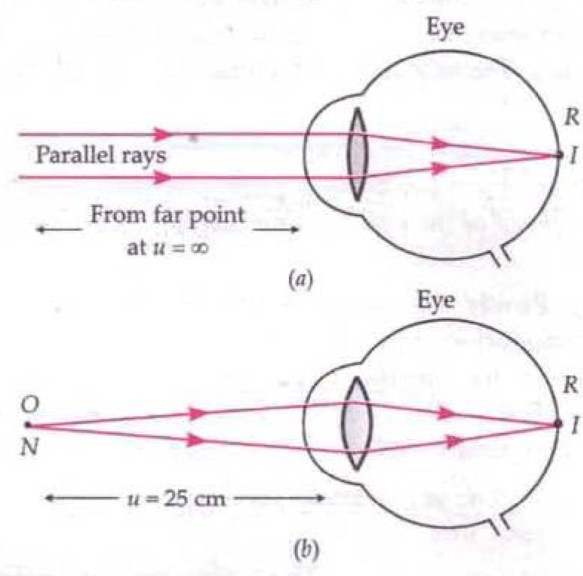


Fig. 9.138 Accomodation of eyelens
(*a*) Focussing parallel rays from infinity *i.e.*, far point
(*b*) Focussing rays from near point *N*.

(b) ***Viewing nearby objects.*** When we look at a nearby object, the ciliary muscles contract, the eyelens bulges out and becomes thick and its focal length is reduced. This focusses the light from the nearby object on the retina, as shown in Fig. 9.138(*b*).

69. *Define the following terms and give their values for a normal eye : (i) range of normal vision, (ii) least distance of distinct vision, (iii) near point of the eye, (iv) far point of the eye, and (v) power of accommodation.*

(*i*) **Range of normal vision.** Due to accommodation property of the lens, a normal eye can clearly see the object situated any where between infinity and 25 cm from it. At distance less than 25 cm, the ciliary muscles cannot bulge the eyelens any more, the object cannot be focussed on the retina and it appears blurred to the eye, as shown in Fig. 9.139. *The distance between infinity and 25 cm point is called the range of normal vision.*

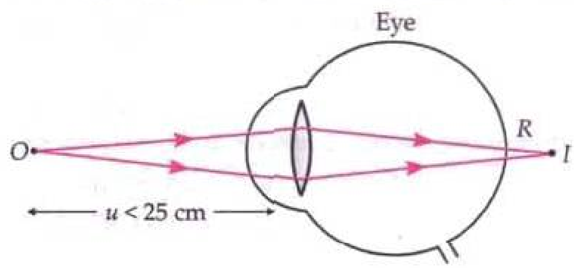


Fig. 9.139 Object *O* within 25 cm from the eye is not focussed on retina and seen blurred.

(*ii*) **Least distance of distinct vision.** *The minimum distance from the eye, at which the eye can see the object clearly and distinctly without any strain is called the least distance of distinct vision.* It is denoted by the letter *D*. *For a normal eye, its value is 25 cm.*

(*iii*) **Near point.** *The nearest point from the eye, at which an object can be seen clearly by the eye is called its near point. The near point of a normal eye is at a distance of 25 cm.*

(*iv*) **Far point.** The farthest point from the eye, at which an object can be seen clearly by the eye is called the far point of the eye. *For a normal eye, the far point is at infinity.*

(*v*) **Power of accommodation.** The power of accommodation of the eye is the maximum variation of its power for focussing on near and far (distant) objects. *For a normal eye, the power of accommodation is about 4 dioptres.*

70. *What do you mean by persistence of vision ? Give an example.*

Persistence of vision. The impression or sensation of an image on the retina remains (or persists) for about (1/16)th of a second even after the removal of the object. *The phenomenon of the continuation of the impression of an image on the retina for some time even after the light from the object is cut off is called persistence of vision.* For example, if there be a picture of a bird on one side of a piece of card-board and a picture of a cage just on the opposite side, then on rapidly revolving the card-board, the two impressions merge and the bird will appear to be inside the cage due to persistence of vision.

71. *What are rods and cones ? How do they differ in their functions ?*

Rods. *These are rod-shaped light sensitive cells of the retina which are responsible for twilight (black-and-white) vision.* These cells are very sensitive to intensity of incident light, that is, the degree of brightness and darkness. The rods cannot distinguish between various colours.

Cones. *These are cone-shaped light sensitive cells of the retina which are responsible for colour vision.* Different cones respond selectively to different colours. Three types of cones *viz.* *R*-cones, *G*-cones and *B*-cones are respectively sensitive to red, green and blue light. When red light falls on the retina, it mainly activates the *R*-cones than the other kinds of cones. However, cone becomes active only in bright light. That is why, we cannot see colours in very dim light.

For Your Knowledge

- **Cinematography.** Cinematography works on the principle of persistence of vision. If photographs of a moving object are taken at intervals of about (1/24)th of a second and then projected on the screen at the same rate, the discontinuous pictures merge or blend together to produce the impression of the moving object on the eyes.
- **Colour blindness.** *A person who cannot distinguish between various colours but can see well otherwise, is said to be colour blind.* Colour blindness is due to the lack of either one type, two types or all the three types of cones in the retina of the eyes. This defect occurs by inheritance. That is colour blindness is a genetic disorder which cannot be cured even today.
- **Cataract.** In old age, the crystalline lens of some people becomes hazy or even opaque due to the development of membrane over it. This results in the development of cataract, which causes a decrease or loss of vision of the eye. The vision can be restored after getting cataract surgery.

9.42 DEFECTS OF VISION AND THEIR CORRECTION*

72. *Mention the common defects of vision of the human eye.*

Defects of vision. A normal eye can see objects clearly at any distance between 25 cm and infinity from the eye. Sometimes, a human eye gradually loses its power of accommodation. Then we cannot see the objects clearly. Our vision becomes defective. There are mainly *four common defects of vision* which can be corrected by the use of suitable eye glasses. These defects are :

1. Myopia or near-sightedness.
2. Hypermetropia or far-sightedness.
3. Presbyopia.
4. Astigmatism.

73. *What is myopia or short-sightedness ? What is its cause ? How can it be remedied ? Explain by ray diagrams.*

Myopia or short-sightedness. *It is a vision defect in which a person can see nearby objects clearly but cannot see the distant objects clearly beyond a certain point.* This defect is common among children.

Cause of myopia. This defect arises due to either of the following two reasons :

(*i*) The *eyeball gets elongated* along its axis so that the distance between the eyelens and the retina becomes larger.

(*ii*) *The focal length of the eyelens becomes too short* due to the excessive curvature of cornea.

As a result of the above causes, the parallel rays coming from a distant object do not meet at the retina but at a point in front of the retina, as shown in Fig. 9.140(*a*) and the distant object is not seen clearly. The object has to be moved closer to the eye to a point *F* to focus it on the retina, as shown in Fig. 9.140(*b*). Thus, the far point of a myopic eye is not at infinity but only a few metres from the eye.

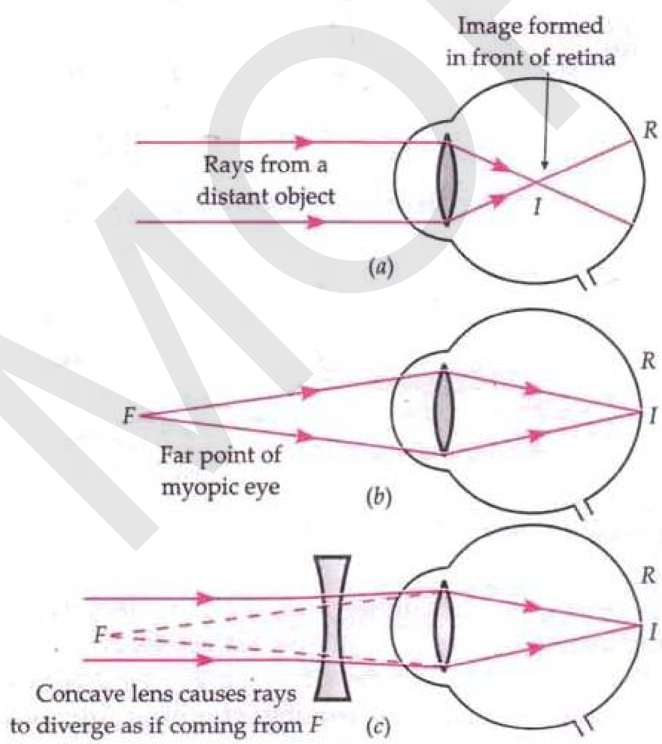


Fig. 9.140 Myopia and its correction.

Correction of myopia. *A myopic eye is corrected by using a concave lens of focal length equal to the distance of the far point F from the eye.* This lens diverges the parallel rays from distant object as if they are coming from the far point *F*. Finally, the eyelens forms a clear image at the retina.

74. *How can we determine the focal length and power of the concave lens required to correct a myopic eye ?*

Calculation of focal length and power of correcting lens in myopia. Let *x* be the distance of the actual far point from the eye and hence from the concave lens placed close to the eye. The rays coming from infinity, after refraction through the concave lens, appear to come from the far point *F*.

$\therefore \quad u = -\infty, \quad v = -x, \quad f = ?$

By lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-x} - \frac{1}{-\infty} = -\frac{1}{x} + 0 = -\frac{1}{x}.$$

$\therefore$ Required focal length, $f = -x$

Required power, $P = \frac{1}{f} = -\frac{1}{x}$

The negative sign shows that the correcting lens is a concave lens.

75. *What is hypermetropia or long-sightedness ? What is its cause ? How can it be corrected ? Explain by ray diagrams.*

Hypermetropia or long-sightedness. *It is a vision defect in which a person can see the distant objects clearly but cannot see the nearby objects clearly.*

Cause of hypermetropia. This defect arises due to either of the following *two* reasons :

(*i*) *The eyeball becomes too small along its axis* so that the distance between the eyelens and the retina is reduced.

(*ii*) *The focal length of the eyelens becomes too large* resulting in the low converging power of the eyelens.

As a result of the above causes, the rays coming from an object placed at 25 cm (normal near point) from the eye meet at a point behind the retina, as shown in Fig. 9.141(*a*). So the object is not seen clearly.

To focus the rays again on the retina, the object has to be moved away from the eyes to a distance greater than 25 cm, as shown in Fig. 9.141(*b*). Thus, *the near point of the eye is not at 25 cm but it has shifted to N′ at a distance greater than 25 cm from the eyes.*

Correction of hypermetropia. *A hypermetropic eye is corrected by using a convex lens of suitable focal length.*

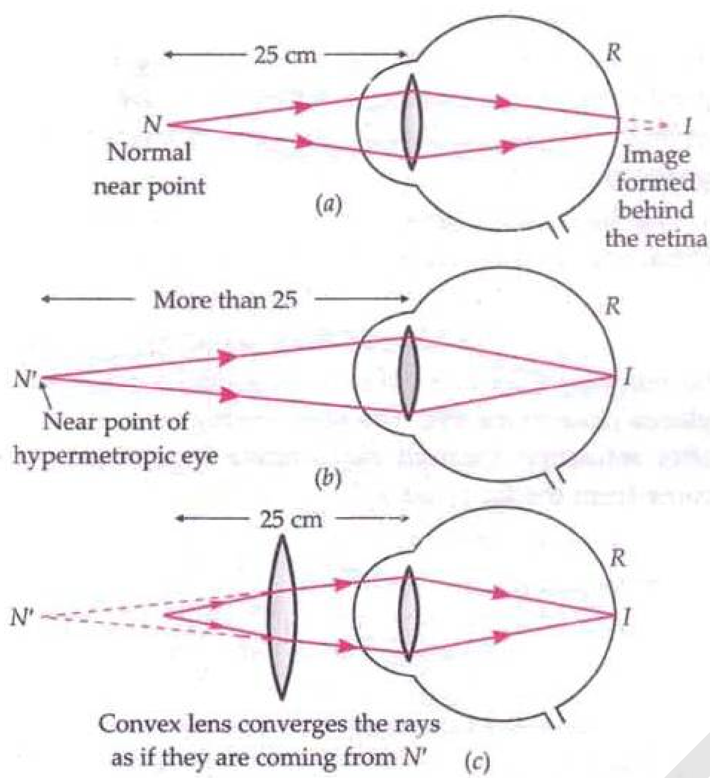


Fig. 9.141 **Hypermetropia and its correction.**

This lens converges the rays such that the rays coming from normal near point N appear to come after refraction, from near point N' of the defected eye. That is a virtual image of the object placed at N is formed at N'. Then the eyelens forms a clear image at the retina, as shown in Fig. 9.141(c).

76. *How can we determine the focal length and power of the convex lens required to correct a hypermetropic eye ?*

Calculation of focal length and power of correcting lens in hypermetropia. Refer to Fig. 9.141(c). Let y = distance of the near point N' from the defective eye. Now the near point N of the normal eye is at distance $D = 25$ cm. The object placed at N forms its virtual image at N' due to the convex lens.

$\therefore \quad u = -D, \quad v = -y, \quad f = ?$

By lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-y} - \frac{1}{-D} = \frac{y-D}{yD}$$

$\therefore$ Required focal length, $f = \dfrac{yD}{y-D}$

Required power, $P = \dfrac{1}{f} = \dfrac{y-D}{yD}$

As $y > D$, so both f and D are positive. That is the correcting lens must be a convex lens.

77. *What is presbyopia ? How does it differ from hypermetropia ?*

Presbyopia. This defect is similar to hypermetropia *i.e.*, a person having this defect cannot see nearby objects distinctly, but can see distant objects without any difficulty. This defect differs from hypermetropia in the cause by which it is produced. It usually occurs in elderly persons. Due to the stiffening of the ciliary muscles, the eyelens loses flexibility and hence the accommodating power of the eyelens decreases. Like hypermetropia, this defect can be corrected by using a convex lens of suitable focal length.

78. *What is astigmatism ? How is it caused ? How is it corrected ?*

Astigmatism. *It is a defect of vision in which a person cannot simultaneously see both the horizontal and vertical views of an object with the same clarity.* This defect can occur alongwith myopia or hypermetropia.

Cause of astigmatism. This defect occurs when the cornea is not perfectly spherical in shape. It may have a large curvature in the vertical plane than in the horizontal plane or vice versa. If one looks at a wire mesh with such a defect in the eyelens, focussing in the vertical plane may not be as sharp as in the horizontal plane or vice versa. Astigmatism results in lines in one direction well focussed while those in perpendicular direction will be distorted or curved, as shown in Fig. 9.142(a).

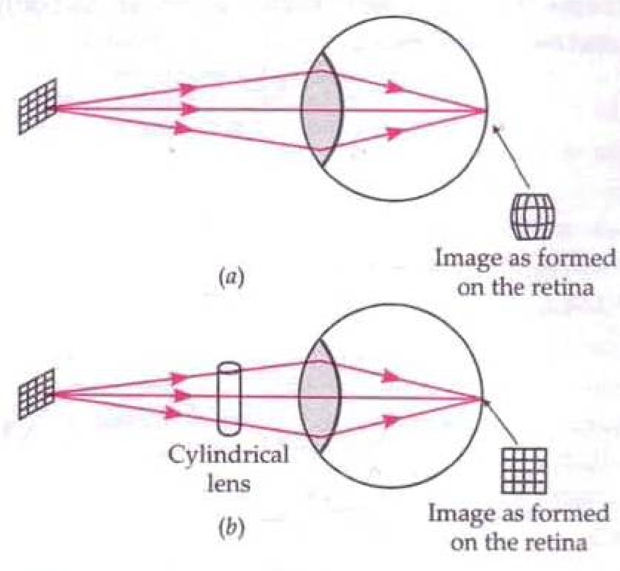


Fig. 9.142 **(a) Astigmatism and (b) its correction.**

Correction of Astigmatism. Astigmatism can be corrected by a lens whose one surface is cylindrical. Such a surface focusses rays in one plane but not in the perpendicular plane. By suitably choosing the radius of curvature and axis direction of the cylindrical surface, astigmatism can be corrected.

Examples based on Defects of Vision

Formulae Used

1. **Correction of myopia or short sightedness.** A concave lens of focal length f equal to the distance x of the far point from the defective eye is used.

$$f = -x \quad \text{and} \quad P = -\frac{1}{x}$$

2. **Correction of hypermetropia or long sightedness.** A convex lens of focal length f is used, where

$$f = \frac{y - D}{y - D}$$

Here

y = distance of the near point from the defective eye and

D = the least distance of distinct vision.

Units Used

Distances f, x, y and D are in metre, and power P in dioptre.

Example 113. *What focal length should the reading spectacles have for a person for whom the least distance of distinct vision is 50 cm ?* [NCERT]

Solution. The reading matter placed at 25 cm from the corrective lens must produce the virtual image at 50 cm. Therefore,

$$u = -25 \text{ cm}, \quad v = -50 \text{ cm}$$

By thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-50} - \frac{1}{-25}$$

$$= \frac{-1+2}{50} = \frac{1}{50}$$

or $\quad f = +\mathbf{50\ cm}$

The positive sign shows that the corrective lens must be a convex lens of focal length 50 cm.

Example 114. *A person wears glasses of power −2.5 D. Is the person far-sighted or near-sighted ? What is the far point of the person without glasses ?*

Solution. Here, $P = -2.5$ D

Negative power shows that the lens is concave, so the person is near-sighted.

$$f = \frac{1}{P} = \frac{1}{-2.5} \text{ m} = -\frac{2}{5} \text{ m} = -40 \text{ cm}$$

The object placed at infinity from the corrective lens must produce the virtual image at the far point. Therefore,

$$u = -\infty, \quad v = ?$$

From thin lens formula,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-40} + \frac{1}{-\infty}$$

$$= \frac{1}{-40} - 0 = -\frac{1}{40}$$

or $\quad v = -\mathbf{40\ cm}$

Thus the far point of the eye is at 40 cm from the eye.

Example 115. (*a*) *The far point of a myopic person is 80 cm in front of the eye. What is the power of the lens required to enable him to see very distant objects clearly ?*

(*b*) *In what way does the corrective lens help the person above ? Does the lens magnify very distant objects ? Explain carefully.* [CBSE D 09C upto part (*b*)]

(*c*) *The person above prefers to remove his spectacles while reading a book. Explain why.* [NCERT]

Solution. (*a*) The remedial lens should make the objects at infinity appear at the far point.

∴ For objects at infinity, $\quad u = -\infty$

Far point distance of the defective eye, $v = -80$ cm

By thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{-80} - \frac{1}{-\infty} = -\frac{1}{80} + 0 = -\frac{1}{80}$$

or $\quad f = -80 \text{ cm} = -0.80 \text{ m}$

Power, $\quad P = \frac{1}{f} = \frac{1}{-0.80 \text{ m}} = \mathbf{-1.25\ D.}$

(*b*) No, the concave lens does not magnify the very distant objects. In fact, it reduces the size of the object (image distance is less than object distance), but the angle subtended by the distant object at the eye is the same as the angle subtended by the image (on the far point) at the eye. The eye is able to see distant objects not because the corrective lens magnifies the object, but because it brings the object (*i.e.*, it produces virtual image of the object) at the far point of the eye which then can be focused by the eye-lens on the retina.

(*c*) The myopic person may have a normal near point *i.e.*, about 25 cm (or even less). In order to read a book with his spectacles (for distant vision), he must keep the book at a greater distance than 25 cm so that the image of the book by the concave lens is produced not closer than 25 cm. The angular size of the book (or its image) at the grater distance is evidently less than the angular size when the book is placed at 25 cm and no spectacles are used. Hence, the person prefers to remove his spectacles while reading.

Example 116. *(a) The near point of a hypermetropic person is 75 cm from the eye. What is the power of the lens required to enable him to read clearly a book held at 25 cm from the eye ?*

(b) In what way does the corrective lens help the person above ? Does the lens magnify objects held near the eye ?

(c) The person above prefers to remove his spectacles while looking at the sky. Explain why. [NCERT]

Solution. (*a*) The book placed at 25 cm from the corrective lens must form the virtual image at 75 cm. Therefore,

$$u = -25 \text{ cm}, \quad v = -75 \text{ cm}$$

By thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{-75} - \frac{1}{-25} = \frac{-1+3}{75} = \frac{2}{75}$$

or $$f = \frac{75}{2} \text{ cm} = \frac{75}{200} \text{ m}$$

$$P = \frac{1}{f} = \frac{200}{75} = \mathbf{+\,2.67\ D}$$

Thus the corrective lens must be a convex lens of power + 2.67 dioptres.

(*b*) The corrective lens produces a virtual image (at 75 cm) of an object at 25 cm. The angular size of this image is the same as that of the object. In this sense the lens does not magnify the object but merely brings the object to the near point of the eye, which then gets focused by the eye lens on the retina. However, the angular size is greater than that of the same object at the near point (75 cm) viewed without the spectacles.

(*c*) The person prefers to remove his spectacles while looking at the sky, because a hypermetropic eye may have normal far point *i.e.*, it may have enough converging power to focus parallel rays from infinity on the retina of the shortened eyeball. Wearing spectacles of converging lenses (used for near vision) will amount to more converging power than needed for parallel rays. The result will be that distant objects may get focused in-front of the retina (like a myopic eye) and will appear blurred.

Example 117. *A 52-year old near-sighted person wears eye-glass with a power of – 5.5 dioptres for distance viewing. His doctor prescribes a correction of + 1.5 dioptres in the near vision section of his bi-focals. This is measured relative to the main part of the lens. (a) What is the focal length of his distance-viewing part of the lens ? (b) What is the focal length of the near-vision section of the lens ?*

Solution. (*a*) Power of the distance-viewing part of the lens,

$$P_1 = -5.5 \text{ D}$$

Focal length of this part,

$$f_1 = \frac{1}{P_1} = \frac{1}{-5.5} \text{ m} = \mathbf{-\,18.73\ cm.}$$

(*b*) As power of the near-vision part is measured relative to the main part of the lens of power – 5.5 D, so we use

$$P_1 + P_2 = P$$

or $$-5.5 + P_2 = +1.5$$

or $$P_2 = +6.5 \text{ D}$$

Focal length of near-vision part,

$$f_2 = \frac{1}{P_2} = \frac{1}{+6.5} \text{ m} = \mathbf{+\,15.4\ cm.}$$

Problems For Practice

1. The far point of a myopic person is 150 cm in front of the eye. Calculate the focal length and the power of the lens required to enable him to see distant objects clearly. (Ans. – 150 cm, – 0.67 D)
2. A person can see clearly up to 3 metre only. Prescribe a lens for him so that he can see clearly up to 12 metre. (Ans. Concave lens, $f = -4$ m)
3. The near point of hypermetropic persion is 50 cm from the eye. What is the power of the lens required to enable the person to read clearly a book held at 25 cm from the eye ? **[CBSE OD 09]** (Ans. + 2 D)
4. A short-sighted person can see most clearly at a distance of 15 cm acquires spectacles enabling him to see clearly objects at a distance of 60 cm. Calculate the focal length of the lens and power of the lens. (Ans. – 20 cm, – 5 D)

HINTS

1. Here, $u = -\infty, \quad v = -150$ cm

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{-150} - \frac{1}{-\infty}$$

$$= -\frac{1}{150} + 0 = -\frac{1}{150}$$

$$f = \mathbf{-\,150\ cm}$$

$$P = \frac{1}{f \text{ (in m)}} = \frac{1}{-1.50 \text{ m}}$$

$$= \mathbf{-\,0.67\ D.}$$

2. Here, $u = -12$ m, $v = -3$ m, $f = ?$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= -\frac{1}{3} + \frac{1}{12} = -\frac{1}{4}$$

or $f = -4$ **m.**

3. The book placed at 25 cm from the corrective lens must form the virtual image at 50 cm.

$\therefore$ $u = -25$ cm, $v = -50$ cm

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = -\frac{1}{50} + \frac{1}{25} = \frac{1}{50}$$

or $f = 50$ cm $= 0.50$ m

Power $P = \frac{1}{f} = \frac{1}{0.50 \text{ m}} = \mathbf{2\ D}$

Thus, the corrective lens must be a converging lens of power 2 D.

9.43 SIMPLE MICROSCOPE

79. *What is a simple microscope ? Give its working principle. Write expressions for its magnifying power when it forms final image at the least distance of distinct vision and at infinity.*

Simple microscope. *A simple microscope or a magnifying glass is just a convex lens of short focal length, held close to the eye.*

Working principle : When the final image is formed at the least distance of distinct vision. When an object *AB* is placed between the focus *F* and optical centre *O* of a convex lens ; a virtual, erect and magnified image *A′B′* is formed on the same side of the lens as the object. Since a normal eye can see an object clearly at the least distance of distinct vision D (=25 cm), the position of the lens is so adjusted that the final image is formed at the distance D from the lens, as shown in Fig. 9.143.

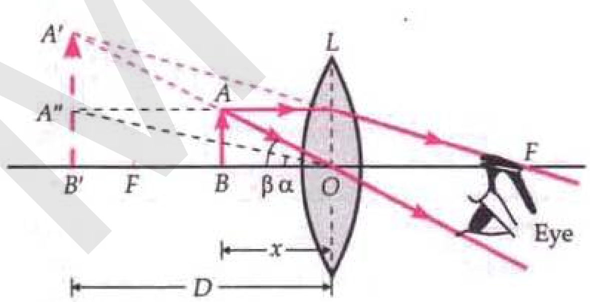


Fig. 9.143 A simple microscope with the eye focussed at the near point.

Magnifying power. *The magnifying power of a simple microscope is defined as the ratio of the angles subtended by the image and the object at the eye, when both are at the least distance of distinct vision from the eye.* Thus,

Magnifying power

$$= \frac{\text{Angle subtended by the image at the least distance of distinct vision}}{\text{Angle subtended by the object at the least distance of distinct vision}}$$

As the eye is held close to the lens, the angles subtended at the lens may be taken to be the angles subtended at the eye. The image *A′B′* is formed at the least distance of distinct vision 'D'. Let $\angle A'OB' = \beta$. Imagine the object *AB* to be displaced to position $A''B'$ at distance D from the lens. Let $\angle A''OB' = \alpha$. Then magnifying power,

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \qquad [\because \alpha, \beta \text{ are small angles}]$$

$$= \frac{AB/OB}{A''B'/OB'} = \frac{AB/OB}{AB/OB'} \qquad [\because A''B' = AB]$$

$$= \frac{OB'}{OB} = \frac{-D}{-x}$$

or $$m = \frac{D}{x}$$

Let f be the focal length of the lens. As the image is formed at the least distance of distinct vision from the lens, so

$$v = -D$$

Using *thin lens formula*,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we get, $$\frac{1}{-D} - \frac{1}{-x} = \frac{1}{f}$$

or $$\frac{1}{x} = \frac{1}{D} + \frac{1}{f}$$

or $$\frac{D}{x} = 1 + \frac{D}{f}$$

$\therefore$ $$m = 1 + \frac{D}{f}$$

Thus shorter the focal length of the convex lens, the greater is its magnifying power.

Working principle : When the final image is formed at infinity. When we see an image at the near point, it causes some strain in the eye. Often the object is placed at the focus of the convex lens, so that parallel rays enter the eye, as shown in Fig. 9.144(a). The image is formed at infinity, which is more suitable and comfortable for viewing by the relaxed eye.

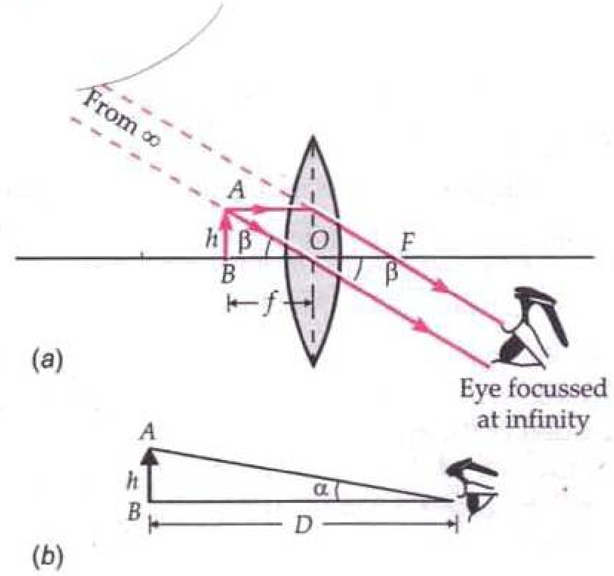


Fig. 9.144 (*a*) With object at *F*, image is at infinity.
(*b*) Object at the near point.

Magnifying power. *It is defined as the ratio of the angle formed by the image (when situated at infinity) at the eye to the angle formed by the object at the eye, when situated at the least distance of distinct vision.*

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \qquad [\alpha, \beta \text{ are small}]$$

From Fig. 9.144(*a*),

$$\tan\beta = \frac{h}{f}$$

From Fig. 9.144(*b*),

$$\tan\alpha = \frac{h}{D}$$

$$\therefore \qquad m = \frac{h/f}{h/D}$$

or $$m = \frac{D}{f}$$

This magnification is one less than the magnification when the image is formed at the near point. But viewing is more comfortable when the eye is focussed at infinity.

Uses of simple microscopes :

1. Watch makers and jewellers use a magnifying glass for having a magnified view of the small parts of watches and the fine jewellery work.
2. In magnifying the printed letters in a book, textures of fibres or threads of a cloth, engravings, details of stamp, etc.
3. Magnifying glass is used in science laboratories for reading vernier scales, etc.

For Your Knowledge

- **Least distance of distinct vision (*D*).** The minimum distance from the eye, at which the eye can see the objects clearly and distinctly without any strain is called the least distance of distinct vision. For a normal eye, its value is 25 cm.
- **Near point.** The nearest point from the eye, at which an object can be clearly seen by the eye is called its near point. The near point of a normal eye is at a distance of 25 cm.
- **Far point.** The farthest point from the eye, at which an object can be seen clearly by the eye, is called the far point of the eye. For a normal eye, the far point is at infinity.
- **Accommodation.** It is the ability of the eyelens due to which it can change its focal length so that images of objects at various distances can be formed on the same retina.
- **Power of accommodation.** The power of accommodation of the eye is the maximum variation of its power for focussing on near and far objects. For a normal eye, the power of accommodation is about 4 dioptres.
- The magnifying power is expressed with a unit X. So if a magnifying glass produces an angular magnification of 10, it is called a 10 X magnifier.
- A simple microscope has a limited maximum magnification of about 10, for realistic focal lengths. For much larger magnifications, we use two convex lenses, one enhancing (compounding) the effect of the other. This is known as the *compound microscope*.

Examples based on
Simple Microscope

Formulae Used

1. When the final image is formed at the least distance of distinct vision, the magnifying power is
$$m = 1 + \frac{D}{f}$$
2. When the final image is formed at infinity, the magnifying power is $m = \frac{D}{f}$

Units Used

Magnification *m* has no units.
$D = 25$ cm, for a normal eye.

Example 118. *A thin convex lens of focal length 5 cm is used as a simple microscope by a person with normal near point (25 cm). What is the magnifying power of the microscope ?* [NCERT]

Solution. Here $f = 5$ cm, $D = 25$ cm

Magnifying power, $m = 1 + \frac{D}{f} = 1 + \frac{25}{5} = \mathbf{6.}$

Example 119. *A simple microscope is a combination of two lenses, in contact, of powers +15 D and +5 D. Calculate the magnifying power of the microscope, if the final image is formed at 25 cm from the eye.*

Solution. Power of combination,

$$P = P_1 + P_2 = 15 + 5 = +20\text{ D}$$

∴ Focal length of combination

$$= \frac{1}{P} = \frac{1}{20}\text{ m} = 5\text{ cm}$$

∴ Magnifying power,

$$m = 1 + \frac{D}{f} = 1 + \frac{25}{5} = \mathbf{6.}$$

Example 120. *An object is to be seen through a simple microscope of power 10 D. Where should the object be placed so as to produce maximum angular magnification ? The least distance for distinct vision is 25 cm.*

Solution. Angular magnification is maximum when the final image is formed at the near point.

$$\therefore \quad v = -25\text{ cm}, \quad f = \frac{1}{P} = \frac{1}{10}\text{ m} = 10\text{ cm}$$

Now $\quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{10} = -\frac{7}{50}$

or $\quad u = -50/7 = \mathbf{-7.1\ cm.}$

Example 121. *A simple microscope is rated 5X for a normal relaxed eye. What will be its magnifying power for a relaxed farsighted eye whose near point is 40 cm ?*

Solution. *For normal eye :* $D = 25$ cm, $m = 5$

As $\quad m = \frac{D}{f} \quad \therefore \quad 5 = \frac{25}{f} \quad$ or $\quad f = 5$ cm

For relaxed farsighted eye : $D' = 40$ cm, $f = 5$ cm

$$\therefore \quad m = \frac{D'}{f} = \frac{40}{5} = 8$$

Thus the magnifying power of the simple microscope is 8 X in the second case.

Example 122. *A converging lens of focal length 6.25 cm is used as a magnifying glass. If the near point of the observer is 25 cm from the eye and the lens is held close to the eye, calculate (i) the distance of the object from the lens and (ii) the angular magnification.*

Find the angular magnification, when the final image is formed at infinity. [ISCE 93]

Solution. Here $f = 6.25$ cm, $v = -D = -25$ cm

(*i*) Using thin lens formula,

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{6.25}$$

$$= -\frac{1}{25} - \frac{4}{25} = -\frac{5}{25} = -\frac{1}{5} \quad \text{or} \quad u = \mathbf{-5\ cm.}$$

(*ii*) Angular magnification,

$$m = 1 + \frac{D}{f} = 1 + \frac{25}{6.25} = 1 + 4 = \mathbf{5}$$

When the final image is formed at infinity, the angular magnification becomes

$$m = \frac{D}{f} = \frac{25}{6.25} = \mathbf{4.}$$

Example 123. *A man with normal near point (25 cm) reads a book with small print using a magnifying glass : a thin convex lens of focal length 5 cm.*

(i) What is the closest and the farthest distance at which he can read the book when viewing through the magnifying glass ?

(ii) What is the maximum and the minimum angular magnification (magnifying power) possible using the above simple microscope ? [NCERT]

Solution. (*i*) *For the closest distance :*

$$v = -25\text{ cm}, \quad f = 5\text{ cm}, \quad u = ?$$

As $\quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\therefore \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{5} = \frac{-1-5}{25} = \frac{-6}{25}$$

or $\quad u = -\frac{25}{6}\text{ cm} = \mathbf{-4.2\ cm}$

This is the closest distance at which the man can read the book.

For the farthest image :

$$v = \infty, \quad f = 5\text{ cm}, \quad u = ?$$

$$\therefore \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{\infty} - \frac{1}{5} = 0 - \frac{1}{5} = -\frac{1}{5}$$

or $\quad u = \mathbf{-5\ cm.}$

This is the farthest distance at which the man can read the book.

(*ii*) Maximum angular magnification

$$= \frac{D}{u_{min}} = \frac{25}{25/6} = \mathbf{6}$$

Minimum angular magnification

$$= \frac{D}{u_{max}} = \frac{25}{5} = \mathbf{5.}$$

Example 124. *A figure divided into squares each of size 1 mm² is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 10 cm) held close to the eye.*

(*i*) *What is the magnification (image size/object size) produced by the lens ? How much is the area of each square in the virtual image ?*

(ii) What is the angular magnification (magnifying power) of the lens ?

(iii) Is the magnification in (i) equal to the magnifying power in (ii) ? Explain. [NCERT ; CBSE D 05]

Solution. (*i*) Here, area of each square (or object)

$$= 1\ \text{mm}^2$$

$$u = -9\ \text{cm}, \quad f = +10\ \text{cm}$$

As $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{9} = \frac{9-10}{90} = -\frac{1}{90}$$

or $v = -90$ cm

Magnitude of magnification,

$$m = \left| \frac{v}{u} \right| = \frac{90}{9} = 10$$

Area of each square in the virtual image

$$= (10)^2 \times 1 = 100\ \text{mm}^2 = \mathbf{1\ cm^2}.$$

(*ii*) Magnifying power,

$$m = \frac{D}{|u|} = \frac{25}{9} = \mathbf{2.8}$$

(*iii*) No. Magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 cm). Thus magnification magnitude is $\left| \dfrac{v}{u} \right|$ and magnifying power is $\dfrac{25}{|u|}$. Only when the image is located at the near point $|v| = 25$ cm, the two quantities are equal.

Example 125. *(i) At what distance should the lens be held from the figure in Example 110 in order to view the squares distinctly with the maximum possible magnifying power ?*

(ii) What is the magnification (image size/object size) in this case ?

(iii) Is the magnification equal to magnifying power in this case ? Explain. [NCERT]

Solution. (*i*) Maximum magnifying power is obtained when the image is formed at the near point (25 cm).

$\therefore \quad v = -25$ cm, $f = +10$ cm, $u = ?$

As $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$$\therefore \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{10} = \frac{-2-5}{50} = \frac{-7}{50}$$

or $u = -\dfrac{50}{7} = \mathbf{-7.14\ cm}$

So lens should be held 7.14 cm away from the figure.

(*ii*) Magnitude of magnification,

$$m = \frac{v}{|u|} = \frac{25}{50/7} = 3.5$$

(*iii*) Magnifying power $= \dfrac{D}{|u|} = \dfrac{25}{50/7} = \mathbf{3.5}$

Yes, the magnifying power is equal to the magnitude of magnification because image is formed at the least distance of distinct vision.

Example 126. *What should be the distance between the object in Example 111 and the magnifying glass if the virtual image of each square in the figure is to have an area 6.25 mm² ? Would you be able to see the squares distinctly with your eyes very close to the magnifier ?* [NCERT]

Solution. Here the magnification in area

$$= \frac{6.25\ \text{mm}^2}{1\ \text{mm}^2} = 6.25$$

$\therefore$ Linear magnification, $m = \sqrt{6.25} = 2.5$

As $m = \dfrac{v}{u} \quad \therefore \quad v = mu = 2.5\,u$

Now $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$$\therefore \quad \frac{1}{2.5\,u} - \frac{1}{u} = \frac{1}{10} \quad \text{or} \quad \frac{1-2.5}{2.5\,u} = \frac{1}{10}$$

or $2.5\,u = -1.5 \times 10$

or $u = -\dfrac{1.5 \times 10}{2.5} = \mathbf{-6\ cm}$

Hence $v = 2.5\,u = 2.5 \times (-6) = \mathbf{-15\ cm}$

As the virtual image is closer than the normal near point (25 cm), it cannot be seen by the eye distinctly.

Problems For Practice

1. What must be the focal length of a lens used as simple microscope of magnifying power 26 ? The final image is formed at the distance of distinct vision. (Ans. 1 cm)
2. A converging lens of power 100 dioptre is used as a simple microscope. What is its magnifying power, if the distance of distinct vision is 25 cm ? (Ans. 26)
3. A converging lens of focal length 6.25 cm is used as a magnifying glass. If the near point of the observer is 25 cm from the eye and the lens is held close to the eye, calculate (*i*) the distance of the object from the lens and (*ii*) the angular magnification. Find the angular magnification when the final image is formed at infinity. [Ans. (*i*) 5 cm (*ii*) 5, 4]

4. The magnifying glass is made of combination of lenses of power + 20 D and – 4 D. If the distance of distinct vision is 25 cm, calculate the size of an object 2 cm high seen through the magnifying glass. (Ans. 10 cm)

5. A magnifying glass is a combination of a convex lens of focal length 5 cm and a concave lens of power – 5 D. If the distance of distinct vision is 20 cm, calculate the magnifying power of the magnifying glass. (Ans. 4)

6. Magnifying power of a simple microscope *A* is 1.25 less than that of a simple microscope *B*. If the power of the lens used in *B* is 25 D, find the power of lens used in *A*. Given that distance of distinct vision is 25 cm. (Ans. 20 D)

7. A child has a near point at 10 cm. What is the maximum angular magnification the child can have with a convex lens of focal length 10 cm ? (Ans. 2)

HINTS

1. Here $m = 26,\ D = 25\text{ cm},\ f = ?$

As $m = 1 + \frac{D}{f}$ $\therefore$ $26 = 1 + \frac{25}{f}$ or $f = \mathbf{1\ cm.}$

2. Use $f = \frac{1}{P}$ and $m = 1 + \frac{D}{f}$.

3. (*i*) Here $f = + 6.25\text{ cm},\ v = -25\text{ cm}$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{6.25}$$

$$= -\frac{1.25}{6.25} = -\frac{1}{5} \quad \text{or} \quad u = -5\text{ cm.}$$

(*ii*) Angular magnification, $m = \frac{D}{u} = \frac{25}{5} = 5.$

The final image is formed at infinity when $u = f$.

$$\therefore \quad m = \frac{D}{f} = \frac{25}{6.25} = 4.$$

4. Power of combination, $P = P_1 + P_2 = 20 - 4 = 16\text{ D}$

Focal length of combination,

$$F = \frac{1}{P} = \frac{1}{16}\text{ m} = \frac{25}{4}\text{ cm}$$

$$\therefore \quad m = 1 + \frac{D}{f} = 1 + \frac{25}{25/4} = 5$$

Size of image, $h_2 = m \times h_1 = 5 \times 2 = \mathbf{10\ cm.}$

7. For maximum angular magnification,

$$v = -10\text{ cm}$$

$$\therefore \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{10} - \frac{1}{10} = -\frac{1}{5}$$

or $u = -5\text{ cm}$

$$m = \frac{v}{u} = \frac{-10}{-5} = 2.$$

9.44 COMPOUND MICROSCOPE

80. *With the help of a ray diagram, explain the construction and working of a compound microscope. Write an expression for its magnifying power.*

Compound microscope. *A compound microscope is an optical device* used to see magnified images of tiny objects. A good quality compound microscope can produce magnification of the order of 1000.

Construction. It consists of two convex lenses of short focal length, arranged co-axially at the ends of *two* sliding metal tubes.

1. ***Objective.*** It is a convex lens of very short focal length f_0 and small aperture. It is positioned near the object to be magnified.

2. ***Eyepiece or ocular.*** It is a convex lens of comparatively larger focal length f_e and larger aperture than the objective ($f_e > f_0$). It is positioned near the eye for viewing the final image.

The distance between the two lenses can be varied by using rack and pinion arrangement.

Working. (*a*) **When the final image is formed at the least distance of distinct vision.** The object *AB* to be viewed is placed at distance u_0, slightly larger than the focal length f_0 of the objective *O*. The objective forms a real, inverted and magnified image *A′ B′*, of the object *AB* on the other side of the lens *O*, as shown in Fig. 9.145. The separation between the objective *O* and the eyepiece *E*, is so adjusted that the image *A′ B′* lies within the focal length f_e of the eyepiece. The image *A′ B′* acts as an object for the eyepiece which essentially acts like a simple microscope. The eyepiece *E* forms a virtual and magnified final image *A″ B″* of the object *AB*. Clearly, the final image *A″ B″* is inverted with respect to the object *AB*.

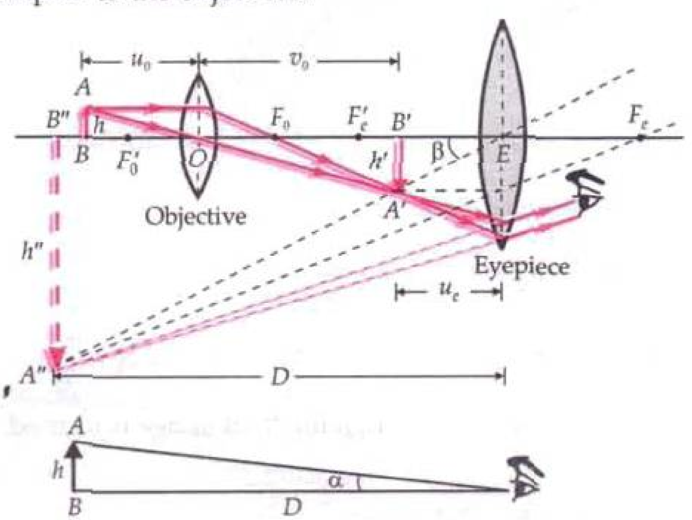


Fig. 9.145 Compound Microscope, final image at *D*.

Magnifying power. *The magnifying power of a compound microscope is defined as the ratio of the angle* subtended at the eye by the final virtual image to the angle subtended at the eye by the object, when both are at the least distance of distinct vision from the eye.

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{h'/u_e}{h/D} = \frac{h'}{h}\cdot\frac{D}{u_e} = m_0 m_e$$

Here $m_0 = \frac{h'}{h} = \frac{v_o}{u_0}$

As the eyepiece acts as a simple microscope, so

$$m_e = \frac{D}{u_e} = 1 + \frac{D}{f_e}$$

$$\therefore \quad m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

As the object AB is placed close to the focus F_0 of the objective, therefore, $u_0 \simeq -f_0$

Also image $A'B'$ is formed close to the eyelens whose focal length is short, therefore $v_0 \simeq L =$ the length of the microscope tube or the distance between the two lenses

$$\therefore \quad m_0 = \frac{v_0}{u_0} = \frac{L}{-f_0}$$

$$\therefore \quad m = -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \quad \text{[for final image at } D\text{]}$$

(*b*) **When the final image is formed at infinity.** When the image $A'B'$ lies at the focus F_e' of the eyepiece *i.e.,* $u_e = f_e$, the image $A''B''$ is formed at infinity, as shown in Fig. 9.146.

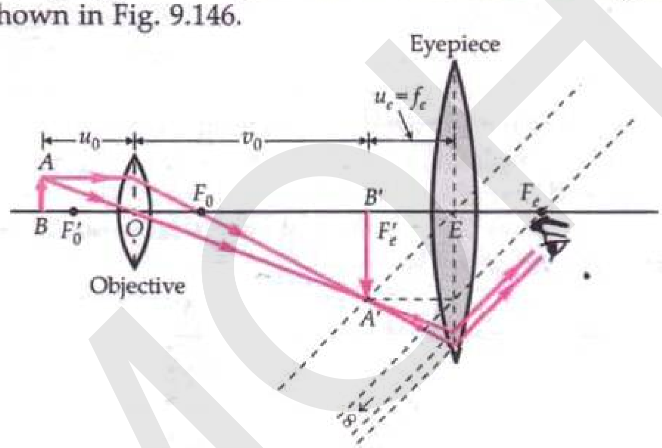


Fig. 9.146 Compound microscope, final image at ∞.

Magnification due to objective, $m_0 = \frac{h'}{h} = \frac{L}{-f_0}$

Angular magnification due to eyepiece, $m_e = \frac{D}{f_e}$

Total magnification when the final image is formed at infinity,

$$m = m_0 \times m_e = -\frac{L}{f_0} \times \frac{D}{f_e}$$

Obviously, magnifying power of the compound microscope is large when both f_0 and f_e are small.

For Your Knowledge

- In a compound microscope, the objective is a convex lens of short focal length and small aperture, while the eyepiece is a convex lens of short focal length and large aperture.
- In actual practice, each of the objective and the eyepiece consists of combination of lenses. To eliminate chromatic aberration, an objective consists of two lenses in contact. To minimise chromatic and spherical aberrations, an eyepiece consists of two lenses separated by a certain distance.
- In a compound microscope, *the objective and the eyepiece are placed a fixed distance apart.* For focussing on an object, the distance of the objective from that object is changed with the help of a rack and pinion arrangement.
- For large magnifying power, both f_0 and f_e have to be small. Also, f_e is taken larger than f_0 so as to increase the field of view of the microscope.
- The visibility and quality of the image can be improved by illuminating the object and by using oil immersion objective.
- When the final image is formed at the least distance D of distinct vision, the length of the compound microscope,
$$L = v_0 + u_e$$
- When the final image is formed at infinity, the length of the compound microscope,
$$L = v_0 + f_e.$$

Examples based on
Compound Microscope

Formulae Used

1. Magnifying power, $m = m_o \times m_e$
2. When the final image is formed at the least distance of distinct vision,
$$m = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$
3. When the final image is formed at infinity,
$$m = \frac{v_0}{u_0}\cdot\frac{D}{f_e} = -\frac{L}{f_0}\cdot\frac{D}{f_e}$$

Units Used

The distances u_0, u_e, v_0, v_e, D and L are all in metre or cm and magnification m has no units. $D = 25$ cm, for a normal eye.

Example 127. *A compound microscope with an objective of* 1.0 *cm focal length and an eyepiece of* 2.0 *cm focal length has a tube length of* 20 *cm. Calculate the magnifying power of the microscope, if the final image is formed at the near point of the eye.* [CBSE D 04]

Solution. Here $f_0 = 1.0$ cm, $f_e = 2.0$ cm, $L = 20$ cm, $D = 25$ cm

When the final image is formed at the near point of the eye, the magnifying power is

$$m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) = \frac{20}{1.0}\left(1 + \frac{25}{2}\right) = 20 \times 13.5 \approx \mathbf{270.}$$

Example 128. *You are given two converging lenses of focal lengths 1.25 cm and 5 cm to design a compound microscope. If it is desired to have a magnification of 30, find out separation between the objective and the eyepiece.* [CBSE OD 15]

Solution. Here $f_0 = 1.25$ cm, $f_e = 5$ cm, $D = 25$ cm, $m = 30$

When the final image is formed at the near point of the eye, the magnifying power of the compound microscope is

$$m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

or $$30 = \frac{L}{1.25}\left(1 + \frac{25}{5}\right)$$

$$L = \frac{30 \times 1.25}{6} = \mathbf{6.25\ cm.}$$

Example 129. *The focal lengths of the eyepiece and the objective of a compound microscope are 5 cm and 1 cm respectively and the length of the tube is 20 cm. Calculate the magnifying power of the microscope, when the final image is formed at infinity. The value of least distance of distinct vision is 25 cm.* [ISCE 98]

Solution. As the final image is being formed at infinity, the image formed by the objective must lie at the focus of the eyepiece *i.e.*, at a distance of 5 cm from the eyepiece.

∴ Image distance for the objective

$$= \text{Tube length } (L) - f_e = 20 - 5 = 15 \text{ cm}$$

Using Cartesian sign convention for the objective,

$$v_0 = +15 \text{ cm}, \quad f_0 = +1 \text{ cm}, \quad u_0 = ?$$

$$\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{15} - \frac{1}{1} = -\frac{14}{15}$$

or $$u_0 = -\frac{15}{14} \text{ cm}$$

When the final image is formed at infinity, the magnifying power of the microscope will be,

$$m = \frac{v_0}{u_0} \times \frac{D}{f_e}$$

$$= \frac{15}{15/14} \times \frac{25}{5} = \mathbf{70.} \qquad \text{[Numerically]}$$

Example 130. *(i) Draw a labelled ray diagram of a compound microscope, showing the formation of image at the near point of the eye. (ii) A compound microscope uses an objective lens of focal length 4 cm and eyelens of focal length 10 cm. An object is placed at 6 cm from the objective lens.*

(a) Calculate magnifying power of the compound microscope, if the final image is formed at the near point.

(b) Calculate the length of the compound microscope also. [CBSE OD 06C]

Solution. (*i*) For ray diagram, see Fig. 9.145

(*ii*) Here $f_0 = 4$ cm, $f_e = 10$ cm,

$$u_0 = -6 \text{ cm}, v_e = -D = -25 \text{ cm}$$

(*a*) As $$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

∴ $$\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{4} + \frac{1}{-6} = \frac{1}{12}$$

or $$v_0 = +12 \text{ cm}$$

Magnifying power of the compound microscope is

$$m = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = \frac{12}{6}\left(1 + \frac{25}{10}\right) = \mathbf{7.}$$

(*b*) $$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{10} = -\frac{7}{50}$$

or $$u_e = -\frac{50}{7} = -7.14 \text{ cm}$$

Length of the compound microscope is

$$L = v_0 + |u_e| = 12 + 7.14 = \mathbf{19.14\ cm}$$

Example 131. *The total magnification produced by a compound microscope is 20, while that produced by the eyepiece alone is 5. When the microscope is focussed on a certain object, the distance between objective and eyepiece is 14 cm. Find the focal length of objective and eyepiece, if distance of distinct vision is 20 cm.* [CBSE D 14]

Solution. Here

$$m = 20, \quad m_e = 5, \quad D = 20 \text{ cm}, \quad v_e = -20 \text{ cm}$$

∴ $$m_0 = \frac{m}{m_e} = \frac{20}{5} = 4$$

As the eyepiece acts as a simple microscope, so

$$m_e = 1 + \frac{D}{f_e}$$

or $$5 = 1 + \frac{20}{f_e}$$

∴ $$f_e = \mathbf{5\ cm}$$

Also, $m_e = \frac{v_e}{u_e}$

or $5 = \frac{-20}{u_e}$

or $u_e = -4$ cm

Distance between the objective and the eyepiece $= 14$ cm

or $|u_e| + |v_o| = 14$

or $4 + v_0 = 14$ or $v_0 = 10$ cm

Now, $m_0 = 1 - \frac{v_0}{f_0} \Rightarrow -4 = 1 - \frac{10}{f_0}$

$\therefore$ $f_0 = $ **2 cm.**

Problems For Practice

1. A convex lens of focal length 5 cm is used as a simple microscope. What is its magnifying power, if final image is formed at the distance of distinct vision *i.e.* 25 cm ? If it is used as an eyepiece in a compound microscope with objective of magnifying power 40, what is the magnifying power of the compound microscope ? (Ans. 6, 240)
2. A compound microscope has a magnification of 30. The focal length of its eyepiece is 5 cm. Assuming the final image to be formed at least distance of distinct vision (25 cm), calculate the magnification produced by the objective. (Ans. 5)
3. The focal lengths of the objective and eye-piece of a compound microscope are 4 cm and 6 cm respectively. If any object is placed at a distance of 6 cm from the objective, what is the magnification produced by the microscope ? Distance of the distinct vision = 25 cm. (Ans. 10.33)
4. The focal lengths of the objective and eyepiece of a microscope are 1.25 cm and 5 cm respectively. Find the position of the object relative to the objective in order to obtain an angular magnification of 30 in normal adjustment. **[CBSE D 12]** (Ans. -1.46 cm)
5. The focal lengths of the objective and the eyepiece of a microscope are 2 cm and 5 cm respectively and the distance between them is 20 cm. Find the distance of the object from the objective when the final image seen by the eye is 25 cm from the eyepiece. Also find the magnifying power. (Ans. $u_0 = -2.3$ cm, $m = 41.5$)
6. A compound microscope is made using a lens of focal length 10 mm as objective and another lens of focal length 15 mm as eyepiece. An object is held at 1.1 cm from the objective and final image is formed at infinity. Calculate distance between objective and eyepiece. (Ans. 12.5 cm)

HINTS

1. For simple microscope, $m = 1 + \frac{D}{f} = 1 + \frac{25}{5} = \mathbf{6}$

   For compound microscope, $m_e = 6, m_0 = 40$

   $\therefore$ $m = m_0\, m_e = 40 \times 6 = \mathbf{240}$.

2. Use $m = m_0 \times m_e = m_0\left(1 + \frac{D}{f_e}\right)$.

3. Here $f_0 = 4\,\text{cm},\ f_e = 6\,\text{cm},\ D = 25\,\text{cm}$

   For objective lens, $-\frac{1}{u_0} + \frac{1}{v_0} = \frac{1}{f_0}$

   Now, $u_0 = -6\,\text{cm},\ f_0 = +4\,\text{cm}$

   $\therefore$ $-\frac{1}{-6} + \frac{1}{v_0} = \frac{1}{+4}$

   or $\frac{1}{v_0} = \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}$

   or $v_0 = 12$ cm

   $$m = \frac{v_0}{u_0}\left[1 + \frac{D}{f_e}\right] = \frac{12}{6}\left[1 + \frac{25}{6}\right]$$

   $$= 2 \times \frac{31}{6} = \frac{31}{3} = \mathbf{10.33}.$$

4. Here $f_0 = 1.25$ cm, $f_e = 5$ cm, $m = 30$

   In normal adjustment, magnification produced by the eyepiece,

   $$m_e = \frac{D}{f_e} = +\frac{25}{5} = 5$$

   Now, $m = m_0 \times m_e$ $\therefore$ $30 = m_0 \times 5$

   or $m_0 = 6$

   As real image is formed by the objective,

   $$m_0 = \frac{v_0}{u_0} = -6 \quad \text{or} \quad v_0 = -6u_0$$

   Using thin lens formula for the objective,

   $$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

   or $\frac{1}{-6u_0} - \frac{1}{u_0} = \frac{1}{1.25}$ or $-\frac{7}{6u_0} = \frac{1}{1.25}$

   or $u_0 = -\frac{7 \times 1.25}{6} = \mathbf{-1.46\ cm}$

   Thus the object should be held at 1.46 cm in front of the objective lens.

5. Proceed as in Exercise 9.11 on page 9.133.

9.45 TELESCOPE

81. *What is a telescope ? What are the different types of telescopes commonly used ?*

Telescope. *A telescope is an optical device which enables us to see distant objects clearly.* It provides an angular magnification of the distant objects.

Different types of telescope. Broadly, the telescopes can be divided into *two* categories :

1. Refracting telescopes. *These make use of lenses to view distant objects.* These are of *two* types :

(*a*) **Astronomical telescope.** It is used to see heavenly objects like the sun, stars, planets, etc. The final image formed is inverted one which is immaterial in the case of heavenly bodies because of their round shape.

(*b*) **Terrestrial telescope.** It is used to see distant objects on the surface of the earth. The final image formed is erect one. This is an essential condition of viewing the objects on earth's surface correctly.

2. Reflecting telescopes. *These make use of converging mirrors to view the distant objects.* For example, Newtonian and Cassegrain telescopes.

9.46 ASTRONOMICAL TELESCOPE

82. *What is an astronomical telescope ? Give its construction. With the help of ray diagrams, explain its working when it forms final image at the least distance of distinct vision and at infinity. Deduce expression for magnifying power in each case.*

Astronomical telescope. *It is a refracting type telescope used to see heavenly bodies like stars, planets, satellites, etc.*

Construction. It consists of two converging lenses mounted co-axially at the outer ends of two sliding tubes.

1. ***Objective.*** It is a convex lens of large focal length and a much larger aperture. It faces the distant object. In order to form bright image of the distant objects, the aperture of the objective is taken large so that it can gather sufficient light from the distant objects.

2. ***Eyepiece.*** It is a convex lens of small focal length and small aperture. It faces the eye. The aperture of the eyepiece is taken small so that whole light of the telescope may enter the eye for distinct vision.

Working. (*a*) **When the final image is formed at the least distance of distinct vision.** As shown in Fig. 9.147, the parallel beam of light coming from the distant object falls on the objective at some angle α. The objective focusses the beam in its focal plane and forms a real, inverted and diminished image $A'B'$. This image $A'B'$ acts as an object for the eyepiece. The distance of the eyepiece is so adjusted that the image $A'B'$ lies within its focal length. The eyepiece magnifies this image so that final image $A''B''$ is magnified and inverted with respect to the object. The final image is seen distinctly by the eye at the least distance of distinct vision.

Magnifying power. *The magnifying power of a telescope is defined as the ratio of the angle subtended at the eye by the final image formed at the least distance of distinct vision to the angle subtended at the eye by the object at infinity, when seen directly.*

As the object is very far off, the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective. Thus

$$\angle A'OB' = \alpha$$

Also, let $\angle A''EB'' = \beta$

$\therefore$ Magnifying power,

$$m = \frac{\beta}{\alpha} \simeq \frac{\tan\beta}{\tan\alpha} \qquad [\because \alpha, \beta \text{ are small}]$$

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}.$$

According to the new Cartesian sign convention,

$OB' = +f_0$ = focal length of the objective

$B'E = -u_e$ = distance of $A'B'$ from the eyepiece, acting as an object for it

$$\therefore \qquad m = -\frac{f_0}{u_e}$$

Again, for the eyepiece :

$$u = -u_e \quad \text{and} \quad v = -D$$

As $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

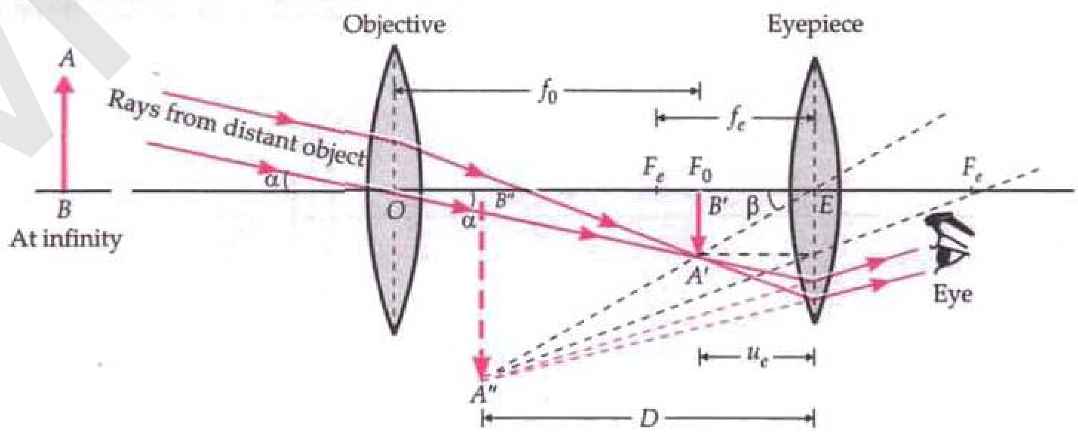


Fig. 9.147 Astronomical telescope focussed for least distance of distinct vision.

$$\therefore \qquad \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or
$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Hence
$$m = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Clearly for large magnifying power, $f_0 >> f_e$. The negative sign for the magnifying power indicates that the final image formed is *real* and *inverted*.

(*b*) When the final image is formed at infinity : Normal adjustment. As shown in Fig. 9.148, when a parallel beam of light is incident on the objective, it forms a real, inverted and diminished image $A'B'$ in its focal plane. The eyepiece is so adjusted that the image $A'B'$ exactly lies at its focus. Therefore, the final image is formed at infinity, and is highly magnified and inverted with respect to the object.

Magnifying power in normal adjustment. *It is defined as the ratio of the angle subtended at the eye by the final image as seen through the telescope to the angle subtended at the eye by the object seen directly, when both the image and the object lie at infinity.*

As the object is very far off, the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective.

Thus
$$\angle A'OB' = \alpha$$

and let
$$\angle A'EB' = \beta$$

$\therefore$ Magnifying power,

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \quad [\because \alpha, \beta \text{ are small angles}]$$

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$$

Applying new Cartesian sign convention,

$OB' = + f_0$ = Distance of $A'B'$ from the objective along the incident light

$B'E = - f_e$ = Distance of $A'B'$ from the eyepiece *against* the incident light

$$\therefore \qquad m = -\frac{f_0}{f_e}$$

Clearly for large magnifying power, $f_0 >> f_e$. The negative sign for m indicates that the image is *real* and *inverted*.

For Your Knowledge

- In a telescope, the objective has large focal length and large aperture while the eyepiece has small focal length and small aperture.
- A telescope is focussed on the distant object by varying distance between the objective and the eye-piece with the help of rack and pinion arrangement.
- The objective of the telescope should have large aperture because then a much wider beam of light is incident on it and is converged into a small cone which, on entering the eye, produces sufficient illumination on the retina. So even two distant faint stars which cannot be seen by naked eyes, become visible through such a telescope.
- In a telescope, the image is not actually magnified. A telescope simply increases the visual angle. The visual angle β for the image is much larger than the visual angle α for the object. Consequently, the angular magnification β / α is quite large.
- In normal adjustment, the distance between the objective and the eyepiece = $f_0 + f_e$.

  When the final image is formed at the least distance of distinct vision, the magnifying power of the telescope is larger than that in the case of normal adjustment because the factor $\left(1 + \frac{f_e}{D}\right) > 1$.
- An astronomical telescope forms an inverted image. As the celestial objects are oval in shape, so it does not matter whether the final image is inverted or erect.

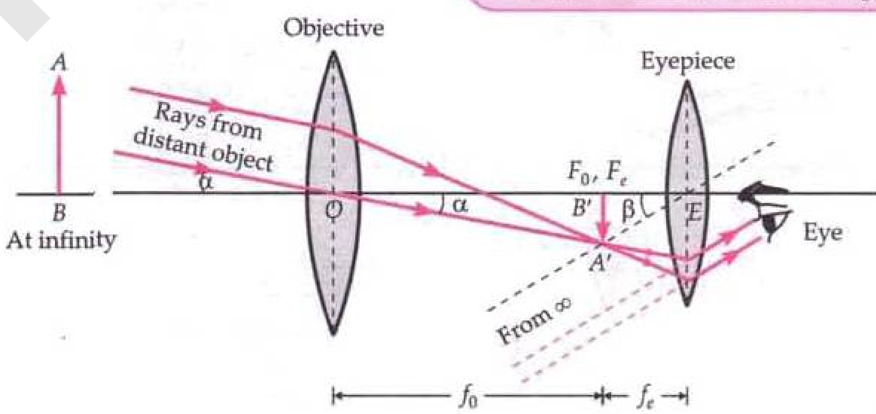


Fig. 9.148 Astronomical telescope in normal adjustment.

9.47 TERRESTRIAL TELESCOPE*

83. *What is a terrestrial telescope ? With the help of a ray diagram, explain its working. Write expression for its magnifying power. State its main drawbacks.*

Terrestrial telescope. *It is a refracting type telescope used to see erect images of distant earthly objects.* It uses an additional convex lens between objective and eyepiece for obtaining an erect image.

As shown in Fig. 9.149, the objective forms a real, inverted and diminished image, $A'B'$ of the distant object in its focal plane. Now the erecting lens is held at twice its focal length from the focal plane of the objective. This lens forms a real, inverted and equal size image $A''B''$ of $A'B'$. This image is now erect with respect to the distant object. The eyepiece is so adjusted that the image $A''B''$ lies at its principal focus. Hence the final image is formed at infinity and is highly magnified and erect with respect to the distant object.

As the erecting lens does not cause any magnification, the angular magnification of the terrestrial telescope is same as that of the astronomical telescope.

When the image is formed at infinity,

$$m = \frac{f_0}{f_e},$$

When the image is formed at the least distance of distinct vision,

$$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Drawbacks :

1. The length of the terrestrial telescope is much larger than the astronomical telescope. In normal adjustment, the length of a terrestrial telescope $= f_0 + 4f + f_e$, where f is the focal length of the erecting lens.
2. Due to extra reflection at the surfaces of the erecting lens, the intensity of the final image decreases.

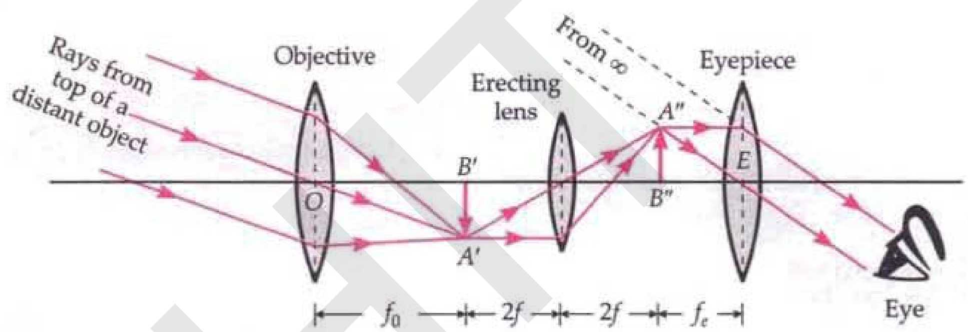


Fig. 9.149 Terrestrial telescope.

9.48 REFLECTING TELESCOPES

84. *With the help of a ray diagram, explain the construction and working of a Newtonian reflecting telescope.*

Newtonian reflecting telescope. The first reflecting telescope was set up by *Newton* in 1668. As shown in Fig. 9.150, it consists of a large concave mirror of large focal length as the objective, made of an alloy of copper and tin.

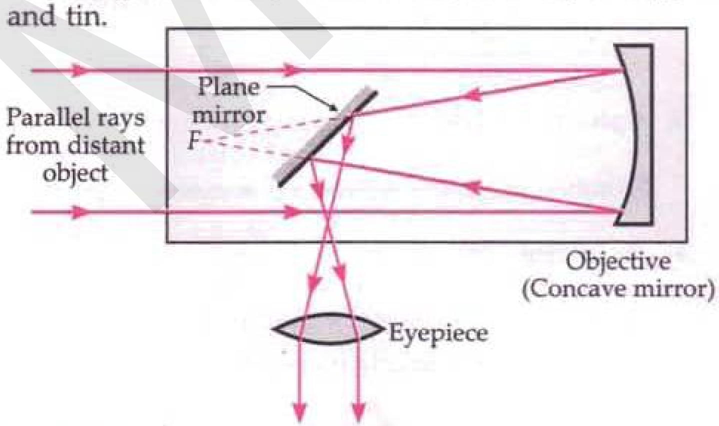


Fig. 9.150 Newtonian reflecting telescope.

A beam of light from the distant star is incident on the objective. Before the rays are focussed at F, a plane mirror inclined at 45° intercepts them and turns them towards an eyepiece adjusted perpendicular to the axis of the instrument. The eyepiece forms a highly magnified, virtual and erect image of the distant object.

85. *With the help of a labelled diagram, explain the construction and working of a Cassegrain reflecting telescope.*

Cassegrain reflecting telescope. Fig. 9.151 shows Cassegrainian type reflecting telescope. It consists of a large concave paraboloidal (primary) mirror having a hole at its centre. There is a small convex (secondary) mirror near the focus of the primary mirror. The eyepiece is placed on the axis of the telescope near the hole of the primary mirror.

The parallel rays from the distant object are reflected by the large concave mirror. Before these rays come to focus at F, they are reflected by the small convex mirror and are converged to a point I just

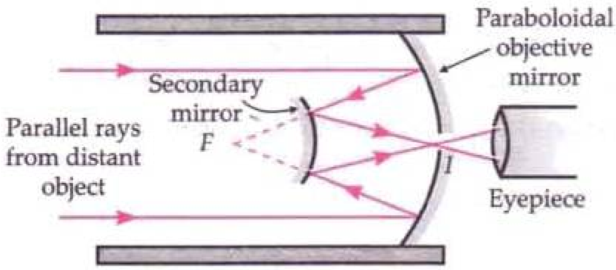


Fig. 9.151 Cassegrain reflecting telescope.

outside the hole. The final image formed at I is viewed through the eyepiece. As the first image at F is inverted with respect to the distant object and the second image I is erect with the respect to the first image F, hence the final image is inverted with respect to the object.

Let f_0 be the focal length of the objective and f_e that of the eyepiece.

For the final image formed at the least distance of distinct vision,

$$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

For the final image formed at infinity,

$$m = \frac{f_0}{f_e} = \frac{R/2}{f_e}.$$

86. *State some important advantages of a reflecting type telescope over a refracting type telescope.*

Advantages of a reflecting type telescope. A reflecting type telescope has the following advantages over a refracting type telescope :

1. A concave mirror of large aperture has high gathering power and absorbs very less amount of light than the lenses of large apertures. The final image formed in reflecting telescope is very bright. So even very distant or faint stars can be easily viewed.
2. Due to large aperture of the mirror used, the reflecting telescopes have ***high resolving power***.
3. As the objective is a mirror and not a lens, it is free from ***chromatic aberration*** (formation of coloured image of a white object).
4. The use of paraboloidal mirror reduces the ***spherical aberration*** (formation of non-point, blurred image of a point object).
5. A mirror requires grinding and polishing of one surface only. So it costs much less to construct a reflecting telescope than a refracting telescope of equivalent optical quality.
6. A lens of large aperture tends to be very heavy and, therefore, difficult to make and support by its edges. On the other hand, a mirror of equivalent optical quality weighs less and can be supported over its entire back surface.

For Your Knowledge

- The largest *refracting telescope* is at the Yerkes Observatory in Wisconsin, USA. It uses an objective lens of diameter 102 cm.
- The largest *reflecting telescopes* in the world are the pair of Keck telescopes in Hawaii, USA. They use reflecting mirrors of diameter 10 m each.
- The largest telescope in India is in Kavalur, Tamilnadu. It is a Cassegrain reflecting telescope having objective of diameter 2.34 m. It was ground, polished, set up and is being used by the Indian Institute of Astrophysics, Bangalore.
- **Prism binocular.** It is a double telescope that uses two sets of totally reflecting prisms. This makes the final image erect which is very desirable for observations on earth. Binoculars are much more compact and easier to use than a refracting telescope, and allow use of both eyes.

Examples based on Telescopes

Formulae Used

1. ***Astronomical telescope.*** (*i*) In normal adjustment,

$$m = \frac{f_0}{f_e}$$

Distance between objective and eyepiece = $f_0 + f_e$

(*ii*) When the final image is formed at the least distance of distinct vision, $m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

Distance between objective and eyepiece

$$= f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$$

2. ***Terrestrial telescope.*** (*i*) In normal adjustment,

$$m = \frac{f_0}{f_e}$$

Distance between objective and eyepiece

$$= f_0 + 4f + f_e,$$

where f = focal length of the erecting lens.

3. ***Galileo's telescope.*** In normal adjustment, $m = \frac{f_0}{f_e}$

Distance between objective and eyepiece = $f_0 - f_e$

4. ***Reflecting telescope.*** $m = \frac{f_0}{f_e} = \frac{R/2}{f_e}$

where f_0 = focal length of concave mirror,

f_e = focal length of eyepiece.

Units Used

Lengths f_0, f_e, f and D are in all in cm or metre.

Example 132. *The magnifying power of an astronomical telescope in the normal adjustment position is 100. The distance between the objective and the eyepiece is 101 cm. Calculate the focal lengths of the objective and the eyepiece.* [CBSE D 04]

Solution. Here $m = \frac{f_0}{f_e} = 100$ or $f_0 = 100\ f_e$

But $f_0 + f_e = 101$ cm

or $100\ f_e + f_e = 101$

$\therefore$ $f_e = \mathbf{1\ cm}$

and $f_0 = \mathbf{100\ cm}$.

Example 133. *An amateur astronomer wishes to estimate roughly the size of the sun using his crude telescope consisting of an objective lens of focal length 200 cm and an eyepiece of focal length 10 cm. By adjusting the distance of the eye-piece from the objective, he obtains an image of the sun on a screen 40 cm behind the eyepiece. The diameter of the sun's image is measured to be 6.0 cm. What is the estimate of the sun's size, given that the average earth-sun distance is* 1.5×10^{11} *m.* [NCERT]

Solution. Here $f_0 = 200$ cm, $f_e = 10$ cm,

$$v_e = +\,40 \text{ cm (for real image)}$$

As $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$$\therefore \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{40} - \frac{1}{10} = \frac{1-4}{40} = \frac{-3}{40}$$

or $u_e = -\frac{40}{3}$ cm

Magnification produced by eyepiece is

$$m_e = \frac{v_e}{|u_e|} = \frac{40}{40/3} = 3$$

$\therefore$ Diameter of the image formed by the objective is

$$d = \frac{6}{3} \text{ cm} = 2 \text{ cm}$$

If D is the diameter of the sun (in m), then the angle subtended by it on the objective will be

$$\alpha = \frac{D}{1.5 \times 10^{11}} \text{ rad}$$

Angle subtended by the image at the objective will be equal to this angle and is given by

$$\alpha = \frac{\text{size of image}}{f_0} = \frac{2}{200} = \frac{1}{100} \text{ rad}$$

$$\therefore \quad \frac{D}{1.5 \times 10^{11}} = \frac{1}{100}$$

or $D = \frac{1.5 \times 10^{11}}{100} = \mathbf{1.5 \times 10^9\ m}$.

Example 134. *A telescope objective of focal length 1 m forms a real image of the moon 0.92 cm in diameter. Calculate the diameter of the moon taking its mean distance from the earth to be* 38×10^4 *km. If the telescope uses an eyepiece of 5 cm focal length, what would be the distance between the two lenses for (i) the final image to be formed at infinity and (ii) the final image (virtual) at 25 cm from the eye.* [ISCE 97]

Solution. Let d be the diameter of the moon. The angle subtended by the moon at the objective of the telescope is

$$\alpha = \frac{\text{Diameter of moon}}{\text{Distance of moon from earth}}$$

$$= \frac{d}{38 \times 10^4 \text{ km}} = \frac{d}{3.8 \times 10^8 \text{ m}}$$

The angle subtended by the image formed by the objective in its focal plane will also be equal to α and is given by

$$\alpha = \frac{\text{Diameter of moon's image}}{\text{Focal length of the objective}}$$

$$= \frac{0.92 \text{ cm}}{100 \text{ cm}} = 0.0092 \text{ rad}$$

$$\therefore \quad \frac{d}{3.8 \times 10^8 \text{ m}} = 0.0092$$

or $d = 3.8 \times 10^8 \times 0.0092 = 3.5 \times 10^6$ m

(*i*) When the image is formed at infinity, the distance between the two lenses is

$$L = f_0 + f_e = 100 \text{ cm} + 5.0 \text{ cm} = \mathbf{105\ cm}.$$

(*ii*) When the image is formed by the eyepiece at the least distance of distinct vision, we have

$$v_e = -\,D = -25 \text{ cm}, \quad f_e = +\,5 \text{ cm}$$

Using thin lens formula,

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5} = -\frac{6}{25}$$

or $u_e = -\frac{25}{6} = -\,4.17$ cm

Therefore, the distance between the two lenses is

$$L' = f_0 + |u_e| = 100 + 4.17 = \mathbf{104.17\ cm}.$$

Example 135. *A telescope has an objective of focal length 50 cm and eyepiece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm away from the object. Calculate (a) the separation between the objective and eyepiece and (b) the magnification produced.* [IIT]

Solution. For the image formed by the objective, we have

$$u_0 = -\,200 \text{ cm}, \quad f_0 = +\,50 \text{ cm}$$

As $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$

$$\therefore \quad \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{50} - \frac{1}{200} = \frac{4-1}{200} = \frac{3}{200}$$

or $$v_0 = \frac{200}{3} \text{ cm}$$

Magnification produced by the objective is

$$m_0 = \frac{v_0}{u_0} = \frac{200}{3 \times (-200)} = -\frac{1}{3}$$

The image formed by objective acts as an object for the eyepiece. So

$$v_e = -25 \text{ cm}, \qquad f_e = +5 \text{ cm}$$

$$\therefore \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5} = \frac{-1-5}{25} = \frac{-6}{25}$$

or $$u_e = -\frac{25}{6} \text{ cm}$$

Magnification produced by the eyepiece,

$$m_e = \frac{v_e}{u_e} = \frac{-25}{-25/6} = 6$$

(*a*) The separation between the objective and eyepiece

$$= v_0 + |u_e| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = \mathbf{70.83 \text{ cm.}}$$

(*b*) Magnification produced,

$$m = m_0 \times m_e = -\tfrac{1}{3} \times 6 = -\mathbf{2}$$

The negative sign indicates that the final image is inverted.

Example 136. *A small telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If this telescope is used to view a 100 m high tower 3 km away, find the height of the final image when it is formed 25 cm away from the eyepiece.* [CBSE D 12]

Solution. Angle subtended by the 100 m tall tower at 3 km away is

$$\alpha \simeq \tan \alpha = \frac{100}{3 \times 10^3} = \frac{1}{30} \text{ rad}$$

Let h be the height of image of tower formed by the objective. Then the angle subtended by this image will also be α and is given by

$$\alpha = \frac{h}{f_0} = \frac{h}{150}$$

$$\therefore \quad \frac{h}{150} = \frac{1}{30} \quad \text{or} \quad h = 5 \text{ cm}$$

Magnification produced by the eyepiece,

$$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

Height of final image

$$= h \times m_e = 5 \times 6 = \mathbf{30 \text{ cm.}}$$

Example 137. *A small telescope has an objective lens of focal length 150 cm and eyepiece of focal length 5 cm. What is the magnifying power of the telescope for viewing distant objects in normal adjustment?*

If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens? [CBSE OD 15]

Solution. Magnifying power of the telescope in normal adjustment,

$$m = \frac{f_0}{f_e} = \frac{150}{5} = \mathbf{30}$$

$$\text{Angular size of the tower } (\alpha) = \frac{100}{3 \times 1000} \text{ rad} = \frac{1}{30} \text{ rad}$$

$$\text{Angular size of the image } (\beta) = \frac{1}{30} \times 30 \text{ rad} = 1 \text{ rad}$$

$$\left[m = \frac{\beta}{\alpha}\right]$$

$$\therefore \quad \text{Height of the image} = 1 \times \frac{5}{100} \text{ m} = \mathbf{0.05 \text{ m.}}$$

Example 138. *A terrestrial telescope has an objective of focal length 180 cm and an eyepiece of focal length 5.0 cm. The erecting lens has a focal length of 3.5 cm. What is the separation between the objective and the eyepiece? What is the magnifying power of the telescope? Can we use the telescope for viewing astronomical objects?* [NCERT]

Solution. In a terrestrial telescope, the inverted image formed by the objective is made erect by positioning it at the $2f$ point of an erecting lens of focal length f.

In normal adjustment, the separation between the objective and the eyepiece is

$$L = f_0 + 4f + f_e$$
$$= 180 + 4 \times 3.5 + 5.0 = \mathbf{199 \text{ cm}}$$

Magnifying power,

$$m = \frac{f_0}{f_e} = \frac{180}{5} = \mathbf{36}$$

Yes, the telescope can be used to view astronomical objects though there is no need to make the 'inverted' image of a star 'upright'. But the final image is less bright than in an equivalent astronomical telescope because of the extra loss of some light due to reflection and absorption by the erecting lens.

Example 139. *(a) A Galilean telescope obtains the final image erect (like in a terrestrial telescope) without an intermediate erecting lens. It does so by using a diverging lens for its eyepiece. Show that the angular magnification of a Galilean telescope is given by the formula : $m = -f_0/f_e$ (negative sign because f_e is negative).*

(*b*) *For a Galilean telescope with* $f_0 = 150$ *cm,* $f_e = -7.5$ *cm, what is the separation between the objective and the eyepiece ?*

(*c*) *What is the main disadvantage of this type of telescope ?*

Solution. (*a*) As shown in the ray diagram of Fig. 9.152, the image formed by the objective lies at a distance f_e from the eyepiece. So the rays refracted by the eyepiece are rendered parallel. The final image is formed at infinity.

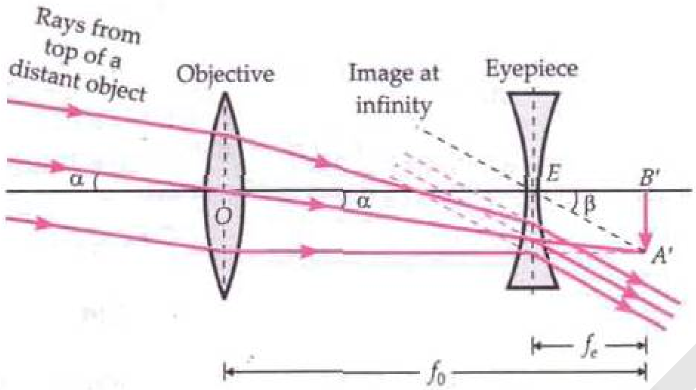


Fig. 9.152 Galileo's telescope (normal adjustment).

In normal adjustment, the angular magnification is given by

$$m = \frac{\beta}{\alpha} = \frac{\tan \beta}{\tan \alpha} \qquad [\because \alpha, \beta \text{ are small angles}]$$

$$= \frac{A'B'/EB'}{A'B'/OB'} = \frac{OB'}{EB'}$$

$$m = -\frac{f_0}{f_e}$$

$$= -\frac{\text{Focal length of the objective}}{\text{Focal length of the eyepiece}}$$

The negative sign has been taken because the focal length f_e of the diverging lens used for the eyepiece is negative.

(*b*) In the normal adjustment, the separation between the objective and the eyepiece is

$$L = f_0 - |f_e| = 150 - |-7.5| = \mathbf{142.5\ cm.}$$

(*c*) The main disadvantage of a Galilean telescope is its limited field of view. This is because the eye cannot be positioned on the location of the eyering between the two lenses.

Example 140. *An eyepiece of a telescope consists of two plano-convex lenses* L_1 *and* L_2 *each of focal length* f *separated by a distance of* $2f/3$. *Where should* L_1 *be placed relative to the focus of the objective lens of the telescope so that the final image through* L_2 *is seen at infinity ?* [NCERT]

Solution. Because the final image through L_2 is seen at infinity, therefore, virtual image of the object produced by L_1 should lie at the focus of L_2.

But distance between L_1 and $L_2 = \frac{2f}{3}$

$\therefore$ Image distance from $L_1 = f - \frac{2f}{3} = \frac{f}{3}$

or $\qquad v = \frac{f}{3}$, and for L_1, $v = -\frac{f}{3}$

As $\qquad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\therefore \qquad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{3}{f} - \frac{1}{f} = -\frac{4}{f}$$

or $\qquad u = -\frac{f}{4}$

So L_1 should be placed at a distance of $\frac{f}{4}$ from the focus of the objective.

Such an arrangement of two plano-convex lenses is preferred to a simple double-convex lens for an eyepiece because it reduces chromatic and spherical aberrations.

Problems For Practice

1. The sum of focal lengths of the two lenses of a refracting telescope is 105 cm. The focal length of one lens is 20 times that of the other. Determine the total magnification of the telescope when the final image is formed at infinity. **[CBSE OD 14C]**
(Ans. 20)

2. An astronomical telescope when in normal adjustment has magnifying power 5. If the distance between two lenses is 24 cm, find the focal length of both the lenses. **[Haryana 04]**
(Ans. 20 cm, 4 cm)

3. A telescope has an objective of focal length 200 cm and eyepiece of focal length 5 cm. Calculate its magnifying power when the final image is formed (*a*) at infinity and (*b*) at distance of distinct vision.
(Ans. 40, 48)

4. A telescope consists of an objective of focal length 50 cm and an eyepiece of focal length 5 cm. In normal adjustment of the telescope, what will be (*i*) the magnifying power and (*ii*) the length of the telescope ? [Ans. (*i*) 10 (*ii*) 55 cm]

5. An astronomical telescope is designed to have a magnifying power of 50 in normal adjustment. If the length of the tube is 102 cm, find the powers of the objective and the eyepiece. (**Ans.** 1 D, 50 D)

6. A telescope has an objective of focal length 30 cm and an eyepiece of focal length 3.0 cm. It is focussed on a scale distant 2.0 m. For seeing with relaxed eye, calculate the separation between the objective and the eyepiece. (**Ans.** 38.3 cm)

7. The focal length of the objective of an astronomical telescope is 1.0 m. If the magnifying power of the telescope is 20, find the focal length of the eyepiece and the length of the telescope for the relaxed eye. (**Ans.** 5 cm, 1.05 m)

8. The diameter of the moon is 3.5×10^3 km and its distance from the earth is 3.8×10^5 km. It is viewed by a telescope which consists of two lenses of focal lengths 4 m and 10 cm. Find the angle subtended at eye by the final image. (**Ans.** 21.1°)

9. On seeing with unaided eye, the visual angle of moon at the eye is 0.06°. The focal lengths of the objective and the eyepiece of a telescope are respectively 200 cm and 5 cm. What will the visual angle on seeing through the telescope ? (**Ans.** 2.4°)

10. A telescope objective has a focal length of 100 cm. When the final image is formed at the least distance of distinct vision, the distance between the lenses is 105 cm. Calculate the focal length of the eyepiece and the magnifying power of the telescope. (**Ans.** 6.25 cm, 20)

11. A refracting telescope has an objective of focal length 1 m and an eyepiece of focal length 20 cm. The final image of the sun 10 cm in diameter is formed at a distance of 24 cm from the eyepiece. What angle does the sun subtend at the objective ? (**Ans.** 0.0455 rad)

12. A reflecting type telescope has a concave reflector of radius of curvature 120 cm. Calculate focal length of eyepiece to secure a magnification of 20. (**Ans.** 3 cm)

HINTS

1. (*i*) Given $f_o + f_e = 105$ and $f_o = 20 f_e$

$\therefore \quad 20 f_e + f_e = 105 \Rightarrow f_e = 5$ cm

$f_0 = 20 \times 5 = 100$ cm

Hence, $m = \dfrac{f_o}{f_e} = \dfrac{100}{5} = \mathbf{20.}$

(*ii*) $m = \dfrac{f_o}{f_e} = 5 \Rightarrow f_o = 5 f_e$

But $f_o + f_e = 24 \quad \therefore \; 6 f_e = 24$

Hence, $f_e = \mathbf{4\ cm}$ and $f_o = \mathbf{20\ cm}$.

6. For seeing with relaxed eye, final image should be formed at infinity. This happens when the image formed by the objective lies at the focus of the eyepiece.

For the objective :

$u_0 = -2.0$ m $= -200$ cm, $\quad f_0 = +30$ cm

$$\therefore \quad \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{30} - \frac{1}{200} = \frac{17}{600}$$

or $\quad v_0 = 35.3$ cm

Distance between the objective and the eyepiece

$= v_0 + f_e = 35.3 + 3.0 = \mathbf{38.3\ cm.}$

8. $m = \dfrac{f_0}{f_e} = \dfrac{4\text{ m}}{10\text{ cm}} = 40.$ Also, $m = \dfrac{\beta}{\alpha}$

$$\therefore \quad \beta = 40\,\alpha = 40 \times \frac{\text{Diameter of moon}}{\text{Radius of lunar orbit}}$$

$$= \frac{40 \times 3.5 \times 10^3}{3.8 \times 10^5} = 36.84 \times 10^{-2} \text{ rad} = \mathbf{21.1°}.$$

9. $m = \dfrac{f_o}{f_e} = \dfrac{\beta}{\alpha}$

$$\therefore \quad \beta = \frac{f_o}{f_e} \cdot \alpha = \frac{200}{5} \times 0.06° = \mathbf{2.4°}.$$

10. Here $f_0 = 100$ cm, $\; f_0 + |u_e| = 105$ cm

$\therefore \quad |u_e| = 5$ cm

For the eyepiece, $u_e = -5$ cm, $\; v_e = -D = -25$ cm

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e} = -\frac{1}{25} + \frac{1}{5} = \frac{4}{25}$$

or $\quad f_e = \mathbf{6.25\ cm}$

$$m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

$$= \frac{100}{6.25}\left(1 + \frac{6.25}{25}\right) = \mathbf{20.}$$

11. Let h_1 and h_2 be the sizes of the images formed by the objective and the eyepiece respectively. Then

$$\tan \alpha = \frac{h_1}{f_o} = \frac{h_1}{100}$$

Also $\quad m_e = \dfrac{h_2}{h_1} = 1 + \dfrac{D'}{f_e}$

or $\quad \dfrac{10}{h_1} = 1 + \dfrac{24}{20} = \dfrac{44}{20}$

or $\quad h_1 = \dfrac{50}{11}$ cm

or $\quad \alpha \simeq \dfrac{50}{11 \times 100} = \dfrac{1}{22} = \mathbf{0.0455\ rad.}$

12. $f_e = \dfrac{f_0}{m} = \dfrac{R_0/2}{m} = \dfrac{120/2}{20} = \mathbf{3\ cm.}$

GUIDELINES TO NCERT EXERCISES

9.1. *A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to receive a sharp image ? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved ?*

Ans. Here, $h_1 = 2.5\text{ cm},\ u = -27\text{cm},\ R = -36\text{ cm}$

$$f = \frac{R}{2} = -18\text{cm}$$

[$\because$ R is – ve for a concave mirror]

By mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-18} + \frac{1}{27} = \frac{-3+2}{54} = -\frac{1}{54}$$

or $v = -\mathbf{54\ cm}$

Thus the screen should be placed at 54 cm from the mirror on the same side as the object.

Magnification,

$$m = \frac{h_2}{h_1} = -\frac{v}{u} = -\frac{-54}{-27} = -2$$

$\therefore$ Size of image,

$$h_2 = -2 \times 2.5 = -\mathbf{5\ cm}$$

Negative sign shows that the image is real and inverted.

If the candle is moved closer to the mirror, the image moves away from mirror, so the screen would have to be moved farther and farther from the mirror. Closer than 18 cm from the mirror (when the focal point is crossed), the image becomes virtual and cannot be taken on screen.

9.2. *A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.*

Ans. Here, $h_1 = 4.5\text{ cm},\ u = -12\text{ cm},\ f = +15\text{ cm}$

[$\because$ f is + ve for a convex mirror]

By mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60} = \frac{3}{20}$$

or $v = +\frac{20}{3} = +\mathbf{6.67\ cm}$

As v is + ve, image is virtual and erect and is formed at 6.67 cm behind the mirror.

Magnification, $m = \frac{h_2}{h_1} = -\frac{v}{u} = -\frac{20}{3 \times (-12)} = \mathbf{\frac{5}{9}}$

Size of image, $h_2 = \frac{5}{9} \times h_1 = \frac{5}{9} \times 4.5 = \mathbf{2.5\ cm}$

As the needle is moved farther from the mirror, the image shifts towards the focus (but never beyond F) and goes on decreasing in size.

9.3. *A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water ? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again ?* **[Himachal 2000 ; CBSE D 09C]**

Ans. *Tank filled with water :*

Real depth = 12.5 cm

Apparent depth = 9.4 cm

Refractive index of water is

$${}^a\mu_w = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{12.5}{9.4} = \mathbf{1.33}$$

Tank filled with liquid :

Real depth = 12.5 cm

$$\text{Refractive index of liquid} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

or $$1.63 = \frac{12.5}{\text{Apparent depth}}$$

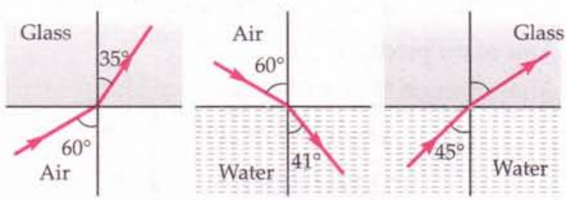
$\therefore$ Apparent depth with liquid

$$= \frac{12.5}{1.63}\text{ cm} = 7.669\text{ cm} \simeq \mathbf{7.7\ cm}$$

Distance through which the microscope has to be moved

$$= 9.4 - 7.7 = \mathbf{1.7\ cm.}$$

9.4. *Figures. 9.199 (a) and (b) show refraction of an incident ray in air at 60° with the normal to a glass air and water air interface, respectively. Predict the angle of refraction of an incident ray in water at 45° with the normal to a water glass interface [Fig. 9.199(c)].*



(a) (b) (c)

Fig. 9.199

Ans. From Fig. 9.199(*a*),

$${}^a\mu_g = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 35°} = \frac{0.8660}{0.5736} = 1.51$$

From Fig. 9.199(*b*),

$$^{a}\mu_{w} = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 41^\circ} = \frac{0.8660}{0.6561} = 1.32$$

From Fig. 9.199(*c*),

$$^{w}\mu_{g} = \frac{^{a}\mu_{g}}{^{a}\mu_{\omega}} = \frac{\sin i}{\sin r}$$

or $$\frac{1.51}{1.32} = \frac{\sin 45^\circ}{\sin r} = \frac{0.7071}{\sin r}$$

or $$\sin r = \frac{1.32 \times 0.7071}{1.51} = 0.6181$$

$\therefore$ $$r \simeq \mathbf{38.2^\circ}.$$

9.5. *A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out ? Refractive index of water is 1.33. Consider the bulb to be a point source.*

Ans. The light rays from the small bulb *S* which are incident at an angle $i > i_c$ are totally internally reflected and cannot emerge out of water surface. The light from the bulb *S* comes out through a circular patch of radius *r* given by

$$\tan i_c = \frac{OA}{OS} = \frac{r}{h} \quad \text{or} \quad r = h \tan i_c$$

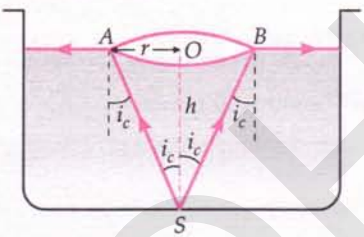


Fig. 9.200

$$\sin i_c = \frac{1}{\mu} = \frac{1}{1.33} = \frac{3}{4}$$

$$\cos i_c = \sqrt{1-\left(\frac{3}{4}\right)^2} = \frac{\sqrt{7}}{4}$$

$$\tan i_c = \frac{3}{4} \times \frac{4}{\sqrt{7}} = \frac{3}{\sqrt{7}}$$

Area of the patch,

$$= \pi r^2 = \pi h^2 \tan^2 i_c$$

$$= 3.14 \times (0.80)^2 \times \frac{9}{7} \text{m}^2$$

$$= 2.58 \text{ m}^2 \simeq \mathbf{2.6 \text{ m}^2}.$$

9.6. *A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism. ? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.*

Ans. *When the prism is placed in air :*

$$\delta_m = 40^\circ, \quad A = 60^\circ$$

$\therefore$ Refractive index of the prism material is

$$^{a}\mu_{g} = \frac{\sin \frac{A+\delta_m}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{60^\circ + 40^\circ}{2}}{\sin \frac{60^\circ}{2}}$$

$$= \frac{\sin 50^\circ}{\sin 30^\circ} = \frac{0.7660}{0.5000} = \mathbf{1.532}$$

When the prism is placed in water :

$$^{\omega}\mu_{g} = \frac{\sin \frac{A+\delta'_m}{2}}{\sin \frac{A}{2}}$$

or $$\frac{^{a}\mu_{g}}{^{a}\mu_{\omega}} = \frac{\sin \frac{60^\circ+\delta'_m}{2}}{\sin \frac{60^\circ}{2}}$$

or $$\frac{1.532}{1.33} = \frac{\sin \frac{60^\circ+\delta'_m}{2}}{\sin 30^\circ}$$

or $$\sin \frac{60^\circ+\delta'_m}{2} = \frac{1.532}{1.33} \times 0.5 = 0.5759$$

$\therefore$ $$30^\circ + \frac{\delta'_m}{2} = \sin^{-1}(0.5759) = 35^\circ 10'$$

or $$\delta'_m = \mathbf{10^\circ 20'}.$$

9.7. *Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length of the lens is to be 20 cm ?* [CBSE OD 14C]

Ans. Here $\mu = 1.55, \quad f = 20 \text{ cm}$

If $R_1 = R$, then $R_2 = -R$

As $$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$\therefore$ $$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$$

or $$\frac{1}{20} = 0.55 \times \frac{2}{R}$$

or $$R = 0.55 \times 2 \times 20 \text{ cm} = \mathbf{22.0 \text{ cm}}.$$

9.8. *A beam of light converges to a point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, (b) a concave lens of focal length 16 cm ?*

[CBSE OD 06]

Ans. Here the point *P* on the right of the lens acts as a virtual object but the image *I* is real, as shown in Figs. 9.201(*a*) and (*b*).

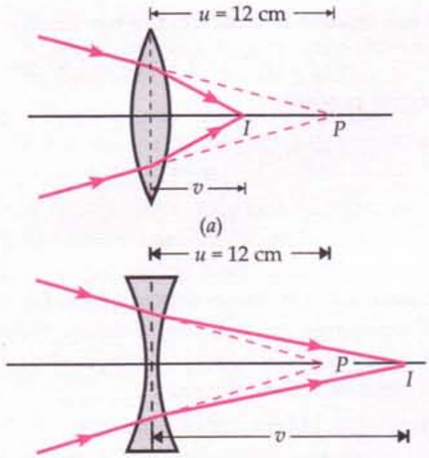


Fig. 9.201

(a) *For convex lens* : $u = +12\text{ cm},\ f = +20\text{ cm}$

Now $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\quad\therefore\quad$ $\frac{1}{v} - \frac{1}{12} = \frac{1}{20}$

or $$\frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$$

or $$v = \frac{15}{2} = \mathbf{7.5\ cm}$$

Thus the beam converges at a point 7.5 cm to the right of the lens.

(b) *For concave lens* : $u = +12\text{ cm},\ f = -16\text{ cm}$

Now $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-16} + \frac{1}{12} = \frac{-3+4}{48} = \frac{1}{48}$$

$\therefore$ $$v = \mathbf{48\ cm}$$

Thus the beam converges at point 48 cm to the right of the lens.

9.9. *An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens ?*

Ans. Here $h_1 = 3\text{ cm},\ u = -14\text{ cm},\ f = -21\text{ cm},\ v = ?$

For a lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore$ $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-21} + \frac{1}{-14} = \frac{-2-3}{42} = \frac{-5}{42}$$

or $$v = -\mathbf{8.4\ cm}$$

Negative v indicates that the image is virtual, erect and is formed at 8.4 cm from the lens on the same side as the object.

As $$m = \frac{h_2}{h_1} = \frac{v}{u}$$

$\therefore$ Size of image,

$$h_2 = \frac{v}{u} \times h_1 = \frac{-8.4}{-14} \times 3\text{ cm} = \mathbf{1.8\ cm}$$

i.e., the image is diminished in size.

As the object is moved away from the lens, the virtual image moves towards the focus of the lens (but never beyond it) and progressively diminishes in size.

9.10. *What is the focal length of a combination of a convex lens of focal length 30 cm and a concave lens of focal length 20 cm ? Is the system a converging or a diverging lens ? Ignore thickness of the lenses.*

Ans. Here, $f_1 = +30\text{ cm}$ (convex lens)

$f_2 = -20$ (concave lens)

Focal length of the combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{-20} = -\frac{1}{60}$$

or $$f = \mathbf{-60\ cm}$$

The negative value of f indicates that the combination behaves as a diverging lens.

9.11. *A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (i) the least distance of distinct vision (25 cm), (ii) infinity ? What is the magnifying power of the microscope in each case ?*

[CBSE OD 08]

Ans. Here $f_0 = 2.0\text{ cm},\ f_e = 6.25\text{ cm},\ u_0 = ?$

(i) *When the final image is obtained at the least distance of distinct vision* :

$$v_e = -25\text{ cm}$$

As $$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$\therefore$ $$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{6.25}$$

$$= \frac{-1-4}{25} = \frac{-5}{25} = -\frac{1}{5}$$

or $$u_e = -5\text{ cm}$$

Now distance between objective and eyepiece

$= 15$ cm

$\therefore$ Distance of the image from objective is

$$v_0 = 15 - 5 = 10\text{ cm}$$

$\therefore$ $$\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} = -\frac{2}{5}$$

or $$u_0 = -\frac{5}{2} = -2.5\text{ cm}$$

$\therefore$ Distance of object from objective = **2.5 cm**

Magnifying power,

$$m = m_0 \times m_e = \frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right)$$

$$= \frac{10}{2.5}\left(1+\frac{25}{6.25}\right) = \mathbf{20}.$$

(*ii*) *When the final image is formed at infinity :*

Here $v_e = \infty, \quad f_e = 6.25\text{ cm}$

As $\frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e} \quad \therefore \quad \frac{1}{\infty}-\frac{1}{u_e}=\frac{1}{f_e}$

or $u_e = -f_e = -6.25\text{ cm}$

Distance between objective and eyepiece = 15 cm

∴ Distance of the objective from the image formed by itself,

$$v_0 = 15 - 6.25 = 8.75\text{ cm}$$

Also $f_0 = +2.0\text{ cm}$

$$\therefore \quad \frac{1}{u_0}=\frac{1}{v_0}-\frac{1}{f_0}=\frac{1}{8.75}-\frac{1}{2}=\frac{2-8.75}{17.5}=\frac{-6.75}{17.5}$$

or $u_0 = -\frac{17.5}{6.75} = -2.59\text{ cm}$

∴ The distance of the object from objective = **2.59 cm**

Magnifying power,

$$m = m_0 \times m_e = \frac{v_0}{u_0}\times\frac{25}{6.25}$$

$$= \frac{27}{8}\times 4 = 13.46 = \mathbf{13.5}.$$

9.12. *A person with a normal near point (25 cm) using a compound microscope with an objective of focal length 8.0 mm and eyepiece of focal length 2.5 cm can bring an object placed 9.0 mm from the objective in sharp focus. What is the separation between the two lenses ? How much is the magnifying power of the microscope ?*

Ans. Here $f_0 = 0.8\text{ cm},\ u_0 = -0.9\text{ cm},\ v_0 = ?$

As $\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$

$$\therefore \quad \frac{1}{v_0}=\frac{1}{f_0}+\frac{1}{u_0}=\frac{1}{0.8}-\frac{1}{0.9}$$

$$=\frac{0.9-0.8}{0.9\times0.8}=\frac{0.1}{0.8\times0.9}$$

or $v_0 = \frac{0.8\times0.9}{0.1} = 7.2\text{ cm}$

Now for the eyepiece, we have

$f_e = 2.5\text{ cm},\ v_e = -D = -25\text{ cm},\ u_e = ?$

$$\therefore \quad \frac{1}{u_e}=\frac{1}{v_e}-\frac{1}{f_e}=-\frac{1}{25}-\frac{1}{2.5}=\frac{-1-10}{25}=\frac{-11}{25}$$

or $u_e = -\frac{25}{11} = -2.27\text{ cm}$

Hence the separation between the two lenses

$$= v_0 + |u_e| = 7.2 + 2.27 = \mathbf{9.47\text{ cm}}$$

Magnifying power,

$$m = m_0 \times m_e = \frac{v_0}{|u_0|}\left(1+\frac{D}{f_e}\right)$$

$$= \frac{7.2}{0.9}\left(1+\frac{25}{2.5}\right) = \mathbf{88}.$$

9.13. *A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope ? What is the separation between the objective and the eyepiece ?*

Ans. Here $f_0 = 144\text{ cm},\quad f_e = 6\text{ cm}$

For the small telescope set in normal adjustment, the magnifying power is

$$m = \frac{f_0}{f_e} = \frac{144}{6} = \mathbf{24}$$

Separation between the objective and the eyepiece

$$= f_0 + f_e = 144 + 6 = \mathbf{150\text{ cm}}.$$

9.14. (*i*) *A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is angular magnification of the telescope ?*

(*ii*) *If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens ? The diameter of the moon is 3.48×10^6 m, and the radius of lunar orbit is 3.8×10^8 m.* [CBSE OD 08, 11 ; D 15]

Ans. Here $f_0 = 15\text{ m},\quad f_e = 1.0\text{ cm} = 0.01\text{ m}$

(*i*) Angular magnification,

$$m = \frac{f_0}{f_e} = \frac{15}{0.01} = \mathbf{1500}.$$

(*ii*) Let d be the diameter of the image in metres. Then angle subtended by the moon will be

$$\alpha = \frac{\text{Diameter of moon}}{\text{Radius of lunar orbit}} = \frac{3.48\times10^6}{3.8\times10^8}$$

Angle subtended by the image formed by the objective will also be equal to α and is given by

$$\alpha = \frac{\text{Diameter of image of moon}}{f_0} = \frac{d}{15}$$

$$\therefore \quad \frac{d}{15} = \frac{3.48\times10^6}{3.8\times10^8}$$

Diameter of image of moon,

$$d = \frac{3.48\times10^6\times15}{3.8\times10^8} = \frac{3.48\times15\times10^{-2}}{3.8} = \mathbf{13.73\text{ cm}}.$$

9.15. *Use the mirror equation to deduce that :*

(*a*) *an object placed between f and 2 f of a concave mirror produces a real image beyond 2 f.* [CBSE D 15]

(b) *a convex mirror always produces a virtual image independent of the location of the object.*

(c) *the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.*

(d) *an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.* [CBSE OD 11]

Ans. (a) From mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Now for a concave mirror, $f < 0$ and for an object on the left, $u < 0$

$$\therefore \quad 2f < u < f \quad \text{or} \quad \frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$$

$$\text{or} \quad -\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f}$$

$$\text{or} \quad \frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f} \quad \text{or} \quad \frac{1}{2f} < \frac{1}{v} < 0$$

This implies that $v < 0$ so that image is formed on left. Also the above inequality implies

$$2f > v$$

$$\text{or} \quad |2f| < |v| \qquad [\because 2f \text{ and } v \text{ are negative}]$$

i.e., the real image is formed beyond 2*f*.

(b) For a convex mirror, $f > 0$ and for an object on left, $u < 0$. From mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

This implies that $\frac{1}{v} > 0$ or $v > 0$

This shows that whatever be the value of *u*, a convex mirror forms a virtual image on the right.

(c) For convex mirror, $f > 0$ and for an object on the left $u < 0$, so mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

implies that $\frac{1}{v} > \frac{1}{f}$ $\left[\because -\frac{1}{u} \text{ is a + ve quantity}\right]$

$$\text{or} \quad v < f$$

This shows that the image is located between the pole and the focus of the mirror.

(d) From mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

For a concave mirror, $f < 0$ and for an object located between the pole and focus of a concave mirror,

$$f < u < 0$$

$$\therefore \quad \frac{1}{f} > \frac{1}{u} \quad \text{or} \quad \frac{1}{f} - \frac{1}{u} > 0 \quad \text{or} \quad \frac{1}{v} > 0$$

i.e., a virtual image is formed on the right.

Also $\frac{1}{v} < \frac{1}{|u|}$ or $v > |u|$ $\therefore$ $|m| = \frac{v}{|u|} > 1$

i.e., image is enlarged.

9.16. *A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table ? Refractive index of glass = 1.5. Does the answer depend on the location of the slab ?*

Ans. The distance through which the pin appears to be raised is

d = Real thickness of slab − Apparent thickness of slab

$$= \text{Real thickness of slab} - \frac{\text{Real thickness of slab}}{\mu}$$

$$= t - \frac{t}{\mu} = t\left(1 - \frac{1}{\mu}\right)$$

Here $t = 15$ cm, $\mu = 1.5$

$$\therefore d = 15\left(1 - \frac{1}{1.5}\right) = 15\left(\frac{1.5 - 1}{1.5}\right) = \mathbf{5\ cm}$$

The answer *does not depend* on the location of the slab.

9.17. *Figure 9.202 shows a cross-section of a 'light- pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place as shown.*

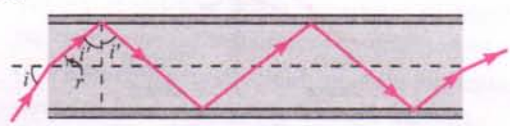


Fig. 9.202

Ans. Given $\mu_2 = 1.68$, $\mu_1 = 1.44$, $\mu = \frac{\mu_2}{\mu_1} = \frac{1}{\sin i_c'}$

$\therefore$ Critical angle i_c' is given by

$$\sin i_c' = \frac{\mu_1}{\mu_2} = \frac{1.44}{1.68} = 0.8571 \quad \Rightarrow i_c' = 59°$$

Total internal reflection will occur if the angle $i' > i_c'$, *i.e.*, if $i' > 59°$ or when $r < r_{max}$, where $r_{max} = 90° - 59° = 31°$. Using Snell's law,

$$\frac{\sin i_{max}}{\sin r_{max}} = 1.68$$

$$\text{or} \quad \sin i_{max} = 1.68 \times \sin r_{max}$$

$$= 1.68 \times \sin 31° = 1.68 \times 0.5150 = 0.8662$$

$$\therefore \quad i_{max} \simeq 60°$$

Thus all incident rays which make angles in the range $0 < i < 60°$ with the axis of the pipe will suffer total internal reflections in the pipe.

9.18. *Answer the following questions :*

(*a*) *You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real image under some circumstances ? Explain.*

(*b*) *A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction ?*

(*c*) *A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is ?*

(*d*) *Does the apparent depth of a tank of water change if viewed obliquely ? If so, does the apparent depth increase or decrease ?*

(*e*) *The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter ?*

Ans. (*a*) Yes, a plane or convex mirror can produce a real image if the object is virtual. As shown in Figs. 9.203(*a*) and (*b*), if a plane or a convex mirror is placed in the path of rays converging to a point, the rays get reflected to a point in front of the mirror. Real image can be obtained on a screen.

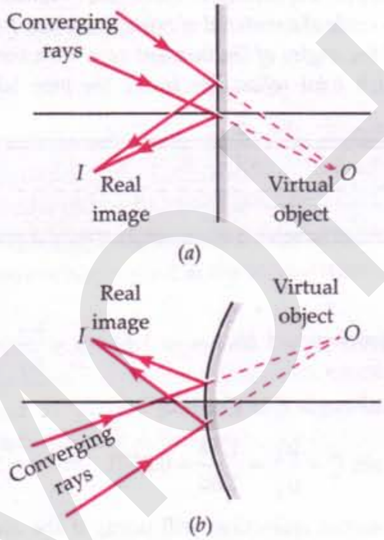


Fig. 9.203

(*b*) When the reflected and refracted rays are divergent, the image is virtual. These rays are converged by the eyelens to form a real image on the retina. The virtual image serves as a virtual object. Also the screen is not located at the position of virtual image. So there is no contradiction.

(*c*) The man looks taller to a diver under water. As the fisherman is in air, the light rays travel from rarer to denser medium. They bend towards the normal and hence appear to come from a larger distance, as shown in Fig. 9.204. It may be noted that the points P and Q are, in fact, so close that the rays through these points can enter the small aperture of the eye of the fish. Here

AB = Real height of the man,

AB' = Apparent height of the man.

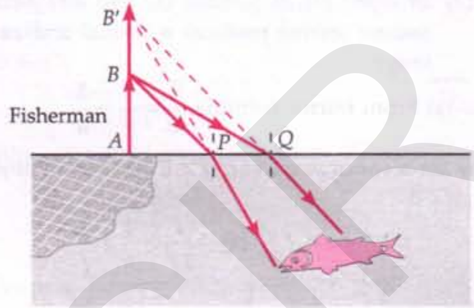


Fig. 9.204

(*d*) Yes. The apparent depth decreases for oblique viewing from its value of normal viewing. This is obvious from the ray diagrams shown in Fig. 9.205.

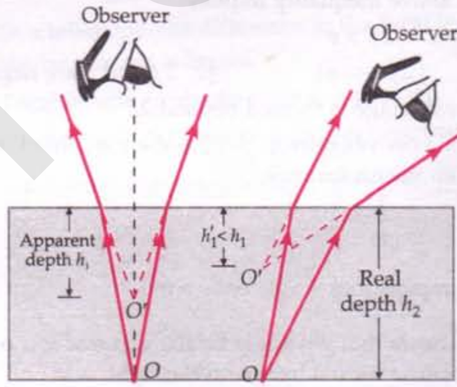


Fig. 9.205

(*e*) Yes. Refractive index of diamond is high ($\mu = 2.42$), so its critical angle is small ($i_c = 24°$). A diamond cutter makes use of this large range of angle of incidence (24° to 90°) to ensure that light entering diamond suffers total internal reflection several times. When light emerges out, it produces sparkling effect.

9.19. *The image of a small electric bulb fixed on the wall to a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose ?*

Ans. The minimum distance (as proved in Problem 14 on page 9.124) between an object and its real image is $4f$.

$$\therefore \quad 4\,f_{max} = D \quad \text{or} \quad f_{max} = \frac{D}{4} = \frac{3\text{ m}}{4} = \mathbf{0.75\ m}.$$

9.20. *A screen is placed 90 cm from an object, the image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.*

Ans. As shown in Fig. 9.206, let O and I be the positions of object and image respectively and L_1 and L_2 be the two conjugate positions of the lens.

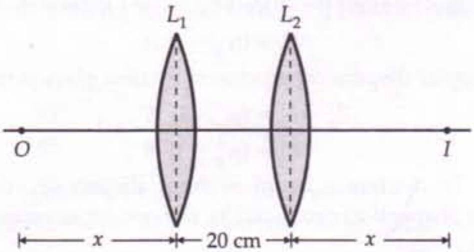


Fig. 9.206

Obviously, $x + 20 + x = 90$ cm or $x = 35$ cm

When the lens is in position L_1, we have

$$u = -x = -35 \text{ cm}, \; v = 20 + x = 20 + 35 = 55 \text{ cm}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{55} + \frac{1}{35} = \frac{7+11}{385} = \frac{18}{385}$$

or $$f = \frac{385}{18} = \mathbf{21.4 \text{ cm.}}$$

9.21. (*a*) *Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side a beam of parallel light is incident ? Is the notion of effective focal length of this system useful at all ?*

(*b*) *An object 1.5 cm in size is placed on the side of the convex lens in the above arrangements. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.*

Ans. (*a*) (*i*) Let a parallel beam of light be incident from the left on the convex lens first. Then

$$f_1 = 30 \text{ cm}$$
$$u_1 = -\infty$$
$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$
$$\therefore \quad \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{30} - \frac{1}{\infty} = \frac{1}{30}$$

or $$v_1 = +30 \text{ cm}$$

This image becomes a *virtual* object for the second lens so that

$$f_2 = -20 \text{ cm},$$
$$u_2 = +(30-8) = +22 \text{ cm}$$

Now $$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2}$$
$$= -\frac{1}{20} + \frac{1}{22} = \frac{-11+10}{220} = \frac{-1}{220}$$

or $$v_2 = \mathbf{-220 \text{ cm}}$$

The parallel incident beam appears to diverge from a point $220 - 4 =$ **216 cm** from the centre of the two-lens system.

(*ii*) Let the parallel beam be incident from the left on the concave lens first. Then

$$f_1 = -20 \text{ cm}, \; u_1 = -\infty$$

As $$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$
$$\therefore \quad \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{-20} + \frac{1}{-\infty} = -\frac{1}{20}$$

or $$v_1 = -20 \text{ cm}$$

This image becomes a *real* object for the second lens so that

$$f_2 = -30 \text{ cm}$$
$$u_2 = -(20+8) = -28 \text{ cm}$$

Now $$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{30} - \frac{1}{28} = \frac{14-15}{420} = \frac{-1}{420}$$

or $$v_2 = \mathbf{-420 \text{ cm}}$$

Thus the parallel incident beam appears to diverge from a point $420 - 4 =$ **416 cm** on the left of the centre of the two-lens system.

Clearly, the answer depends on which side of the lens system the parallel beam is incident. The notion of effective focal length, therefore, does not seem to be meaningful for this system.

(*b*) Here $u_1 = -40$ cm, $f_1 = 30$ cm

As $$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$
$$\therefore \quad \frac{1}{v_1} + \frac{1}{40} = \frac{1}{30}$$

or $$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{40} = \frac{4-3}{120} = \frac{1}{120}$$

or $$v_1 = 120 \text{ cm}$$

Magnitude of magnification due to the first (convex) lens is

$$m_1 = \frac{v}{|u|} = \frac{120}{40} = 3$$

This image becomes a virtual object for the second lens so that

$$u_2 = +(120-8) = +112 \text{ cm}$$
$$f_2 = -20 \text{ cm}$$

Now $$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2}$$
$$= -\frac{1}{20} + \frac{1}{112} = \frac{-112+20}{112 \times 20} = \frac{-92}{112 \times 20}$$

or $$v_2 = -\frac{112 \times 20}{92} \text{ cm} = -24.9 \text{ cm}$$

Magnitude of magnification due to the second (concave) lens is

$$m_2 = \frac{|v_2|}{u_2} = \frac{112 \times 20}{92 \times 112} = \frac{20}{92}$$

Net magnitude of magnification due to the two-lens system is

$$m = m_1 \times m_2$$
$$= \frac{3 \times 20}{92} = 0.652$$

Size of image,

$$h_2 = mh_1 = 0.652 \times 1.5 = \mathbf{0.98\ cm.}$$

9.22. *At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face ? The refractive index of the prism is 1.524.*

Ans. The refracted ray QR will just suffer total internal reflection if it is incident at the critical angle i_c.

Thus $\quad r_2 = i_c$

Now $\quad \sin i_c = \frac{1}{\mu} = \frac{1}{1.524} = 0.6542$

$\therefore \quad i_c = \sin^{-1}(0.6542) \simeq 41°$

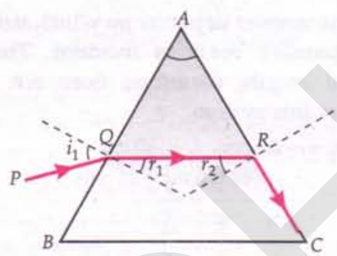


Fig. 9.207

But $\quad r_1 + r_2 = A$

$\therefore \quad r_1 = A - r_2 = A - i_c = 60° - 41° = 19°$

From Snell's law,

$$\mu = \frac{\sin i_1}{\sin r_1}$$

$\therefore \quad \sin i_1 = \mu \sin r_1 = 1.524 \times \sin 19°$

$\quad = 1.524 \times 0.3256 = 0.4962$

Hence $\quad i_1 = \sin^{-1}(0.4962) \simeq \mathbf{30°}$.

9.23. *You are given prisms made of crown glass and flint glass with a variety of angles. Suggest a combination of prisms which will (a) deviate a pencil of white light without much dispersion, (b) disperse (and displace) a pencil of white light without much deviation.*

Ans. Two identical prisms made of the same material placed with their base on opposite sides (or the incident white light) and faces touching (or parallel) will neither deviate nor disperse, but will merely produce a parallel displacement of the beam.

Now, angular dispersion produced by crown glass prism is

$$\delta_b - \delta_r = (\mu_b - \mu_r) A$$

Mean deviation produced by crown glass prism is

$$\delta_y = (\mu_y - 1) A$$

Angular dispersion produced by flint glass prism is

$$\delta'_b - \delta'_r = (\mu'_y - 1) A'$$
$$\delta'_y = (\mu'_y - 1) A$$

(*a*) To deviate a beam without dispersion, the net angular dispersion produced by the combination must be zero *i.e.,*

$$(\mu_b - \mu_r) A + (\mu'_b - \mu'_r) A' = 0$$

or $$A' = -\frac{(\mu_b - \mu_r)}{(\mu'_b - \mu'_r)} A$$

Negative sign shows that the two prisms must be placed with their bases on opposite sides. As $(\mu'_b - \mu'_r)$ for flint glass is more than $(\mu_b - \mu_r)$ for crown glass, therefore, a flint glass prism of smaller refracting angle should be combined with a crown glass prism so that the dispersion due to the first is nullified by the second as shown in Fig. 9.208(*a*).

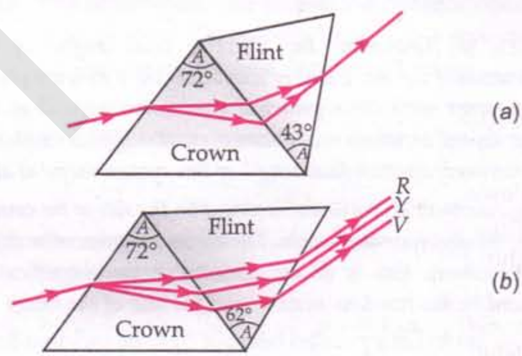


Fig. 9.208

(*b*) To produce dispersion without deviation, the net mean deviation should be zero, *i.e.,*

$$(\mu_y - 1) A + (\mu'_y - 1)A' = 0$$

or $$A' = -\frac{\mu_y - 1}{\mu'_y - 1} A$$

We take a crown glass prism of certain angle and go on increasing the angle of flint glass prism till the deviations due to the two prisms are equal and opposite. However, the flint glass prism angle will still be smaller than that of crown glass because flint glass has higher refractive index than that of crown glass as shown in Fig. 9.208(*b*).

Due to the adjustments involved for many colours, the above combinations are not very accurate arrangements for the purposes required.

9.24. *For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about*

40 dioptres, and the least converging power of the eyelens behind the cornea is about 20 diopters. From this rough data estimate the range of accommodation (i.e., range of converging power of its eyelens) of a normal eye.

Ans. To see objects at infinity, the eye uses its least converging power

$$= 40 + 20 = 60 \text{ dioptres}$$

$\therefore$ Approximate distance between the retina and the cornea eyelens

= focal length of the eyelens

$$= \frac{100}{P} = \frac{100}{60} = \frac{5}{3} \text{ cm}$$

To focus an object at the near point on the retina, we have

$$u = -25 \text{ cm}, \quad v = \frac{5}{3} \text{ cm}$$

$\therefore$ Focal length f should be given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{3}{5} + \frac{1}{25} = \frac{15+1}{25} = \frac{16}{25}$$

$$f = \frac{25}{16} \text{ cm}$$

$\therefore$ Corresponding converging power

$$= 64 \text{ dioptres}$$

Power of the eyelens = 64 − 40 = 24 dioptres

Thus the range of accommodation of the eyelens is roughly 20 to 24 dioptres.

9.25. *Does short-sightedness (myopia) or long-sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation ? If not, what might cause these defects of vision ?*

Ans. No, it does not imply necessarily that the eye has lost its ability of accommodation. A person may have normal ability of accommodation of the eye lens and yet may be myopic or hyperopic. Myopia arises when the eye ball from front to back gets too elongated, hypermetropia arises when it gets too shortened. In practice, in addition the eye lens may also lose some of its ability of accommodation. When the eye ball has normal length but the eyelens loses partially its ability of accommodation (as happens with increasing age for normal eye), the defect is called *presbyopia* and is corrected in the same manner as hypermetropia.

9.26. *A myopic person has been using spectacles of power −1.0 dioptre for distant vision. During old age he also needs to use separate reading glass of power +2.0 dioptres. Explain what may have happened.*

Ans. Here, $P = -1.0$ dioptre

$$\therefore \quad f = \frac{100}{P} = \frac{100}{-1} = -100 \text{ cm}$$

Thus, the far point of the person is 100 cm, on the other hand, his near point may have been normal (about 25 cm).

The objects at infinity produce virtual images at 100 cm (using spectacles).

To see closer objects, *i.e.*, those which are (or whose images using the spectacles are) between 100 cm and 25 cm, the person uses the ability of accommodation of his eyelens. This ability usually gets partially lost in old age (presbyopia). The near point of the person recedes to 50 cm.

So to view the objects at 25 cm clearly, we have

$$u = -25 \text{ cm}, \quad v = -50 \text{ cm}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= -\frac{1}{50} + \frac{1}{25} = \frac{-1+2}{50} = \frac{1}{50}$$

or $\quad f = 50$ cm

Hence, $\quad P = \frac{100}{f} = \frac{100}{50} = \mathbf{+2}$ **dioptres**

Thus the person needs a converging lens of power + 2 dioptres.

9.27. *A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to ? How is such a defect of vision corrected ?*

Ans. This defect is called *astigmatism*. It arises because the curvature of the cornea plus eyelens refracting system is not the same in different planes. The eyelens is usually spherical, *i.e.*, has the same curvature in different planes but the cornea is not spherical in case of an astigmatic eye. In the present case, the curvature in the vertical plane is enough, so sharp images of vertical lines can be formed on the retina. But the curvature is insufficient in the horizontal plane, so horizontal lines appear blurred. The defect can be corrected by using a cylindrical lens with its axis along the vertical. Clearly, parallel rays in the vertical plane will suffer no extra refraction, but those in the horizontal plane can get the required extra convergence due to refraction by the curved surface of the cylindrical lens if the curvature of the cylindrical surface is chosen appropriately.

9.28. *A man with normal near point (25 cm) reads a book with small print using a magnifying glass : a thin convex lens of focal length 5 cm.*

(a) What is the closest and the farthest distance at which he can read the book when viewing through the magnifying glass ?

(b) What is the maximum and the minimum angular magnification (Magnifying power) possible using the above simple microscope ?

Ans. *For the closest distance :*

$$v = -25 \text{ cm}, \quad f = 5 \text{ cm}, \quad u = ?$$

As $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\therefore \quad \frac{1}{u}=\frac{1}{v}-\frac{1}{f}=\frac{1}{-25}-\frac{1}{5}=\frac{-1-5}{25}=\frac{-6}{25}$

or $\quad u=-\frac{25}{6}\text{ cm}=\mathbf{-4.2\ cm}$

This is the closest distance at which the man can read the book.

For the farthest image :

$$v=\infty,\quad f=5\text{ cm},\quad u=?$$

$\therefore \quad \frac{1}{u}=\frac{1}{v}-\frac{1}{f}$

$$=\frac{1}{\infty}-\frac{1}{5}=0-\frac{1}{5}=-\frac{1}{5}$$

$$u=\mathbf{-5\ cm}$$

This is the farthest distance at which the man can read the book.

(*b*) Maximum angular magnification is

$$\frac{D}{u_{min}}=\frac{25}{25/6}=\mathbf{6}$$

Minimum angular magnification is

$$\frac{D}{u_{max}}=\frac{25}{5}=\mathbf{5.}$$

9.29. *A card sheet divided into squares each of size 1 mm² is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 10 cm) held close to the eye.*

(*a*) *What is the magnification produced by the lens ? How much is the area of each square in the virtual image ?*

(*b*) *What is the angular magnification (magnifying power) of the lens ?*

(*c*) *Is the magnification in (a) equal to the magnifying power in (b) ? Explain.*

Ans. (*a*) Here, area of each square (or object)

$$=1\text{ mm}^2$$

$$u=-9\text{ cm},\quad f=+10\text{ cm}$$

As $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\therefore \quad \frac{1}{v}=\frac{1}{f}+\frac{1}{u}$

$$=\frac{1}{10}-\frac{1}{9}=\frac{9-10}{90}=-\frac{1}{90}$$

or $\quad v=-90\text{ cm}$

Magnitude of magnification is

$$m=\frac{v}{|u|}=\frac{90}{9}=10$$

Area of each square in the virtual image

$$=(10)^2\times 1=100\text{ mm}^2=\mathbf{1\ cm^2}$$

(*b*) Magnifying power, $M=\frac{D}{|u|}=\frac{25}{9}=\mathbf{2.8.}$

(*c*) No. Magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 cm). Thus magnification magnitude is $\left|\frac{v}{u}\right|$ and magnifying power is $\frac{25}{|u|}$.

Only when the image is located at the near point $|v|=25$ cm, the two quantities are equal as will be seen in the next exercise.

9.30. *(a) At what distance should the lens be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power ?*

(b) What is the magnification in this case ?

(c) Is the magnification equal to the magnifying power in this case ? Explain.

Ans. (*a*) Maximum magnifying power is obtained when the image is at the near point (25 cm). Thus

$$v=-25\text{ cm},\quad f=+10\text{ cm},\quad u=?$$

As $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\therefore \quad \frac{1}{u}=\frac{1}{v}-\frac{1}{f}=-\frac{1}{25}-\frac{1}{10}=\frac{-2-5}{50}=\frac{-7}{50}$

or $\quad u=-\frac{50}{7}=\mathbf{-7.14\ cm}$

So lens should be held 7.14 cm away from the figure.

(*b*) Magnitude of magnification is

$$m=\frac{v}{|u|}=\frac{25}{50/7}=\mathbf{3.5.}$$

(*c*) Magnifying power $=\frac{D}{|u|}=\frac{25}{50/7}=\mathbf{3.5}$

Yes, the magnifying power is equal to the magnitude of magnification because image is formed at the least distance of distinct vision.

9.31. *What should be the distance between the object in Exercise 9.30 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm² ? Would you be able to see the squares distinctly with your eyes very close to the magnifier ?*

Ans. Here, the magnification in area

$$=\frac{6.25\text{ mm}^2}{1\text{ mm}^2}=6.25$$

$\therefore$ Linear magnification, $m=\sqrt{6.25}=2.5$

As $\quad m=\frac{v}{u} \quad \therefore \quad v=mu=2.5\,u$

Now $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$\therefore$ $$\frac{1}{2.5\,u}-\frac{1}{u}=\frac{1}{10}$$

or $$\frac{1-2.5}{2.5\,u}=\frac{1}{10}$$

or $$2.5\,u=-1.5\times 10$$

or $$2.5\,u=-1.5\times 10$$

or $$u=-\frac{1.5\times 10}{2.5}=\mathbf{-6\ cm}$$

Hence $v=2.5\,u=2.5\times(-6)=\mathbf{-15\ cm}$

As the virtual image is closer than the normal near point (25 cm), it cannot be seen by the eye distinctly.

9.32. *Answer the following questions :*

(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification ?

(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back ?

(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power ?

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths ? [CBSE OD 10]

(e) When viewing through a compound microscope, our eyes should be positioned not on the eye-piece but a short distance away from it for best viewing. Why ? How much should be that short distance between the eye and eyepiece ?

Ans. (*a*) It is true that the angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. When a magnifying glass is not used, an object has to be placed at a distance of 25 cm. But the use of a magnifying glass allows us to place the object much closer to the eye than at 25 cm. The closer object has larger angular size than the same object at 25 cm. It is in this sense that a magnifying lens produces angular magnification.

(*b*) Yes, the angular magnification decreases slightly if the eye is moved back. This is because angle subtended at the eye would be slightly less than the angle subtended at the lens. The effect is negligible when image is at much larger distance.

(*c*) First, grinding lenses of very small focal lengths is not easy. More important, if we decrease focal length, both spherical and chromatic abberations become large. So in practice we cannot get a magnifying power of more than 3 or so with a simple convex lens. However using an aberration corrected lens system, one can increase this limit by a factor of 10 or so.

(*d*) The magnifying power of a compound microscope is given by

$$m=m_o\times m_e=\frac{v_o}{u_o}\times\left(1+\frac{D}{f_e}\right)$$

$$=\frac{f_o}{u_o-f_o}\times\left(1+\frac{D}{f_e}\right)$$

Angular magnification (m_0) of objective will be large when u_o is slightly greater than f_o. Since microscope is used for viewing very close objects, so u_o is small. Consequently f_o has to be small.

Moreover, the angular magnification (m_e) of the eyepiece will be large if f_e is small.

(*e*) Refer to the solution of Problem 30(*a*) on page 9.129.

9.33. *An angular magnification (magnifying power) of 30 X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope ?*

Ans. We assume the microscope in common usage, *i.e.*, the final image is formed at the least distance of distinct vision,

$$D=25\text{ cm},\quad f_e=5\text{ cm}$$

$\therefore$ Angular magnification of the eyepiece is

$$m_e=1+\frac{D}{f_e}=1+\frac{25}{5}=6$$

As total magnification, $m=m_e\times m_0$

$\therefore$ Angular magnification of the objective is

$$m_0=\frac{m}{m_e}=\frac{30}{6}=5$$

As real image is formed by the objective, therefore,

$$m_0=\frac{v_0}{u_0}=-5\quad\text{or}\quad v_0=-5\,u_0$$

$$f_0=1.25\text{ cm}$$

Now $$\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$$

or $$\frac{1}{-5\,u_0}-\frac{1}{u_0}=\frac{1}{1.25}$$

or $$\frac{-6}{5\,u_0}=\frac{1}{1.25}$$

or $$u_0=-\frac{6\times 1.25}{5}=\mathbf{-1.5\ cm}$$

Thus the object should be held at 1.5 cm in front of the objective lens.

Also $v_0=-5\,u_0=-5\times(-1.5)=7.5$ cm

As $$\frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e}$$

$$\therefore \quad \frac{1}{u_e}=\frac{1}{v_e}-\frac{1}{f_e}=\frac{1}{-25}-\frac{1}{5}$$

$$[v_e=-D=-25\text{ cm}]$$

$$=\frac{-1-5}{25}=-\frac{6}{25}$$

or $$u_e=\frac{-25}{6}=-4.17\text{ cm}$$

$\therefore$ Separation between the objective and the eyepiece

$$=|u_e|+|v_0|$$

$$=4.17+7.5=\mathbf{11.67\text{ cm.}}$$

9.34. *A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when*

(a) *the telescope is in normal adjustment (i.e., when the final image is at infinity),*

(b) *the final image is formed at the least distance of distinct vision (25 cm) ?* [CBSE OD 13C]

Ans. Here $f_0=140\text{ cm}, \quad f_e=5.0\text{ cm}$

(a) In normal adjustment :

Magnifying power,

$$m=\frac{f_0}{f_e}=\frac{140}{5}=\mathbf{28}$$

(b) When the final image is formed at the least distance of distinct vision (25 cm) :

$$m=\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$$

$$=\frac{140}{5}\left(1+\frac{5}{25}\right)=28\times1.2=\mathbf{33.6.}$$

9.35. *(a) For the telescope described in Exercise 9.34(a), what is the separation between the objective lens and the eyepiece ?* [CBSE OD 13C]

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens ?

(c) What is the height of the final image of the tower if it is formed at 25 cm ?

Ans. (*a*) In normal adjustment, the separation between objective and eyelens

$$=f_0+f_e=140+5=\mathbf{145\text{ cm.}}$$

(*b*) Angle subtended by the 100 m tall tower at 3 km away is

$$\alpha\simeq\tan\alpha=\frac{100}{3\times10^3}=\frac{1}{30}\text{ rad}$$

Let h be the height of the image of tower formed by the objective. Then angle subtended by the image produced by the objective will also be equal to α and is given by

$$\alpha=\frac{h}{f_0}=\frac{h}{140}$$

$$\therefore \quad \frac{h}{140}=\frac{1}{30}$$

or $$h=\frac{140}{30}=\frac{14}{3}=\mathbf{4.67\text{ cm.}}$$

(*c*) Magnification produced by the eyepiece is

$$m_e=1+\frac{D}{f_e}=1+\frac{25}{5}=6$$

$\therefore$ Height of the final image

$$=h\times m_e=\frac{14}{3}\times6=\mathbf{28\text{ cm.}}$$

9.36. *A Cassegrain telescope uses two mirrors as shown in Fig. 9.151. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be ?*

Ans. The image formed by the larger (concave) mirror acts as a virtual object for the smaller (convex) mirror. Parallel rays coming from the object at infinity will focus at 110 mm from the larger mirror. The distance of the virtual object for the smaller mirror = 110 – 20 = 90 mm.

For the small convex mirror, we have

$$u=-90\text{ mm}, \quad f=-70\text{ mm}, \quad v=?$$

Using mirror formula,

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-70}-\frac{1}{-90}=-\frac{1}{315}$$

$$\therefore \quad v=-315\text{ mm}$$

Thus the image is formed at 315 mm from the smaller mirror.

9.37. *Light incident normally on plane mirror attached to a galvanometer coil retraces backward as shown. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away ?*

Ans. When the mirror is turned through angle θ, from position M to M', the reflected ray turns through angle 2θ, so that the reflected spot moves on the screen from position P to Q and

$$\angle POQ=2\theta=2\times3.5°=7°$$

Now $$\tan 2\theta=\frac{d}{1.5}$$

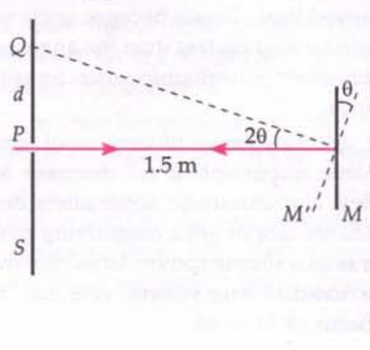


Fig. 9.209

∴ Displacement of reflected spot on the screen is

$$d = 1.5 \tan 2\theta$$
$$= 1.5 \times \tan 7°$$
$$= 1.5 \times 0.1228 \text{ m}$$
$$= 0.1842 \text{ m} = \mathbf{18.4 \text{ cm}}.$$

9.38. *Figure 9.210 shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid ?*

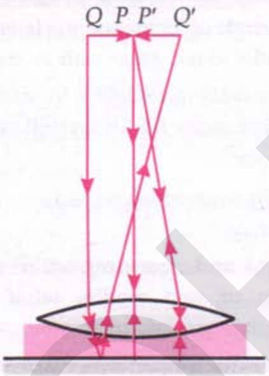


Fig. 9.210

Ans. Distance of the needle from the lens in the first case

= Focal length F of the combination of the convex lens and planoconcave lens formed by the liquid

i.e., $F = 45 \text{ cm}$

Distance measured in second case

= Focal length of the convex lens

i.e., $f_1 = +30 \text{ cm}$

The focal length f_2 of the plano-concave lens is given by

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$$

or
$$\frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1}$$
$$= \frac{1}{45} - \frac{1}{30}$$
$$= \frac{2-3}{90} = -\frac{1}{90}$$

∴
$$f_2 = -90 \text{ cm}$$

Now for the equiconvex lens, we have

$$R_1 = R,\ R_2 = -R,\ f = 30 \text{ cm},\ \mu = 1.5$$

Using Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or
$$\frac{1}{30} = (1.5 - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$$
$$= 0.5 \times \frac{2}{R}$$

or
$$R = 0.5 \times 2 \times 30 \text{ cm} = 30 \text{ cm}.$$

For plano-concave lens, $f = -90 \text{ cm}$,

For concave surface, $R_1 = -R = -30 \text{ cm}$,

For plane surface, $R_2 = \infty$

As
$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

∴
$$\frac{1}{-90} = (\mu - 1)\left[\frac{1}{-30} - \frac{1}{\infty}\right]$$

or
$$\mu - 1 = \frac{-30}{-90} = +\frac{1}{3}$$

or
$$\mu = 1 + \frac{1}{3} = \mathbf{1.33}.$$

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. To which wavelength of light is our eye most sensitive ? In which region does this wavelength lie ?
2. A ray of light falls normally on a mirror. What are the values of angle of incidence and angle of reflection ?
3. A person moves with velocity v towards a plane mirror. With what velocity does his image move towards him ?
4. A mirror is turned through 10°. By what angle will the reflected ray turn ?

5. What is the focal length (or radius of curvature) of a plane mirror ? [Himachal 97 ; CBSE D 01C]
6. What is the number of images of an object held between two parallel plane mirrors ?
7. An object is held between two plane parallel mirrors inclined at 45° to each other. How many images do you expect to see ?
8. What is the minimum size of a plane mirror which can enable a man to see his full image ?
9. How many images of himself can an observer see in a room whose ceiling and two adjacent walls are mirrors ?
10. What is a spherical mirror ? What are its two types ?
11. Which spherical mirror is called a divergent mirror-cancave or convex ?
12. Define principal focus of a spherical mirror.
13. Which spherical mirror has (*i*) a real focus (*ii*) a virtual focus ?
14. Which spherical mirror always forms a virtual, erect and diminished image of an object ?
15. One wants to see an enlarged image of an object in a mirror. Which type of mirror one should use ?
16. Which type of spherical mirror can form a real and diminished image of an object ?
17. When an object is placed between f and $2f$ of a concave mirror, would the image formed be (*i*) real or virtual and (*ii*) diminished or magnified ? [CBSE D 15C]
18. Can we obtain image of an object formed by convex mirror on a screen ? If not, why ?
19. Can we photograph a virtual image ?
20. A concave mirror has focal length 20 cm. Where should the object be placed in front of the mirror so that area of image equal to the size of the object is formed ?
21. What is the angle of incidence, when a ray of light falls on the spherical mirror from its centre of curvature ?
22. Starting from a large distance, a flame is moved towards a convex mirror. Comment on how the size and position of the image change ?
23. A concave mirror, of aperture 4 cm, has a point object placed on its principal axis at a distance of 10 cm from the mirror. The image, formed by the mirror, is not likely to be a sharp image. State the likely reasons for the same. [CBSE Sample Paper 13]
24. What is refraction ? [Haryana 01]
25. Define refractive index.
26. Define refractive index in terms of wavelength of light.
27. What is meant by relative refractive index of medium ?
28. State the factors on which the refractive index of a medium depends.
29. State Snell's law of refraction of light. [Punjab 2000, 04]
30. When does Snell's law of refraction fail ?
31. For which material the value of refractive index is : (*i*) minimum and (*ii*) maximum ?
32. What is lateral shift in refraction ?
33. On what factors does the lateral shift depend ?
34. For what angle of incidence, the lateral shift produced by a parallel sided glass slab is zero ?
35. For what angle of incidence, the lateral shift produced by a parallel sided glass slab is maximum ?
36. Light of wavelength 6000 Å in air enters a medium of refractive index 1.5. What will be its frequency in the medium ? [CBSE D 94]
37. When light undergoes refraction, what happens to its frequency ? [CBSE OD 2000C]
38. When light undergoes refraction at the surface of separation of two media, what happens to its wavelength ? [CBSE OD 2000C]
39. How does the frequency of a beam of ultraviolet light change when it goes from air into glass ? [CBSE D 01]
40. Define the term critical angle for a pair of media. [CBSE D 15C]
41. Can total internal reflection occur when light travels from a rarer to a denser medium ?
42. Velocity of light in glass 2×10^8 m/s and in air is 3×10^8 m/s. If the ray of light passes from glass to air, calculate the value of critical angle. [CBSE F 15]
43. Write the value of critical angle for a material of refractive index $\sqrt{2}$ [Himachal 93 ; CBSE F 94]
44. A substance has a critical angle of 45° for yellow light. What is its refractive index ? [ISCE 97 ; Haryana 2000]
45. Which one has a greater critical angle–diamond or water ?
46. A good plane mirror reflects about 95% of light. What is the percentage of light reflected when total internal reflection occurs ?
47. Write the relation between the refractive index and critical angle for a given pair of optical media. [CBSE OD 09 ; D 13]
48. When light is incident on a rarer medium from a denser medium, write the relation between the critical angle and refractive indices of two media. [CBSE D 07C]

49. State the conditions under which total internal reflection occurs. [ISCE 99 ; CBSE D 10, 13 ; F 09]
50. What is an optical fibre ? [Haryana 04]
51. Which of the two main parts of an optical fibre has a higher value of refractive index ?
52. Name the physical principle on which the working of optical fibres is based. [ISCE 95, 2000 ; Punjab 2000, 02]
53. What is the main use of optical fibres ?
54. If ${}^{a}\mu_{g} = \frac{3}{2}$ and ${}^{a}\mu_{w} = \frac{4}{3}$, then what will be the value of ${}^{w}\mu_{g}$?
55. What is a lens ?
56. Define optical centre of a lens.
57. What is the deviation produced by a thin lens of a ray passing through its optical centre ?
58. What type of a lens is a tumbler filled with water ?
59. Can a lens be used in a medium of which it is made of ?
60. A lens always forms virtual and erect image of the object irrespective of the position of the object. What type of lens is this ?
61. What should be the position of an object relative to a biconvex lens so that it behaves like a magnifying glass ?
62. Where should an object be placed from a convex lens to form an image of the same size ? Can it happen in case of concave lens ?
63. Define power of a lens. Give its SI units. [CBSE F 09]
64. Define one dioptre.
65. If the power of a lens is + 5 dioptre, what is its focal length ? [CBSE D 94 C]
66. A lens has a power of – 2.5 D. What is the focal length and nature of the lens ?
67. Two thin lenses of power + 4D and – 2 D are in contact. What is the focal length of the combination ? [CBSE OD 09]
68. An object is held at the principal focus of a concave lens of focal length *F*. Where is the image formed ? [CBSE OD 03, 08]
69. The central portion of a lens is covered with a black paper. Will the lens form full image of an object ?
70. In Fig. 9.211 given below, path of a parallel beam of light passing through a convex lens of refractive index μ_g kept in a medium of refractive index μ_m is shown. Is (*i*) $\mu_g = \mu_m$ or (*ii*) $\mu_g > \mu_m$ or (*iii*) $\mu_g < \mu_m$? [CBSE D 02C]

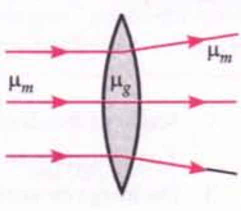


Fig. 9.211

71. A double convex lens, made from a material of refractive index μ_1, is immersed in a liquid of refractive index μ_2, where $\mu_2 > \mu_1$. What change, if any, would occur in the nature of the lens ? [CBSE Sample Paper 08]
72. A glass lens of reflective index 1.5 is placed in a trough of liquid. What must be the refractive index of the liquid in order to make the lens disappear ? [CBSE D 08, 10]
73. A converging lens of refractive index 1.5 is kept in a liquid medium having same refractive index. What would be the focal length of the lens in this medium ? [CBSE D 08]
74. How does the power of a convex lens vary, if the incident red light is replaced by violet light ? [CBSE D 08]
75. A diverging lens of focal length *F* is cut into two identical parts each forming a plano-concave lens. What is the focal length of each part. [CBSE OD 08]
76. Draw a plot showing the variation of power of a lens, with the wavelength of the incident light. [CBSE OD 08]
77. Use lens maker's formula to write an expression for the refractive index, μ of the material in terms of its focal length *f*, and the radii of curvature R_1 and R_2 of its two surfaces. [CBSE OD 07C]
78. Define linear magnification produced by a lens/mirror.
79. Three lenses with magnifications 2, 3 and 10 form a combination. What is its total magnification ? [CBSE F 94 C]
80. What is the purpose of adding 'blue' to clothes ?
81. What is a prism ?
82. Define angle of the prism.
83. Define angle of deviation.
84. What is the effect on a ray of light passing through a prism ?
85. Name the factors on which the angle of deviation produced by a prism depends.
86. Define angle of minimum deviation.
87. Write the relationship between angle of incidence '*i*', angle of prism '*A*' and angle of minimum deviation for a triangular prism. [CBSE D 13]
88. Write the relation for the refractive index of the prism in terms of the angle of minimum deviation and the angle of prism. [CBSE OD 03C ; D 10]
89. What is dispersion of light ? [CBSE D 93C ; Punjab 04]
90. State the factors on which dispersive power of a prism depends.

91. What are the factors on which angular dispersion of a prism depends ?

92. For which colour, the refractive index of prism material is (*i*) minimum and (*ii*) maximum ? [D 10]

93. Which colour is deviated (*i*) most (*ii*) least, on passing through a prism ?

94. Does the angle of minimum deviation produced by a prism depend on wavelength ?

95. Out of red and blue lights, for which colour is the refractive index of glass greater ? [CBSE OD 99C]

96. A glass prism is held in water. How is the angle of minimum deviation affected ?

97. When does a ray passing through a prism deviate away from its base ?

98. Define dispersive power (for light) of a medium.

99. A monochromatic ray of light is made to fall on a normal 60° prism under minimum deviation condition. What is the relation between the angle of incidence and the angle of emergence ? [ISCE 98]

100. Draw a properly labelled graph between the angle of incidence and the angle of deviation for a prism and show the point of minimum deviation. [ISCE 96 ; CBSE OD 09]

101. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced with red light ? [CBSE OD 08]

102. How does the angle of minimum deviation of a glass prism of refractive index 1.5 change, if it is immersed in a liquid of refractive index 1.3 ? [CBSE OD 08]

103. Violet colour is seen at the bottom of the spectrum when white light is dispersed by a prism. [D 10]

104. State Rayleigh's law of scattering.

105. What is monochromatic light ? Give one example of a source of monochromatic light.

106. What is angular size of an object or image ?

107. What is simple microscope ?

108. What is the magnification produced by a single convex lens used as a simple microscope in normal use ? [ISCE 95]

109. If the final image is formed at infinity, what is the magnification of a simple microscope ?

110. What is the eye-ring of a microscope or a telescope ?

111. What is the nature of the final image in a compound microscope ?

112. What can we say about the length of a compound microscope if the final image is formed at infinity ?

113. What do you mean by normal adjustment of telescope ?

114. What is the distance between the objective and the eyepiece of a telescope in normal adjustment ?

Or

What is the length of the telescope in normal adjustment ? [CBSE OD 03]

115. Express the angular magnification of an astronomical telescope in terms of the focal length of the objective and the eyepiece. [ISCE 94]

116. An astronomical telescope set for normal adjustment has a magnifying power 10. If the focal length of the objective is 1.2 m, what is the focal length of the eyepiece ? [ISCE 01]

117. In which device–microscope or telescope, the difference in the focal lengths of the two lenses is larger ?

118. A convex lens of focal length f_1 is kept in contact with a concave lens of focal length f_2. Find the focal length of the combination. [CBSE OD 13]

119. A biconvex lens made of a transparent material of refractive index 1.25 is immersed in water of refractive index 1.33. Will the lens behave as a converging or a diverging lens ? Give reason. [CBSE OD 14]

120. Redraw the diagram given below and mark the position of the centre of curvature of the spherical mirror used in the given set up. [CBSE SP 15]

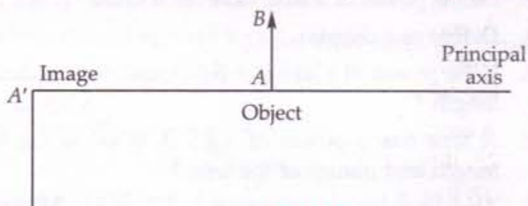


Fig. 9.212

121. A biconvex lens made of a transparent material of refractive index 1.5 is immersed in water of refractive index 1.33. Will the lens behave as a converging or a diverging lens ? Give reason. [CBSE OD 14]

122. A convex lens is placed in contact with a plane mirror. A point object at a distance of 20 cm on the axis of this combination has its image coinciding with itself. What is the focal length of the lens ? [CBSE D 14]

Answers

1. Our eye is most sensitive to wavelength λ = 5500 Å. This wavelength lies in the yellow-green region of the visible spectrum.

2. Angle of incidence = 0°, Angle of reflection = 0°.

3. The image moves towards the person with velocity 2 *v*.

4. 20°, because the reflected ray turns through twice the angle through which the plane mirror is rotated.
5. Infinity.
6. For parallel plane mirrors, $\theta = 0°$, therefore,
$$n = \frac{360}{\theta} - 1 = \infty$$
7. $n = \frac{360}{45} - 1 = 8 - 1 = 7$ **images**.
8. The minimum size (vertical length) of the plane mirror should be equal to half the height of the man.
9. Six images. The two adjacent walls inclined at 90° will make three images and the ceiling will repeat them.
10. A spherical mirror is a reflecting surface which forms part of a hollow sphere. Spherical mirrors are of two types
   (*i*) cancave mirror and (*ii*) convex mirror.
11. A convex mirror is called a divergent mirror because it diverges a parallel beam of light incident on it.
12. A narrow beam of light parallel to the principal axis either actually converges to or appears to diverge from a point *F* on the principal axis after reflection from the spherical mirror. This point is called principal focus of the mirror.
13. (*i*) A concave mirror has a real focus.
   (*ii*) A convex mirror has a virtual focus.
14. Convex mirror.
15. A concave mirror, because it forms an erect and enlarged image when the object is placed between the focus and the mirror.
16. A cancave mirror, when the object is placed beyond 2*F*, it forms a real and diminished image.
17. (*i*) Real and
   (*ii*) magnified.
18. No. A convex mirror always forms a virtual image which cannot be obtained on a screen.
19. Yes, because the rays diverging from the virtual image are real and can be focused.
20. The object should be placed at 40 cm from the mirror.
21. A ray of light from the centre of curvature falls normally on the spherical mirror. So its angle of incidence is 0°.
22. Size of the image increases and image shifts towards the pole of the mirror.
23. For the concave mirror of aperture as large as 4 cm, all the incident rays are not likely to be paraxial.
24. Refraction is the phenomenon of the change in path of light as it passes from one transparent medium to another.
25. The refractive index of a medium for a light of given wavelength may be defined as the ratio of the speed of light in vacuum to its speed in that medium.
$$\text{Refractive index} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$
or $$\mu = \frac{c}{v}.$$
26. The ratio of the wavelength of light in vacuum to its wavelength in a medium is called refractive index of that medium.
$$\mu = \frac{\lambda_{vac}}{\lambda_{med}}.$$
27. The relative refractive index of medium 2 with respect to medium 1 is defined as the ratio of speed of light (v_1) in medium 1 to the speed of light (v_2) in medium 2. It is given by
$$\mu_2 = \frac{v_1}{v_2}.$$
28. Refractive index of a medium depends on
   (*i*) Nature of the medium
   (*ii*) Wavelength of light used
   (*iii*) Temperature
   (*iv*) Nature of surrounding medium.
29. According to Snell's law, the ratio of the sine of the angle of incidence and the sine of the angle of refraction is constant for a given pair of media. This constant is called *refractive index* (μ) of second medium w.r.t. first medium. Mathematically,
$$\frac{\sin i}{\sin r} = \mu, \text{ a constant.}$$
30. Snell's law of refraction fails when light is incident normally on the surface of a refracting medium. In such a situation $i = 0$ and also $r = 0$. The ratio $\sin i / \sin r$ becomes meaningless.
31. (*i*) Refractive index is minimum for vacuum ($\mu = 1$).
   (*ii*) Refractive index is maximum for diamond.
32. The sidewise shift in the path of light on emerging from a refracting medium with parallel faces is called lateral shift.
33. Lateral shift depends on angle of incidence, the refractive index and thickness of the refracting medium.
34. For $i = 0°$, lateral shift is zero.

35. For $i = 90°$, lateral shift is maximum and is equal to the thickness of the slab.

$$d = \frac{t \sin (i - r)}{\cos r}$$

$$d_{max} = \frac{t \sin (90° - r)}{\cos r} = \frac{t \cos r}{\cos r} = t.$$

36. Frequency in air,

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14} \text{ Hz}$$

The frequency of light remains same as it travels form air to the given medium.

∴ Frequency in medium = $\mathbf{5 \times 10^{14}}$ **Hz.**

37. The frequency does not change when light undergoes refraction.

38. The wavelength changes when light undergoes refraction from one medium to another.

39. Frequency of the ultraviolet light remains unchanged.

40. The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° is called critical angle (i_c) of the denser medium.

41. No, it cannot occur.

42. $${}^a\mu_g = \frac{\text{Speed of light in air}}{\text{Speed of light in glass}} = \frac{3 \times 10^8}{2 \times 10^8} = \frac{3}{2}$$

$$\sin i_c = \frac{1}{{}^a\mu_g} = \frac{2}{3}$$

$$\therefore \quad i_c = \sin^{-1}\left(\frac{2}{3}\right) = \mathbf{41°49'}.$$

43. $\sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$

∴ Critical angle, $i_c = \mathbf{45°}$

44. $\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2}.$

45. Water.

46. 100%.

47. $\mu = \frac{1}{\sin i_c}$.

48. $i_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$.

49. The necessary conditions for total internal reflection are

(i) Light must travel from denser to rarer medium.

(ii) The angle of incidence in the denser medium must be greater than the critical angle for the two media.

50. An optical fibre is a thread-like structure of quality glass or quartz which enables a beam of light to travel through several kilometres without any appreciable loss of intensity.

51. The value of the refractive index of the core material is higher than that of the cladding.

52. The working of an optical fibre is based on the phenomenon of total internal reflection.

53. The optical fibres are mainly used for transmitting optical frequencies.

54. ${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$.

55. A lens is a piece of refracting medium bounded by two surfaces at least one of which is a curved surface.

56. It is a point situated within the lens through which a ray of light passes undeviated.

57. 0°.

58. It behaves like a biconvex lens.

59. No, it cannot be used as a lens because there would be no refraction of light.

60. Concave lens.

61. The object should be placed between the optical centre and the focus of the biconvex lens.

62. The object should be placed at a distance equal to $2f$ from the lens. This cannot happen in a concave lens which always forms a diminished image.

63. The power of a lens is defined as the reciprocal of its focal length expressed in metres.

$$P = \frac{1}{f \text{ (in m)}} = \frac{100}{f\text{(in cm)}}$$

64. One dioptre is the power of a lens whose principal focal length is 1 metre.

65. Focal length, $f = \frac{1}{P} = \frac{1}{+5} = +\,\mathbf{0.2\ m}$

66. Focal length, $f = \frac{1}{P} = \frac{1}{-2.5 \text{ m}} = -\,\mathbf{40\ cm}$.

The negative sign shows that the lens is concave.

67. $P = P_1 + P_2 = +4 - 2 = +2$ D

$$f = \frac{1}{P} = \frac{1}{+2} \text{ m} = +\,\mathbf{50\ cm}.$$

68. Image is formed at infinity.

69. Yes, each part of the lens will form full image. But the intensity of the image is reduced.

70. $\mu_g < \mu_m$.

71. The lens would behave as a diverging lens when immersed in the liquid.

$$\frac{1}{f_l} = \left(\frac{\mu_1}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When $\mu_2 > \mu_1$, f_l is negative.

72. The refractive index of liquid must be equal to 1.5 *i.e.*, equal to that of glass lens.

73. $\frac{1}{f} = ({}^1\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1.5}{1.5} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$

or $f = \infty$.

74. Power of the convex lens increases, because $P \propto (\mu - 1)$ and $\mu_V > \mu_R$.

75. Focal length of each part will be $2F$.

For original concave lens,

$$\frac{1}{F} = (\mu - 1)\left(-\frac{1}{R} - \frac{1}{R}\right) = -\frac{2(\mu - 1)}{R}$$

For each half lens,

$$\frac{1}{F'} = (\mu - 1)\left(-\frac{1}{R} - \frac{1}{\infty}\right) = -\frac{(\mu - 1)}{R}$$

On dividing,

$$\frac{F'}{F} = 2 \quad \text{or} \quad F' = 2F.$$

76. Power of a lens, $P \propto (\mu - 1)$. As μ of a material decreases with the increase in wavelength λ, so the graph between P and λ is of type shown in Fig. 9.213.

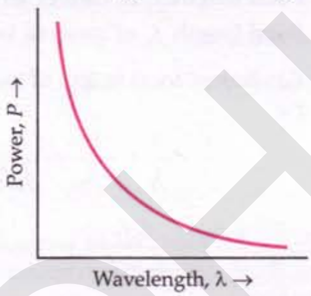


Fig. 9.213

77. By lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\therefore \quad \mu = 1 + \frac{1}{f\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}.$$

78. The ratio of the size of the image to the size of the object is called linear magnification (m).

$$m = \frac{\text{size of image}}{\text{size of object}} = \frac{h_2}{h_1}$$

79. Total magnification,

$$M = m_1 \times m_2 \times m_3 = 2 \times 3 \times 10 = \mathbf{60}.$$

80. When washed, the clothes get a yellowish tint. Blue and yellow are complimentary colours and they give white colour.

81. Any portion of a refracting medium bounded by two plane faces inclined to each other at a certain angle is called a prism.

82. The angle between the refracting faces of a prism is called angle of the prism.

83. The angle between the incident ray and the emergent ray is called angle of deviation (δ).

84. It bends towards the base of the prism.

85. The angle of deviation produced by prism depends on (*i*) Angle of incidence (*ii*) Material of the prism (*iii*) Wavelength of light used (*iv*) Angle of the prism.

86. The minimum value of the angle of deviation suffered by a ray of light on passing through a prism is called angle of minimum deviation (δ_m).

87. Angle of minimum deviation,

$$\delta_m = 2i - A$$

88. $\mu = \dfrac{\sin \dfrac{A + \delta_m}{2}}{\sin \dfrac{A}{2}}$

89. Dispersion is the phenomenon of splitting of white light into the constituent colours on passing through a prism.

90. (*i*) Nature of the prism material (*ii*) Choice of extreme colours for which dispersive power is to be measured.

91. (*i*) Angle of prism (*ii*) Nature of prism material.

92. Refractive index of prism material is (*i*) minimum for red colour (*ii*) maximum for violet colour.

93. (*i*) Violet colour is deviated most (*ii*) Red colour is deviated least, on passing through a prism.

94. Yes, it depends on the wavelength of light.

95. $\mu_B > \mu_R$, because $\lambda_B < \lambda_R$.

96. When the prism is held in water,

$${}^w\mu_g = \frac{\sin \dfrac{A + \delta_m}{2}}{\sin \dfrac{A}{2}}$$

As ${}^w\mu_g < {}^a\mu_g$, so the angle of minimum deviation decreases in water.

97. This happens when the prism is immersed in a transparent medium having refractive index greater than that of the prism material.

98. Dispersive power is defined as the ratio of the angular dispersion to the mean deviation.

$$\omega = \frac{\delta_V - \delta_R}{\delta}.$$

99. In the minimum deviation condition,

Angle of incidence = Angle of emergence.

100. See Fig. 9.117(*b*) on page 9.67.

101. As $\delta = (\mu - 1)A$ and $\mu_R < \mu_V$, so the angle of minimum deviation decreases when incident violet light is replaced with red light.

102. In air, $^a\mu_g = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}} = 1.5$

In water $^w\mu_g = \dfrac{\sin\dfrac{A+\delta'_m}{2}}{\sin\dfrac{A}{2}} = \dfrac{1.5}{1.3}$

As $^w\mu_g < {^a\mu_g}$, so $\delta'_m < \delta_m$ *i.e.*, angle of minimum deviation decreases when the prism is immersed in a liquid of $\mu = 1.3$.

103. The deviation produced by a small angled prism, $\delta = (\mu - 1)A$. As μ_V has maximum value, so violet colour is bent must by the glass prism.

104. According to Rayleigh's law of scattering, the intensity of light of wavelength λ present in the scattered light is inversely proportional to wavelength λ.

Mathematically,

$$I \propto \frac{1}{\lambda^4}.$$

105. A light of single wavelength is called monochromatic light. The commonly used source of monochromatic light is a sodium lamp.

106. It is the angle subtended by the object or image at the eye when placed at the least distance of distinct vision.

107. A simple microscope is a convex lens of short focal length. It forms a magnified image when the object is placed between its focus and optical centre.

108. $m = 1 + \dfrac{D}{f}$.

109. $m = \dfrac{D}{f}$.

110. The image of the objective in the eyepiece is known as eye-ring. All the rays from the object refracted by the objective go through the eye-ring. So it is an ideal position for our eyes for viewing.

111. In a compound microscope, the final image is inverted with respect to the object. It is virtual and magnified.

112. The length of the compound microscope is greater than $f_0 + f_e$.

113. When the final image is formed at infinity, the telescope is said to be in normal adjustment.

114. Length of telescope in normal adjustment

$$= f_0 + f_e.$$

115. When the final image is formed at the least distance of distinct vision,

$$m = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

When the final image is formed at infinity,

$$m = -\frac{f_0}{f_e}.$$

116. As $m = \dfrac{f_0}{f_e}$ (magnitude in normal adjustment)

$$\therefore \quad f_e = \frac{f_0}{m}$$

$$= \frac{1.2 \text{ m}}{10} = \mathbf{0.12 \text{ m}}.$$

117. In a telescope, the difference in the focal lengths of the two lenses is larger.

118. Focal length f_1 of convex lens is positive.

Focal length f_2 of concave lens in negative.

Equivalent focal length of the combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{-f_2} = \frac{f_2 - f_1}{f_1 f_2}$$

$$\therefore \quad f = \frac{f_1 f_2}{f_2 - f_1}$$

119. Diverging lens, because the light rays diverge on refraction from rarer to denser medium.

120. The required ray diagram is shown below :

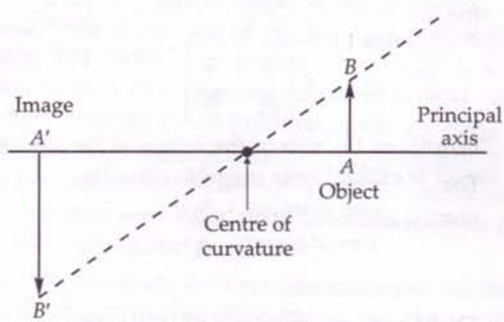


Fig. 9.214

121. Converging lens. Light rays get converged on refraction from denser to a rarer medium.

122. 20 cm. For explanation, refer to the solution of Example 83 on page 9.63.

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. What is optics ? What are its two main branches ?
2. Prove that for a concave mirror the radius of curvature is twice the focal length. [CBSE OD 96]
3. With the help of a suitable ray diagram, derive the mirror formula for a concave mirror. [CBSE OD 09]
4. What is meant by linear magnification of an image ? Using Cartesian sign convention, write the expression for the linear magnification in terms of the object and image distance for (*a*) a concave mirror and (*b*) a convex mirror. What is the meaning of sign of magnification ?
5. What is spherical aberration ? How can it be removed ?
6. Give some practical applications of spherical and parabolic mirrors.
7. Define refractive index of a material. Give its physical significance.
8. Distinguish between absolute refractive index and relative refractive index of a material. Write a relation between these refractive indices.
9. Explain the cause of refraction of light.
10. A ray of light bends towards normal as it passes from air to glass. Give reason.
11. State the principle of reversibility of light. Hence prove that ${}^1\mu_2 = \frac{1}{{}^2\mu_1}$.
12. Discuss the refraction through a glass-slab and show that the emergent ray is parallel to the incident ray but laterally displaced. [Himachal 96]
13. A ray of light is incident at angle i on a rectangular slab of thickness t and refractive index μ. Show that the lateral displacement of the emergent ray is

$$x = t \sin i \left[1 - \frac{\cos i}{(\mu^2 - \sin^2 i)^{1/2}}\right]$$

Can x exceed t ?

14. For a ray of light suffering refraction through a combination of three media, show that

$${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1$$

15. Deduce the relation, $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$.
16. Explain why does a water tank appear shallower ?
17. An object placed at the bottom of a beaker containing water appears to be raised. Why ?
18. Explain, with the help of a diagram, how is the phenomenon of total internal reflection used in
    (*i*) an optical fibre
    (*ii*) a prism that inverts an image without changing its size. [CBSE Sample Paper 15]
19. The sun near the horizon appears flattened at sunset and sunrise. Why ?
20. State the conditions for total internal reflection of light to take place at an interface separating two transparent media. Hence derive the expression for the critical angle in terms of the speeds of light in the two media. [CBSE D 2000]
21. What is critical angle ? Give one application of total internal reflection. [Haryana 02]
22. What are optical fibres ? How are light waves propagated in them ? Write their any two uses. [Himachal 02, 04 ; Haryana 02, 04]
23. Explain briefly, with a ray diagram, how a mirage is formed in deserts. [Haryana 01 ; CBSE D 98C]
24. Why does a diamond sparkle ? Is it a source of light ? [Punjab 01, 02]
25. Give four advantages of totally reflecting prisms over plane mirrors.
26. Give reasons for the following observations made from the earth : (*i*) Sun is visible before the actual sunrise. (*ii*) Sun looks reddish at sunset or sunrise. [CBSE D 2000, 02 ; OD 06C]
27. Draw a ray diagram to show the formation of the image of an object placed between f and $2f$ of a thin convex lens. Deduce the relationship between the object distance, image distance and focal length under the conditions stated.
28. Two thin convex lenses L_1 and L_2 of focal lengths f_1 and f_2 respectively, are placed coaxially in contact. An object is placed at a point beyond the focus of lens L_1. Draw a ray diagram to show the image formation by the combination and hence derive the expression for the focal length of the combined system. [CBSE OD 15]
29. Derive the lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a concave lens, using the necessary ray diagram. [CBSE OD 08]
30. Derive the expression for the angle of deviation for a ray of light passing through an equilateral prism of refracting angle 'A'. [CBSE D 93]
31. (*a*) A ray of light falls on a triangular glass prism in such a way that the deviation of the emergent ray is minimum for that prism. Draw the ray diagram for this case and write the relation between the angle of incidence and angle of emergence.

(b) A ray of light falls on a transparent right-angled isosceles prism made from a glass of refractive index $\sqrt{2}$. Draw the ray diagram for this prism when the incident ray falls normally on one of the equal sides of this prism. [CBSE D 08C]

32. Derive the expression for the refractive index of the material of the prism in terms of the angle of the prism and angle of minimum deviation. [CBSE D 06C]

33. Write the relation between the angle of incidence (i), the angle of emergence (e), the angle of prism (A) and the angle of deviation (δ) for rays undergoing refraction through a prism. What is the relation between $\angle i$ and $\angle e$ for rays undergoing minimum deviation ? Using this relation, write the expression for the refractive index (μ) of the material of a prism in terms of $\angle A$ and the angle of minimum deviation (δ_m). [CBSE Sample Paper 08]

34. Draw a graph to show the variation of the angle of deviation 'δ' with that of the angle of incidence 'i' for a monochromatic ray of light passing through a glass prism of refracting angle 'A'. Hence deduce the relation

$$\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

[Haryana 04 ; CBSE D 02C, 04C ; OD 03]

35. Derive an expression for the angle of deviation of a small prism in terms of the refractive index and the angle of the prism. [ISCE 96]

36. Draw an appropriate ray diagram to show the passage of a 'white ray', incident on one of the two refracting faces of a prism. State the relation for the angle of deviation, for a prism of small refracting angle. [CBSE Sample Paper 13]

37. What is dispersion of light ? Explain it with a ray diagram. Also explain the cause of dispersion of light. [Punjab 99C, 02]

38. Define the term angular dispersion. Draw the path of a ray of white light passing through prism and mark angular dispersion on it. [CBSE SP 97]

39. Explain the terms angular dispersion and dispersive power. How are the two related ? [Haryana 01]

40. Write the conditions for observing a rainbow. Show, by drawing suitable diagrams, how one understands the formation of a rainbow. [CBSE OD 14C]

41. Draw a neat ray diagram of a simple microscope. Deduce the formula for its angular magnification when the image is formed at the least distance of distinct vision. [ISCE 2000]

42. With the help of a ray diagram, explain the working of a simple microscope when the image is formed at infinity. Write an expression for its magnifying power.

43. Draw a ray diagram showing the image formation by a compound microscope. Obtain expression for total magnification when the images is formed at infinity. [CBSE OD 14C, 15C]

44. (a) Draw a ray diagram showing the image formation by a compound microscope.
(b) Derive expression for total magnification when the image is formed at infinity.
(c) Why is the objective of a compound microscope of short aperture and short focal length ? Give reason. [CBSE D 13, F 13]

45. Draw the course of rays in an astronomical telescope, when the final image is formed at the least distance of distinct vision. Also define and write an expression for the magnifying power in this position. [CBSE OD 09, 13]

46. (a) Draw a labelled diagram of refraction type telescope in normal adjustment.
(b) Give its two shortcomings over reflection type telescope.
(c) Why is eyepiece of a telescope of short focal length, while objective is of large focal length ? Explain. [CBSE D 08 ; OD 04 ; F 13]

47. Draw a labelled ray diagram of an astronomical telescope of the near point adjustment. You are given three lenses of power 0.5 D, 4 D, 10 D. State, with reason, which two lenses will you select for constructing a good astronomical telescope. [CBSE D 06C]

48. Two monochromatic rays of light are incident normally on the face AB of an isosceles right-angled prism ABC. The refractive indices of the glass prism for the two rays '1' and '2' are respectively 1.35 and 1.45. Trace the path of these rays after entering through the prism. [CBSE OD 14]

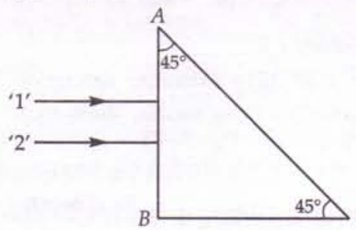


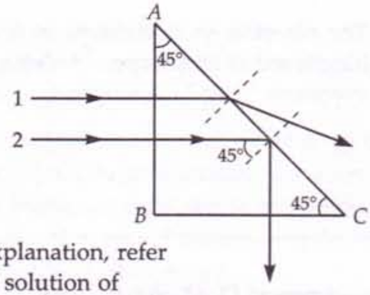
Fig. 9.215

49. Draw a labelled ray diagram to show the image formation in a reflecting type telescope. Write its two advantages over a refracting type telescope. On what factors does its resolving power depend ? [CBSE D 06, 08 ; OD 12]

50. (*a*) Draw a labelled diagram of a reflecting type telescope.
(*b*) Write two important advantages justifying why reflecting type telescopes are preferred over refracting telescopes.
(*c*) The objective of a telescope is of larger focal length and of larger aperture (compared to the eyepiece). Why ? Give reasons. [CBSE F 13]

Answers

1. Refer answer to Q. 1 on page 9.1.
2. Refer answer to Q. 6 on page 9.3.
3. Refer answer to Q. 9(*a*) on page 9.5.
4. Refer answer to Q. 11 on page 9.7.
5. Refer answer to Q. 12 on page 9.8.
6. Refer answer to Q. 13 on page 9.8.
7. Refer answers to Q. 16 and Q. 19 on page 9.15.
8. Refer answer to Q. 16 on page 9.14. The relative refractive index of any medium with respect of vacuum is called its absolute refractive index.
9. Refer answer to Q. 18 on page 9.15.
10. Refer solution to Problem 14 on page 9.103.
11. Refer answer to Q. 20 on page 9.15.
12. Refer answer to Q. 21 on page 9.16.
13. Refer answer to Q. 22 on page 9.16.
14. Refer answer to Q. 23 on page 9.17.
15. Refer answer to Q. 24 on page 9.17.
16. Refer answer to Q. 24 on page 9.17.
17. Refer answer to Q. 24 on page 9.17.
18. (*i*) See Fig. 9.39(*a*) and its explanation.
(*ii*) See Fig. 9.37 and its explanation.
19. Refer answer to Q. 26 on page 9.18.
20. Refer to points 21 and 22 of Glimpses and the solution of Problem 12 on page 9.112.
21. The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° is called critical angle. Diamonds sparkle due to the phenomenon of total internal reflection.
22. Refer answer to Q. 31 on page 9.24 and Q. 32 on page 9.25.
23. Refer answer to Q. 28 on page 9.23.
24. Refer answer to Q. 28 on page 9.23. No, diamond is not a source of light. It accumulates light due to multiple internal reflections.
25. Refer answer to Q. 30 on page 9.24.
26. (*i*) Refer answer to Q. 25 on page 9.17.
(*ii*) Refer solution to Problem 66 on page 9.108.
27. Refer answer to Q. 42 on page 9.47.
28. Refer answer to Q. 48 on page 9.57.
29. Refer answer to Q. 44 on page 9.48.
30. Refer answer to Q. 51 on page 9.67.
31. (*a*) See Fig. 9.175 (*b*) See Fig. 9.178.
32. Refer answer to Q. 52 on page 9.67.
33. Refer answer to Q. 52 on page 9.67.
34. Refer answer to Q. 52 on page 9.67.
35. Refer answer to Q. 53 on page 9.68.
36. See Fig. 9.118 on page 9.68.
Angle of deviation,
$$\delta = (\mu - 1) A.$$
37. Refer answer to Q. 54 on page 9.68.
38. Refer answer to Q. 56 on page 9.69.
39. Refer answer to Q. 56 on page 9.69.
40. Refer answer to Q. 65 on page 9.80.
41. Refer answer to Q. 79 on page 9.87.
42. Refer answer to Q. 79 on page 9.87.
43. Refer answer to Q. 80 on page 9.91.
44. (*a*) See Fig. 9.146 on page 9.92.
(*b*) Refer answer to Q. 80 (*b*) on page 9.92.
(*c*) The objective of smaller aperture produces a highly bright image while the objective of short focal length produces large angular magnification.
45. Refer answer to Q. 82 on page 9.95.
46. (*a*) See Fig. 9.149 on page 9.97.
(*b*) **Drawbacks of astronomical telescopes :**
(*i*) The large objective lens used is very heavy, which is difficult to make and support by its edges.
(*ii*) It is difficult and expensive to make large size lenses free from chromatic aberration and distortions.
(*c*) When $f_0 >> f_e$, the telescope will have large magnifying power.
47. See Fig 9.147 on page 9.95.
For constructing astronomical telescope, the lens of 0.5 D should be used as objective because of its larger focal length and lens of 10 D should be used as eyepiece because of its smallest focal length.

48. 

For explanation, refer to the solution of Problem 3 on page 9.121.

Fig. 9.216

49. Refer answer to Q. 84 on page 9.97 and Q. 86 on page 9.98.

50. (*a*) See Fig. 9.51 on page 9.98.

(*b*) Refer answer to Q. 86 on page 9.98.

(*c*) The objective of larger focal length produces high angular magnification while that of larger aperture has a high resolving power.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. (*a*) With the help of a suitable ray diagram, derive the mirror formula for a concave mirror. [CBSE OD 09]

(*b*) Draw a ray diagram to show the image formation by a concave mirror when the object is kept between its focus and the pole. Using this diagram, derive the magnification formula for the image formed. [CBSE D 11]

2. By stating the sign conventions and assumptions made, derive mirror formula for a convex mirror. [Punjab 99, 2000 ; CBSE OD 97]

3. (*a*) For a ray of light travelling from a denser medium of refractive index n_1 to a rarer medium of refractive index n_2, prove that $\frac{n_2}{n_1} = \sin i_c$, where i_c is the critical angle of incidence for the media.

(*b*) Explain with the help of a diagram, how the above principle is used for transmission of video signals using optical fibres.

4. With the help of a ray diagram explain the phenomenon of total internal reflection. Obtain the relation between critical angle and the refractive index of the medium.

Draw ray diagrams to show how a right angled isosceles prism can be used to

(*i*) deviate the ray through 180°,

(*ii*) deviate the ray through 90°, and

(*iii*) invert the ray. [CBSE D 01C]

5. A spherical surface of radius of curvature R, separates a rarer and a denser medium as shown in Fig. 9.217.

Complete the path of the incident ray of light, showing the formation of a real image. Hence derive the relation connecting object distance 'u', image distance 'v', radius of curvature R and the refractive indices n_1 and n_2 of the two media.

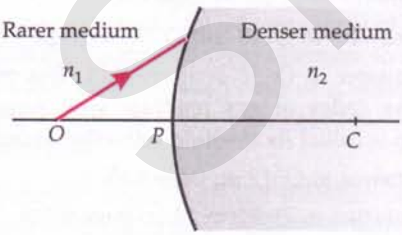


Fig. 9.217

Briefly explain, how the focal length of a convex lens changes, with increase in wavelength of incident light. [CBSE OD 04]

6. A spherical surface 'of radius of curvature R' and of refractive index μ_2 is placed in a medium of refractive index μ_1, where $\mu_1 < \mu_2$. The surface produces a real image of an object kept in front of it. Using appropriate assumptions and sign conventions, derive a relationship between the object distance, image distance, R, μ_1 and μ_2. Under what conditions this surface diverges a ray incident on it ? [CBSE Sample Paper 03]

7. (*a*) A point object 'O' is kept in a medium of refractive index n_1 in front of a convex spherical surface of radius of curvature R which separates the second medium of refractive index n_2 from the first one, as shown in the figure.

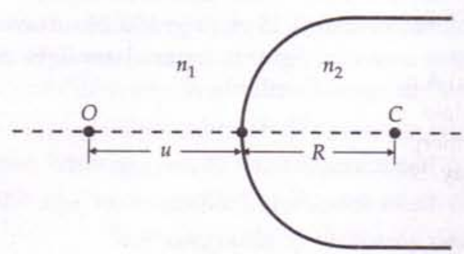


Fig. 9.218

Draw the ray diagram showing the image formation and deduce the relationship between the object distance and the image distance in terms of n_1, n_2 and R.

(*b*) When the image formed above acts as a virtual object for a concave spherical surface separating the medium n_2 from n_1 ($n_2 > n_1$), draw this ray diagram and write the similar (similar to (*a*)) relation. Hence obtain the expression for the lens maker's formula. [CBSE D 15]

8. Draw a ray diagram showing the formation of the image by a point object on the principal axis of a spherical convex surface separating two media of refractive indices n_1 and n_2, when a point source is kept in rarer medium of refractive index n_1. Derive the relation between object and image distance in terms of refractive index of the medium and radius of curvature of the surface. Hence obtain the expression for lens-maker's formula in the case of thin convex lens. [CBSE D 09, 14, 14C]

9. Derive expression for the lens maker's formula, *i.e.*,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

where the symbols have their usual meanings.State the assumptions used and the convention of signs used. [CBSE OD 96C ; Punjab 03 ; Himachal 99 ; Haryana 01]

10. A point object is placed in front of a double convex lens (of refractive index $n = n_2 / n_1$ with respect to air) with its spherical faces of radii of curvature R_1. and R_2. Show the path of rays due to refraction at first and subsequently at the second surface to obtain the formation of the real image of the object.

Hence obtain the lens-marker's formula for a thin lens. [CBSE F 09, 13 ; OD 14 C]

11. Draw a ray diagram to show the formation of image of an object placed between the optical centre and focus of the convex lens. Write the characteristics of image formed. Using this diagram, derive the relation between object distance, image distance and focal length of the convex lens. Write the assumptions and convention of signs used. Draw the graph showing the variation of v and u. [CBSE D 99C, 03]

12. (*a*) A ray '*PQ*' of light is incident on the face *AB* of a glass prism *ABC* (as shown in Fig. 9.219) and emerges out of the face *AC*. Trace the path of the ray. Show that

$$\angle i + \angle e = \angle A + \angle \delta$$

where δ and e denote the angle of deviation and angle of emergence respectively.

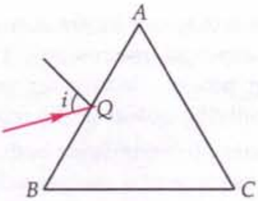


Fig. 9.219

Plot a graph showing the variation of the angle of deviation as a function of angle of incidence. State the condition under which $\angle\delta$ is minimum.

(*b*) Find out the relation between the refractive index (μ) of the glass prism and $\angle A$ for the case when the angle of prism (A) is equal to the angle of minimum deviation (δ_m). Hence obtain the value of the refractive index for angle of prism $A = 60°$. [CBSE OD 15]

13. (*a*) Draw a ray diagram to show refraction of a ray of monochromatic light passing through a glass prism.

Deduce the expression for the refractive index of glass in terms of angle of prism and angle of minimum deviation.

(*b*) Explain briefly how the phenomenon of total internal reflection is used in fibre optics. [CBSE D 11]

14. Trace the path of a monochromatic ray of light through a prism of refracting angle '*A*'. Draw a graph to show the variation of angle of deviation 'δ' with the variation of angle of incidence '*i*'.

Deduce the relation

$$\mu = \frac{\sin\frac{\delta_m + A}{2}}{\sin\frac{A}{2}}$$

where terms μ, δ_m have their usual meaning. [CBSE F 08]

15. What is rainbow ? Differentiate between primary rainbow and secondary rainbow with a diagram. Why two observers do not see the same rainbow ? [Punjab 01]

16. You are given two convex lenses of short aperture having focal lengths 4 cm and 8 cm respectively. Which one of these will you use as an objective and which one as an eyepiece for constructing a compound microscope ? Draw a ray diagram to show the formation of the image of a small object due to a compound microscope. Derive an expression for its magnifying power. [CBSE D 01C]

17. (a) Draw a diagram for the formation of image by a compound microscope. Define its magnifying power. Deduce the expression for the magnifying power of the microscope.
(b) Explain : (i) Why must both the objective and the eyepiece of a compound microscope have short focal lengths ? (ii) While viewing through a compound microscope, why should our eyes be positioned not on the eyepiece but a short distance away from it for best viewing.
[CBSE F 08 ; D 09 ; OD 10]

18. Draw a ray diagram for a compound microscope. Derive an expression for the magnifying power when the final image is formed at the least distance of distinct vision. State the expression for the magnifying power when the image is formed at infinity. Why is the focal length of the objective lens of a compound microscope kept quite small ?
[CBSE Sample Paper 11]

19. Draw a ray diagram to show the working of a compound microscope. Deduce an expression for the total magnification when the final image is formed at the near point.
In a compound microscope, an object is placed at a distance of 1.5 cm from the objective of focal length 1.25 cm. If the eye piece has a focal length of 5 cm and the final image is formed at the near point, estimate the magnifying power of the microscope.
[CBSE D 10]

20. With the help of a ray diagram, explain the formation of image in an astronomical telescope for a distant object. Define the term magnifying power of a telescope. Derive an expression for its magnifying power when the final image is formed at the least distance of distinct vision. [CBSE OD 2000C]

21. Draw a ray diagram for the formation of image of a distant object by an astronomical telescope in normal adjustment position. Deduce the expression for its magnifying power. Write two basic features which can distinguish between a telescope and a compound microscope. [CBSE D 03C ; OD 04C, 14C ; F 09]

22. Draw a ray diagram showing the image formation of a distant object by a refracting telescope. Define its magnifying power and write the two important factors considered to increase the magnifying power.
Describe briefly the two main limitations and explain how far these can be minimized in a reflecting telescope. [CBSE F 15]

Answers

1. (a) Refer answer to Q. 9(b) on page 9.6.
(b) See Fig. 9.14 on page 9.6.

$\Delta MPF \sim A'B'F'$, therefore

$$\frac{A'B'}{MP} = \frac{FB'}{FP}$$

or $$\frac{A'B'}{AP} = \frac{FP + PB'}{FP}$$

Applying the new Cartesian sign convention, we get

$$A'B' = +h_2, \quad AB = +h_1, \quad FP = -f, \quad PB' = v$$

$$\therefore \quad \frac{h_2}{h_1} = \frac{-f+v}{-f}$$

or $$m = \frac{h_2}{h_1} = \frac{f-v}{f} = -\frac{v}{u}$$

(Using mirror formula)

2. Refer answer to Q. 10 on page 9.6.
3. (i) Refer to the solution of Problem 13 on page 9.113.
(ii) Refer answer to Q. 21 on page 9.16.
4. Refer answer to Q. 27 on page 9.22 and Q. 29 on page 9.23.
5. Refer answer to Q. 36(i) on page 9.32. Also refer to solution of Problem 24(ii) on page 9.115.
6. Refer answer to Q. 36(i) on page 9.32. When $\mu_1 > \mu_2$, the given surface diverges the rays incident on it.
7. (a) Refer answer to Q. 37 on page 9.34.
(b) Refer answer to Q. 38 on page 9.40.
8. For refraction at a spherical surface, refer answer to Q. 36 on page 9.31.
For lens maker's formula, refer answer to Q. 38 on page 9.40.
9. Refer answer to Q. 38 on page 9.40.
10. Refer answer to Q. 38 on page 9.40.
11. Refer answer to Q. 43 on page 9.47. For graph between u and v, see Fig. 9.82.
12. (a) Refer answer to Q. 51 and see Fig. 9.117(b) on page 9.67.
(b) Refer answer to Q. 52 on page 9.67.
13. (a) Refer answer to Q. 52 on page 9.67.
(b) Refer answer to Q. 21 on page 9.16.
14. Refer answer to Q. 52 on page 9.67.
15. Refer answer to Q. 65 on page 9.80.
Two observers cannot see the same rainbow. Primary rainbow is seen when the rays emerging from the water droplets subtend a mean angle of 41° [= (40° + 42°) / 2] and secondary rainbow is seen

when the emerging rays subtend a mean angle of $53.5° [= (52° + 55°)/2]$. The positions of such droplets which send these rays, depend on the position of the observer. Hence two observers at two different positions do not see the rainbow formed by the same set of droplets.

16. The lens of 4 cm focal length should be used as objective and the lens of 8 cm focal length should be used as eyepiece of the compound microscope. Refer answer to Q. 80 on page 9.91.

17. (*a*) Refer answer to Q. 80 on page 9.91.

(*b*) Refer answer to Exercise 9.32 on page 9.141.

18. Refer answer to Q. 80 on page 9.91.

$$m = m_0 \times m_e = \frac{v_0}{u_0}\left(1+\frac{D}{f_0}\right) = \frac{f_0}{u_0 - f_0}\left(1+\frac{D}{f_e}\right)$$

Angular magnification (m_0) of objective will be large when u_0 is slightly greater than f_0. Now a compound microscope is used for viewing very close objects, so u_0 is small. Consequently, f_0 has to be small.

19. Refer answer to Q. 80 on page 9.91.

Numerical. Here

$$f_0 = 1.25 \text{ cm}, \quad f_e = 5 \text{ cm},$$
$$u_0 = -1.5 \text{ cm}, \quad v_e = -D = -25 \text{ cm}$$

As $$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

$\therefore$ $$\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{1.25} - \frac{1}{1.5}$$
$$= \frac{100}{125} - \frac{10}{15} = \frac{300-250}{375} = \frac{50}{375}$$
$$v_0 = \frac{375}{50} = 7.5 \text{ cm}$$

Magnifying power of the compound microscope,

$$m = \frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right) = \frac{7.5}{-1.5}\left(1+\frac{25}{5}\right) = -5 \times 6 = -30.$$

20. Refer answer to Q. 82 on page 9.95.

21. Refer answer to Q. 82 on page 9.95.

The two important differences between a telescope and a compound microscope are :

(*i*) The aperture of the objective of a microscope is very small while that of the telescope is large.

(*ii*) Both the lenses of a compound microscope have short focal length while the objective of a telescope has large focal length.

22. See Fig. 9.148 on page 9.96.

Magnifying power in the normal adjustment of the telescope is defined as the ratio of the angle subtended at the eye by the final image as seen through the telescope to the angle subtended at the eye by the object seen directly, when both the image and the object lie at infinity.

$$m = \frac{f_0}{f_e}$$

Factors for increasing the magnifying power :

(*i*) Increasing focal length of the objective

(*ii*) Decreasing focal length of the eyepiece.

Limitations of a refracting telescope :

(*i*) Suffers from chromatic aberration

(*ii*) Suffers from spherical aberration

(*iii*) Small magnifying power

(*iv*) Small resolving power.

Advantages of a reflecting telescope :

(*i*) No chromatic aberration, because mirror objective is used.

(*ii*) Spherical aberration can be removed by paraboloidal mirror.

(*iii*) Image is bright because there is no loss of energy due to refraction.

(*iv*) Large mirror provides an easier mechanical support over its entire back surface.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Two students of class XII brought three big plane mirrors in their classroom for science fair. They fixed the three mirrors : one at the ceiling and the other two on the adjacent walls of the room. Every student was able to see six images of himself/herself. Students of other classes also came to see this and felt happy. A student of class X was determined to know the reason behind it. She went to the library, consulted other students and next day came up with the correct answer.

(*a*) What values were depicted by the student of class X ?

(*b*) Give the reason for seeing six images.

2. One day Chetan's mother developed a severe stomach ache all of a sudden. She was rushed to the doctor who suggested for an immediate endoscopy test and gave an estimate of expenditure for the same. Chetan immediately contacted his class teacher and shared the information with her. The

class teacher arranged for the money and rushed to the hospital. On realising that Chetan belonged to a below average income group family, even the doctor offered concession for the test fee. The test was conducted successfully.

Answer the following questions based on the above information :

(*a*) Which principle in optics is made use of in endoscopy ?

(*b*) Briefly explain the values reflected in the action taken by the teacher.

(*c*) In what way do you appreciate the response of the doctor on the given situation ?

[CBSE OD 13]

3. Rama was watching a programme on Moon on the Discovery Channel. He came to know from the observation recorded on the surface of the moon that sunrise and sunset are abrupt there and the sky appears dark from there. He was surprised and determined to know the reason behind it. He discussed it with his Physics teacher next day, who explained him the reason behind it.

(*a*) What were the values being displayed by Rama ?

(*b*) Why are sunrise and sunset are abrupt on the surface of the moon ?

(*c*) Why does the sky appear dark from the moon ?

4. Amit's uncle was finding great difficulty in reading a book placed at normal place. He was not going to the doctor because he could not afford the cost. When Amit came to know of it, he took his uncle to the doctor. After thoroughly checking his eyes, the doctor prescribed the proper lenses for him. Amit bought the spectacles for his uncle from his pocket money. By using spectacles he could now read with great ease. For this he expressed his gratitude to his nephew.

Based on the above paragraph, answer the following :

(*a*) (*i*) Why does least distance of distinct vision increase with age ?

(*ii*) What type of lens is required to correct this defect ?

(*b*) What, according to you, are the two values displayed by Amit towards his uncle ?

[CBSE D 13C]

5. Satish was seeing a person wearing a shirt with a pattern comprising of vertical and horizontal lines. He was able to see the vertical lines more clearly than the horizontal ones. He shared his problem with his friend Ramesh. Ramesh suggested him to get his eyes checked-up by a doctor immediately.

(*a*) What value is being displayed by Ramesh here ?

(*b*) What is this defect due to ?

(*c*) How is such a defect of vision corrected ?

Answers

1. (*a*) Determination and critical thinking.

(*b*) The mirrors on two adjacent walls inclined at 90° will make three images and the ceiling mirror will repeat them.

2. (*a*) Endoscopy is based on the phenomenon of total internal reflection of light. A light pipe, a bundle of optical fibres, is inserted into stomach. Light transmitted through outer fibres is scattered by various parts of the stomach. The reflected light coming out of inner fibres produces a final image with excellent details.

(*b*) Empathy, charity, helping and caring.

(*c*) Doctor displayed sympathy and social responsibility by offering concession to Chetan's poor family.

3. (*a*) Keen observer and curiosity.

(*b*) Moon has no atmosphere. There is no refraction of light. Sunlight reaches moon straight covering shortest distance. Hence sunrise and sunset are abrupt.

(*c*) Moon has no atmosphere. So there is nothing to scatter sunlight towards the moon. No skylight reaches the moon surface. Sky appears dark in the day time as it does at night.

4. (*a*) (*i*) Due to stiffening of the ciliary muscles, the eye lens of elderly persons loses flexibility and hence the accommodating power of the eye lens decreases.

(*ii*) By using a convex lens of suitable focal length.

(*b*) Compassion for others, charity and caring

5. (*a*) Empathy.

(*b*) This defect is called astigmatism in which a person cannot simultaneously see both the horizontal and vertical views of an object with the same clarity. It is due to the irregular curvature of the cornea.

(*c*) Astigmatism can be corrected by using a cylindrical lens.

COMPETITION SECTION

Ray Optics and Optical Instruments

GLIMPSES

1. Optics. It is the branch of physics that deals with the study of nature, production and propagation of light. It has *two* sub-branches : ray optics and wave optics.
2. Ray or geometrical optics. It concerns itself with the particle nature of light and is based on (*i*) the rectilinear propagation of light and (*ii*) the laws of reflection and refraction of light.
3. Wave or physical optics. It concerns itself with the wave nature of light and is based on the phenomena like (*i*) interference (*ii*) diffraction and (*iii*) polarisation of light.
4. Laws of reflection of light. (*i*) The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

   (*ii*) The angle of incidence '*i*' is equal to the angle of reflection '*r*' *i.e.,* $\angle i = \angle r$.
5. Properties of images formed by plane mirrors.
   (*i*) The image formed by a plane mirror is virtual, erect and laterally reversed.
   (*ii*) The size of the image is equal to the size of the object.
   (*iii*) The image is as far behind the mirror as the object is in front of it.
   (*iv*) The line joining the object and the image is normal to the plane mirror.
   (*v*) When a plane mirror is rotated through a certain angle, the reflected ray turns through twice this angle.
6. Images formed by inclined mirrors. When two planes mirrors are kept facing each other at an angle θ and an object is placed between them, a number of images are formed due to multiple reflections.

   If θ is a submultiple of 180°, then the number of images formed is $n = \frac{360}{\theta} - 1$.

   If θ is not a submultiple of 180°, then the number of images formed is the integer next higher than $\left(\frac{360}{\theta} - 1\right)$. For two parallel plane mirrors, $n = \frac{360}{0} = \infty$.
7. Spherical mirror. It is a mirror whose reflecting surface forms part of a hollow sphere. Spherical mirrors are of *two* types :
   (*i*) ***Concave mirror*** in which the reflection of light takes place from the inner hollow surface.
   (*ii*) ***Convex mirror*** in which the reflection of light takes place from the outer bulged surface.
8. Definitions in connection with spherical mirrors.
   (*i*) ***Pole.*** *It is the middle point P of the spherical mirror.*
   (*ii*) ***Centre of curvature.*** *It is the centre C of the sphere of which the mirror forms a part.*
   (*iii*) ***Radius of curvature.*** *It is radius (R) of the sphere of which the mirror forms a part.*
   (*iv*) ***Principal axis.*** *The line PC passing through the pole and the centre of curvature of the mirror is called its principal axis.*
   (*v*) ***Linear aperture.*** *It is the diameter of the circular boundary of the spherical mirror.*

(*vi*) ***Angular aperture.*** *It is the angle subtended by the boundary of the spherical mirror at its centre of curvature C.*

(*vii*) ***Principal focus.*** *A narrow beam of light parallel to the principal axis either actually converges to or appears to diverge from a point F on the principal axis after reflection from the spherical mirror. This point is called the principal focus of the mirror.* A concave mirror has a real focus while a convex mirror has a virtual focus.

(*viii*) ***Focal length.*** *It is the distance* ($f = PF$) *between the focus and the pole of the mirror.*

(*ix*) ***Focal plane.*** *The vertical plane passing through the principal focus and perpendicular to the principal axis is called focal plane.* When a parallel beam of light is incident on a concave mirror at a small angle to the principal axis, it is converged to a point in the focal plane of the mirror.

9. New cartesian sign convention for spherical mirrors.

(*i*) All ray diagrams are drawn with the incident light travelling from left to right.

(*ii*) All distances are measured from the pole of the mirror.

(*iii*) All distances measured in the direction of incident light are taken positive.

(*iv*) All distances measured in the opposite direction of incident light are taken to be negative.

(*v*) Heights measured upwards and perpendicular to the principal axis are taken positive.

(*vi*) Heights measured downwards and perpendicular to the principal axis are taken as negative.

10. Relation between focal length and radius of curvature of a spherical mirror.

$$\text{Focal length} = \frac{1}{2} \times \text{Radius of curvature}$$

or $$f = \frac{R}{2}$$

In new cartesian sign convention, the focal length and radius of curvature are taken negative for a concave mirror and positive for a convex mirror.

11. Spherical mirror formula. This gives relation between object distance u, image distance v and the focal length f a spherical mirror.

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

12. Linear or transverse magnification. It is the ratio of the height of the image to that of the object.

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{h_2}{h_1} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

(*i*) If $|m| > 1$, the image is *magnified*.

(*ii*) If $|m| < 1$, the image is diminished.

(*iii*) If $|m| = 1$, the image is of the same size as the object.

(*iv*) If m is positive, the image is *virtual* and *erect*.

(*v*) If m is negative, the image is real and inverted.

13. Refraction of light. It is the phenomenon of bending of light from its straight path when it passes at an angle from one transparent medium to another.

14. Laws of refraction of light :

First law. The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.

Second law. The ratio of the sine of the angle of incidence and the sine of the angle of refraction is constant for a given pair of media. This law is also known as ***Snell's law of refraction***

$$\frac{\sin i}{\sin r} = \mu, \text{ a constant.}$$

The constant μ is called refractive index of second medium w.r.t. first medium.

15. Refractive index. Refractive index of a medium for a light of given wavelength may be defined as the ratio of the speed of light in vacuum to its speed in that medium.

$$\mu = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}} = \frac{c}{v}$$

It may also be defined as the ratio of the wavelength of light in vacuum to its wavelength in that medium.

$$\mu = \frac{\lambda_{vac}}{\lambda_{med}}$$

The refractive index of a medium with respect to vacuum is also called ***absolute refractive index.***

16. Relative refractive index. The relative refractive index of medium 2 w.r.t. medium 1 is the ratio of speed of light (v_1) in medium 1 to the speed of light (v_2) in medium 2.

$$^1\mu_2 = \frac{v_1}{v_2}$$

Also $\quad ^1\mu_2 = \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \text{constant}$

or $\quad \mu_1 \sin i = \mu_2 \sin r$

17. Principle of reversibility of light. This principle states that if the final path of ray of light after it has suffered several reflections and refractions is reversed, it retraces its path exactly. It follows from this principle that

$$^1\mu_2 = \frac{1}{^2\mu_1}$$

i.e., the refractive index of medium 2 w.r.t. medium 1 is reciprocal of the refractive index of medium 1 w.r.t. medium 2.

18. Refraction through a rectangular glass slab. A ray of light on refraction through a glass slab does not suffer any deviation, *i.e.*, the incident and emergent rays are parallel, but the emergent ray is laterally displaced w.r.t. the incident ray. The lateral displacement x on passing through a glass slab of thickness t and refractive index μ is given by

$$x = \frac{t}{\cos r}\sin(i - r) = t \sin i\left[1 - \frac{\cos i}{(\mu^2 - \sin^2 i)^{1/2}}\right]$$

where i is angle of incidence

$$x_{max} = t \sin 90° = t$$

Thus the displacement of the emergent ray cannot exceed the thickness of the glass slab.

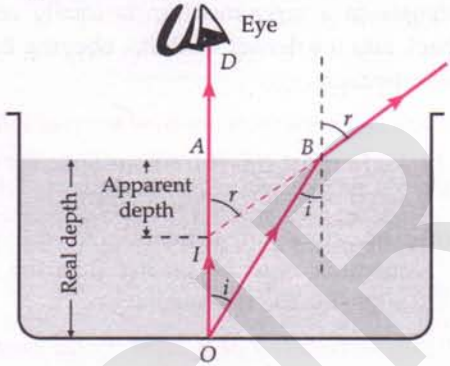
19. Refraction through a combination of media. When a ray of light passes through a combination of media, the quantity $\mu \sin i$ remains constant, where μ is the absolute refractive index of the medium and i the angle of incidence in that medium. Thus

$$\mu_{air} \times \sin i_{air} = \mu_{glass} \times \sin i_{glass} = \mu_{water} \times \sin i_{water}$$

Also $\quad ^a\mu_\omega \times {}^\omega\mu_g \times {}^g\mu_a = 1$

and $\quad ^\omega\mu_g = \frac{^a\mu_g}{^a\mu_\omega}$

20. Relation between real depth and apparent depth. Due to refraction of light, the apparent depth of an object placed in a denser medium is



less than the real depth. When an object O, in a denser medium of thickness t and refractive index μ is seen through a rarer medium, its image is seen at I. It is seen that

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{AO}{AI}$$

Also, apparent depth, $AI = \frac{t}{\mu}$

The height through which an object appears to be raised in a denser medium is called ***normal shift***.

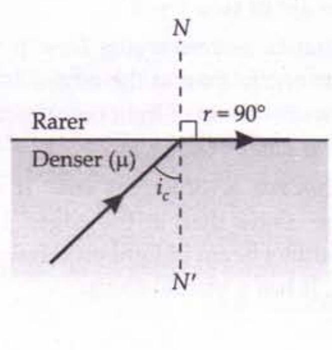
$\therefore$ Normal shift, $d = IO = AO - AI = t\left(1 - \frac{1}{\mu}\right)$

Total normal shift for compound media

$$= t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right) + \ldots$$

21. Critical angle and total internal reflection. The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° is called ***critical angle*** of the denser medium and is denoted by i_c. When $i = i_c$, $r = 90°$.

As $\quad \frac{\sin i_c}{\sin 90°} = \frac{1}{\mu} \quad$ or $\quad \mu = \frac{1}{\sin i_c}$



Total internal reflection is the phenomenon in which a ray of light travelling at an angle of incidence greater than the critical angle from a denser to a rarer medium is totally reflected back into the denser medium, obeying the laws of reflection.

22. Necessary conditions for total internal reflection.

(*i*) Light must travel from an optically denser to an optically rarer medium.

(*ii*) The angle of incidence in the denser medium must be greater than the critical angle for the two media.

23. Relation between critical angle and refractive index. $\mu = \frac{1}{\sin i_c}$.

24. Totally reflecting prisms. A right angled isosceles prism, *i.e.*, a $45° - 90° - 45°$ prism is called a totally reflecting prism. It can be used to deviate rays through 90° or 180°.

25. Mirage. It is an optical illusion observed in deserts or over hot extended surfaces like a coaltarred road due to which a traveller sees a shimmering pond of water some distance ahead him and in which the surrounding objects like tree, etc. appear inverted.

26. Optical fibres. Optical fibres consist of thousands of fine strands of quality glass, coated with a material of lower refractive index. Light entering the fibres at one end undergoes several total internal reflections and finally emerges out without any appreciable change in intensity. A bundle of optical fibres is called a light pipe, used in medical and optical examination and in receiving and transmitting signals in telecommunication.

27. Lens. A lens is a piece of a refracting medium bounded by two surfaces, at least one of which is a curved surface.

Lenses are of two types :

(*i*) ***Convex or converging lens.*** It is thicker at the centre than at the edges. It converges a parallel beam of light on refraction through it. It has a real focus.

(*ii*) ***Concave or diverging lens.*** It is thinner at the centre than at the edges. It diverges a parallel beam of light on refraction through it. It has a virtual focus.

28. Definitions in connection with spherical lenses :

(*i*) ***Centre of curvature.*** The centre of curvature of the surface of a lens is centre of the sphere of which it forms a part. Because a lens has two surfaces, so it has two centres of curvature.

(*ii*) ***Radius of curvature.*** The radius of the surface of a lens is the radius of the sphere of which the surface forms a part.

(*iii*) ***Principal axis.*** It is the line passing through the two centres of curvature of the lens.

(*iv*) ***Principal focus.*** A narrow beam of light parallel to the principal axis either converges to a point or appears to diverge from a point on the principal axis after refraction through the lens. This point is called principal focus. A lens has two principal focii.

(*v*) ***Optical Centre.*** It is the point situated within the lens through which a ray of light passes undeviated.

(*vi*) ***Focal length.*** It is the distance between the principal focus and the optical centre of the lens.

(*vii*) ***Aperture.*** It is the diameter of the circular boundary of the lens.

29. New Cartesian sign convention for spherical lenses :

(*i*) All distances are measured from the optical centre of the lens.

(*ii*) The distances measured in the direction of incident light are taken as positive.

(*iii*) The distances measured in the opposite direction of incident light are taken as negative.

(*iv*) Heights measured upwards and perpendicular to the principal axis are taken as positive.

(*v*) Heights measured downwards and perpendicular to the principal axis are taken as negative.

In this sign convention, the focal length of a converging lens is positive and that of a diverging lens is negative.

30. Refraction through a spherical surface. A surface which forms part of a sphere of a transparent refracting material is called a spherical refracting surface.

(*i*) *Refraction from rarer to denser medium.* When a ray of light travels from a rarer medium of refractive index μ_1 to a denser medium of refractive index μ_2 of a spherical surface of radius of curvature R, the relation between object distance u and image distance v is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

If the rarer medium is air, then $\mu_1 = 1$ and $\mu_2 = \mu$, we have $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$

(*ii*) *Refraction from denser to rarer medium.* When the object is placed in a denser medium, the relation between u and v can be obtained by interchanging μ_1 and μ_2.

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

31. Power of a spherical refracting surface. It is given by $P = \frac{\mu_2 - \mu_1}{R} = \frac{\mu - 1}{R}$ (for air)

where R is measured in metre. The power of a convex surface is positive and that of a concave surface is negative.

32. Principal focal lengths of a spherical surface.

(*i*) *First principal focal length.* It is the distance of a point from the pole of the surface at which if an object is placed, the image is formed at infinity.

First principal focal length, $f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$

(*ii*) *Second principal focal length.* It is the distance of a point from the pole of the surface at which the image of an object at infinity is formed.

Second principal focal length, $f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$

33. Lens maker's formula. This formula relates the focal length f to the refractive index μ and the radii of curvature R_1, R_2 of its spherical surfaces.

$$\frac{1}{f} = \left[\frac{\mu_2 - \mu_1}{\mu_1}\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For the lens placed in air,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

34. Thin lens formula. This formula gives relationship between object distance u, image distance v and focal length f a spherical lens (convex or concave) of small aperture.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

35. Linear magnification produced by a lens. It is the ratio of the size of the image formed by a lens to the size of the object.

$$\text{Magnification} = \frac{\text{size of image}}{\text{size of object}}$$

or $$m = \frac{h_2}{h_1} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}.$$

When m is positive (or v is negative), the image is virtual and erect. When m is negative (or v is positive), the image is real and inverted.

36. Power of a lens. The power of a lens is defined as the reciprocal of its focal length, expressed in metres.

$$P = \frac{1}{f\,(\text{m})}$$

SI unit of power is m^{-1}, also called dioptre (D). One dioptre is the power of a lens whose principal focal length is 1 metre.

$$P = \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

37. Lens combinations. When lenses are used in combination, each lens magnifies the image formed by the preceding lens. The total magnification is equal to the product of the magnifications produced by the individual lenses.

$$m = m_1 \times m_2 \times m_3 \times ....$$

The combined focal length f of two thin lenses of focal lengths f_1 and f_2 placed in contact is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

For n thin lenses in contact,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + ... + \frac{1}{f_n} \quad \text{or} \quad P = P_1 + P_2 + ... + P_n$$

When the two thin lenses are separated by a distance d, their equivalent focal length f is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{d}{f_1 f_2}$$

or Power, $P = P_1 + P_2 + d \times P_1 \times P_2$

38. Prism. A *prism* is a portion of a refracting medium bounded by two plane faces inclined to each other at a certain angle. The two plane faces inclined to each other are called *refracting faces.* The line along which the two refracting faces meet is called *refracting edge of the prism.* The third face of the prism opposite to the refracting edge is called *base of the prism*. The angle included between the two refracting faces is called *angle of prism*.

39. Refraction through a prism. When a ray of light is refracted through a prism, the sum of the angle of incidence i and the angle of emergence i' is equal to the sum of the angle of the prism A and the angle of deviation δ.

$$A + \delta = i + i' \quad \text{and} \quad A = r + r'$$

where r and r' are the corresponding angles of refraction at the two faces.

40. Relation between the refractive index and angle of minimum deviation. The minimum value of the angle of deviation suffered by a ray on passing through a prism is called the angle of minimum deviation and is denoted by δ_m. When a ray of light suffers minimum deviation.

$$i = i', \ r = r' \quad \text{and} \quad \delta = \delta_m$$

$$\therefore \quad A + \delta_m = i + i = 2i \quad \text{or} \quad i = \frac{A + \delta_m}{2}$$

$$\text{and} \quad A = r + r = 2r \quad \text{or} \quad r = \frac{A}{2}$$

$$\text{Refractive index, } \mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$

41. Deviation produced by a prism of small angle. It does not depend on the angle of incidence and is given by $\delta = (\mu - 1) A$.

42. Dispersion. The splitting of white light into its constituent colours when it passes through a glass prism is called dispersion. The dispersion of light occurs because refractive index of prism material is different for different wavelengths.

43. Angular dispersion. The angular separation between the two extreme colours (violet and red) in the spectrum is called angular dispersion.

Angular dispersion

$$= \delta_V - \delta_R = (\mu_V - 1) A - (\mu_R - 1) A = (\mu_V - \mu_R) A$$

44. Dispersion power. It is the ability of the prism material to cause dispersion and is defined as the ratio of the angular dispersion to the mean deviation.

$$\text{Dispersion power} = \frac{\text{Angular dispersion}}{\text{Mean deviation}}$$

$$\omega = \frac{\delta_V - \delta_R}{\delta} = \frac{(\mu_V - 1) A - (\mu_R - 1) A}{(\mu - 1) A} = \frac{\mu_V - \mu_R}{\mu - 1}$$

$$\text{Here} \quad \delta = \frac{\delta_V + \delta_R}{2} \quad \text{and} \quad \mu = \frac{\mu_V + \mu_R}{2}$$

45. Pure and impure spectra. The spectrum in which the component colours of the spectra of different rays overlap each other and the various colours are not distinctly seen is called an impure spectrum. A spectrum in which there is no overlapping of colours and different colours are distinctly seen is called the pure spectrum.

46. Spectroscope or spectrometer. It is an optical device used for producing and studying the spectrum of various light sources. It consists of three main parts : (*i*) collimator, (*ii*) prism table and (*iii*) telescope.

47. Spherical aberration. The inability of a lens or spherical mirror of large aperture to bring the paraxial and marginal rays of a wide beam of light to focus at a single point is called spherical aberration.

48. Chromatic aberration. The inability of a lens to bring the light rays of different colours to focus at a single point is called chromatic aberration.

Longitudinal chromatic aberration of a lens

= Dispersive power × focal length of the lens for mean colour

$$\text{or} \quad f_R - f_V = \omega \times f$$

49. Blue colour of the sky. According to ***Rayleigh's law of scattering***, the intensity of light of wavelength λ present in the scattered light is inversely proportional to the fourth power of wavelength : $I \propto \frac{1}{\lambda^4}$

So, blue colour of sunlight is scattered more by the atmospheric molecules, due to which the sky appears blue.

50. Rainbow. It is nature's most spectacular display of the spectrum of light produced by refraction, dispersion and total internal reflection of sunlight by several raindrops. It is observed when the sun shines on rain drops after a shower. An observer standing with his back towards the sun observes it in the form of concentric circular arcs of different colours in the horizon.

Primary rainbow is brighter with its inner edge violet and outer edge red, subtending $41° - 43°$ angle at the observer's eye. Secondary rainbow is fainter with its inner edge red and outer edge violet, subtending $51° - 54°$ angle at the observer's eye.

51. Human eye. It is most important and sensitive sense organ. The essential parts of a human eye are sclerotic, cornea, choroid, iris, pupil, crystalline lens, ciliary muscles, aqueous humour, vitreous humour and retina. It is a convex lens of focal length about 2.5 cm.

52. Accommodation. It is the ability of the eyelens due to which it can change its focal length so that images of objects at various distances can be formed on the same retina.

53. Range of normal vision. The distance between infinity and 25 cm point is called the range of normal vision.

54. Least distance of distinct vision (D). The minimum distance from the eye, at which the eye can see the objects clearly and distinctly without any strain is called the least distance of distinct vision. For a normal eye, its value is 25 cm.

55. Near point. The nearest point from the eye, at which an object can be seen clearly by the eye is called its near point. The near point of a normal eye is at a distance of 25 cm.

56. Far point. The farthest point from the eye, at which an object can be seen clearly by the eye is called the far point of the eye. For a normal eye, the far point is at infinity.

57. Power of accommodation. The power of accommodation of the eye is the maximum variation of its power for focussing on near and far objects. For a normal eye, the power of accommodation is about 4 dioptres.

58. Persistence of vision. The phenomenon of the continuation of the impression of an image on the retina for some time even after the light from the object is cut off is called persistence of vision. The impression of the image remains on the retina for about (1/16)th of a second. Cinematography works on the principle of persistence of vision.

59. Rods. These are rod-shaped cells of the retina that are sensitive to the intensity of light.

60. Cones. These are cone-shaped cells of the retina that are sensitive to the colours of light.

61. Colour blindness. A person who cannot distinguish between various colours but can see well otherwise, is said to be colour-blind. It is due to lack of some cones in the retina of the eyes.

62. Cataract. It is due to the development of hazy or opaque membrane over the eyelens which results in the decrease or loss of vision. It can be cured by surgery.

63. Common defects of vision. There are mainly four common defects of vision which can be corrected by the use of suitable eye glasses. These are (*i*) myopia or near sightedness (*ii*) hypermetropia or far-sightedness (*iii*) presbyopia (*iv*) astigmatism.

64. Myopia or short-sightedness. In this defect a person can see nearby objects clearly but cannot see far off objects clearly. Here, either the eyeball becomes too longer or the focal length of the eyelens becomes too short. It can be corrected by using a concave lens of suitable focal length.

Focal length of the correcting lens

= Distance of the far point from the eye.

65. Long-sightedness or hypermetropia. In this defect a person can see the far off objects clearly but he cannot see nearby objects distinctly. Here, either the eyeball becomes too short or the focal length of the eyelens becomes too large. It can be corrected by using convex lens of suitable focal length.

$$\text{Focal length of correcting lens} = \frac{yD}{y-D}$$

where y = distance of the near point from the defective eye.

66. Presbyopia. In this defect, a person in old age cannot read correctly due to the stiffening of the ciliary muscles and the decrease in flexibility of the eyelens.

67. Astigmatism. It is defect of vision in which a person cannot simultaneously see both the horizontal and vertical views of an object with the same clarity. It is due to the irregular curvature of the cornea. It can be corrected by using a cylindrical lens.

68. Simple microscope. It is a convex lens of short focal length. When the object is placed between the lens and its focus and the eye is held just behind the lens, a virtual, erect and enlarged image is seen. When the final image is formed at the least distance of distinct vision (D), the magnifying power of the simple microscope is

$$m = \frac{\text{Angle subtended by the image at the least distance of distinct vision}}{\text{Angle subtended by the object at the least distance of distinct vision}}$$

or $m = \frac{\beta}{\alpha} = 1 + \frac{D}{f}$

When the final image is formed at infinity, $m = \frac{D}{f}$, viewing is more comfortable when the eye is focussed at infinity.

69. Visual angle. The angle subtended by an object on the eye is called visual angle. Larger the visual angle, larger is the apparent size of an object.

70. Compound microscope. It is an optical device used to see magnified images of tiny objects. The objective is a convex lens of very short focal length and of small aperture. The eyepiece is a convex lens of relatively larger focal length and of larger aperture. The difference between the focal lengths of the eyepiece and the objective is small. Its magnifying power is given by

$$m = m_o \times m_e$$

When the final image is formed at the least distance of distinct vision,

$$m = \frac{\text{Angle subtended by final virtual image at distance } D \text{ from the eye}}{\text{Angle subtended by the object at distance } D \text{ from the eye}}.$$

or $$m = \frac{\beta}{\alpha} = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

When the final image is formed at infinity,

$$m = \frac{L}{f_0} \times \frac{D}{f_e}$$

where L is the distance between the objective and the eyepiece.

71. Astronomical telescope. It is used to view heavenly bodies. The objective is a convex lens of large focal length and large aperture. The eyepiece is convex lens of small focal length and small aperture. The difference in the focal lengths of the two lenses is large. The eyepiece forms a real, inverted and diminished image. The eyepiece magnifies this image. The final image is inverted w.r.t. the object.

When the final image is formed at the least distance of distinct vision,

$$m = \frac{\text{Angle subtended by the image at distance } D \text{ from the eye}}{\text{Angle subtended by the object at infinity}}$$

or $$m = \frac{\beta}{\alpha} = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Length of telescope, $L = f_0 + u_e = \frac{f_e D}{f_e + D}$

When the final image is formed at infinity (normal adjustment),

$$m = \frac{\text{Angle subtended by the final image formed at } \infty}{\text{Angle subtended by the object at } \infty} \quad \text{or} \quad m = \frac{\beta}{\alpha} = -\frac{f_0}{f_e}$$

Length of the telescope in normal adjustment,

$$L = f_0 + f_e$$

For large magnifying power of a telescope, clearly

$$f_0 >> f_e.$$

72. Terrestrial telescope. It is used to see the erect images of distant earthly objects. It uses an additional convex lens between the objective and the eyepiece for erecting the image.

When the final image is formed at infinity, its magnifying power, $m = \frac{f_0}{f_e}$

Length of telescope $= f_0 + 4f + f_e$

where f is the focal length of the erecting lens.

When the final image is formed at the least distance of distinct vision, $m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$.

73. Galileo's telescope. It uses a concave lens for the eyepiece to obtain an erect image of the distant object. The real, inverted and diminished image formed by the objective lies at the focus of the eyepiece. The final image is formed at infinity and is erect and magnified.

In normal adjustment, $m = \frac{f_0}{f_e}$

Length of telescope, $L = f_0 - f_e$.

74. Reflecting telescope. It uses a concave paraboloidal mirror of large aperture to view the distant objects. Both spherical and chromatic aberrations are minimum.

When the final image is formed at the least distance of distinct vision,

$$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

When the final image is formed at infinity,

$$m = \frac{f_0}{f_e} = \frac{R/2}{f_e}.$$